Basics of Neural Network Programming

Binary Classification

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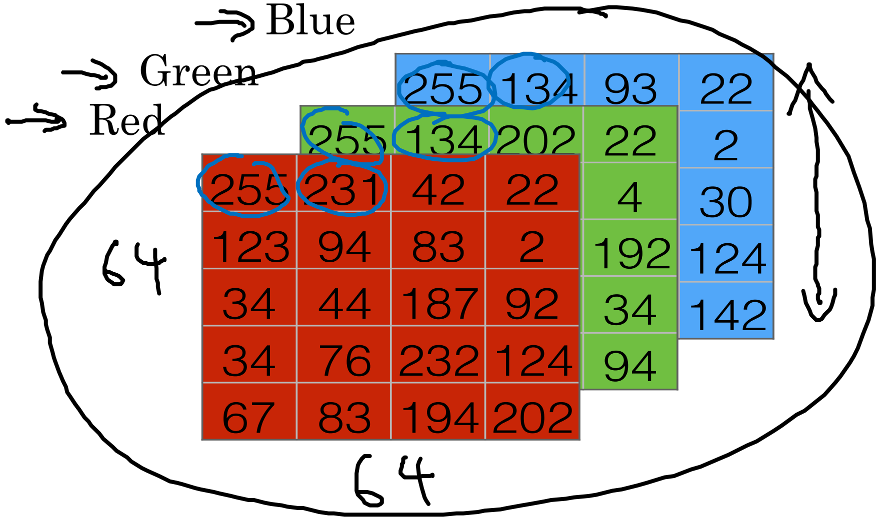
Binary Classification

64

64

$\longrightarrow$ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

$x \longrightarrow y$

Notation

$(x, y) \quad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(\underline{x}^{(1)}, \underline{y}^{(1)}), (\underline{x}^{(2)}, \underline{y}^{(2)}), \ldots, (\underline{x}^{(m)}, \underline{y}^{(m)})\}$

$M = M_{train}$ $\qquad M_{test}$ = #test examples.

$$X = \begin{array}{c} \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \ldots & x^{(m)} \\ | & | & & | \end{bmatrix} \\ \longleftarrow m \longrightarrow \end{array} \updownarrow n_x \qquad \cancel{\begin{bmatrix} — x^{(1)T} — \\ \vdots \\ — x^{(m)T} — \end{bmatrix}}$$

$X \in \mathbb{R}^{n_x \times m}$ $\qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

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Basics of Neural Network Programming

Logistic Regression

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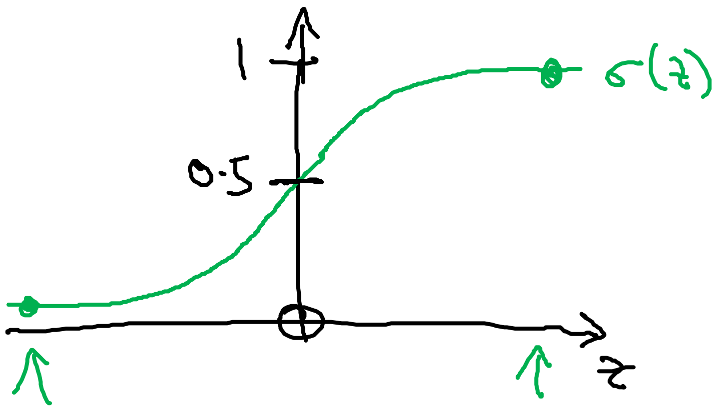
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\qquad 0 \le \hat{y} \le 1$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$X_0 = 1, \quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \vdots\\ \theta_{n_x}\end{bmatrix} \begin{matrix} \}\, b \\ \left.\begin{matrix}\\ \\ \\ \\ \end{matrix}\right\} w \end{matrix}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Logistic Regression cost function

Logistic Regression cost function

$\to \hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

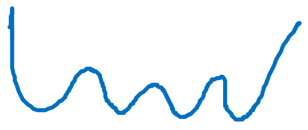
$z^{(i)} = w^T x^{(i)} + b$

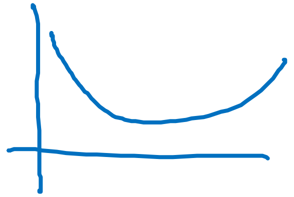
Given $\{(x^{(1)}, y^{(1)}),\dots,(x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$
$y^{(i)}$
$z^{(i)}$

i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$





$\boxed{\mathcal{L}(\hat{y}, y)} = -(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y})) \leftarrow$

If $y=1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y} \leftarrow$ Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y=0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y}) \leftarrow$ Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

Basics of Neural Network Programming

Gradient Descent

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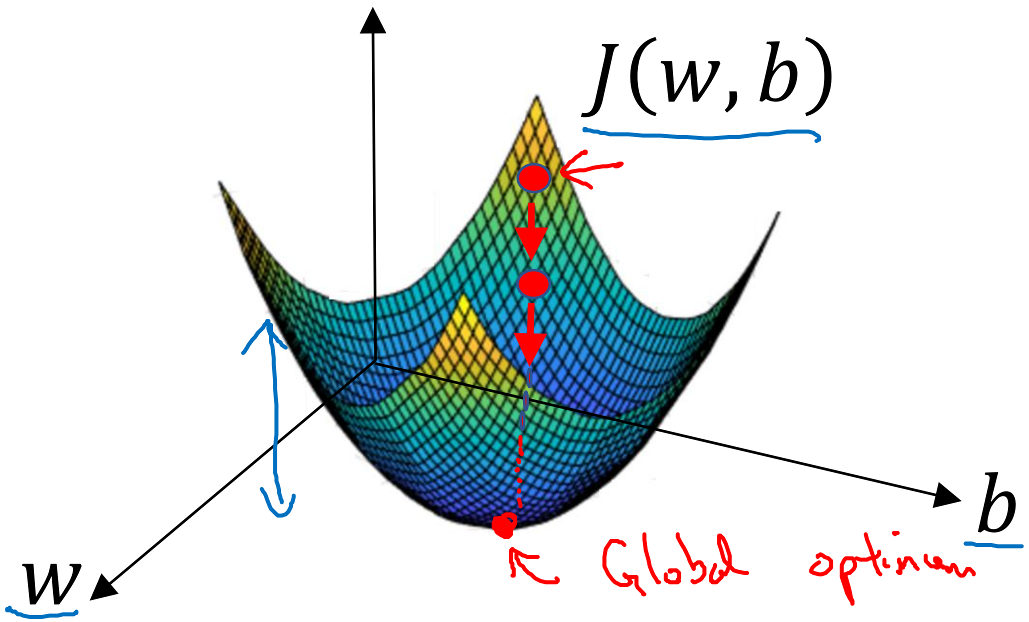
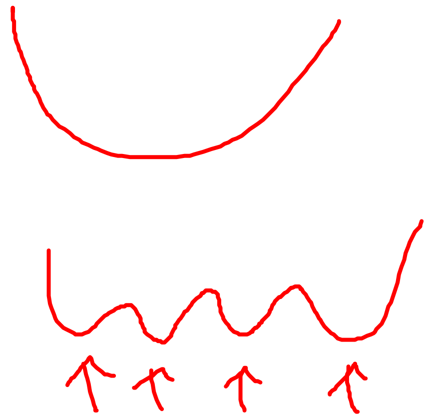
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

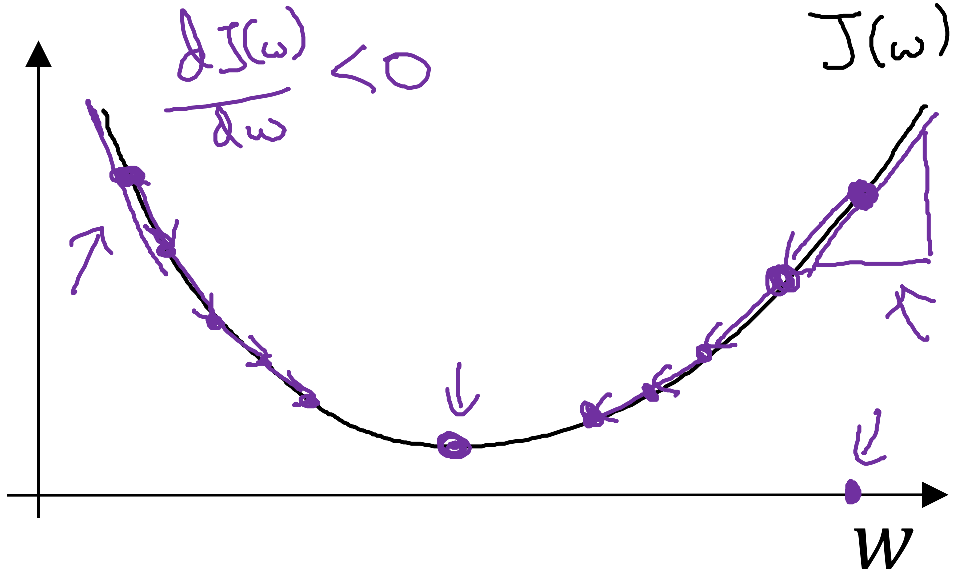
$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

}

"dw"

$w := w - \alpha \, dw$

$$\underbrace{\frac{dJ(w)}{dw}}\ = ?$$

$J(w, b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \qquad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \qquad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d J

Basics of Neural Network Programming

Derivatives

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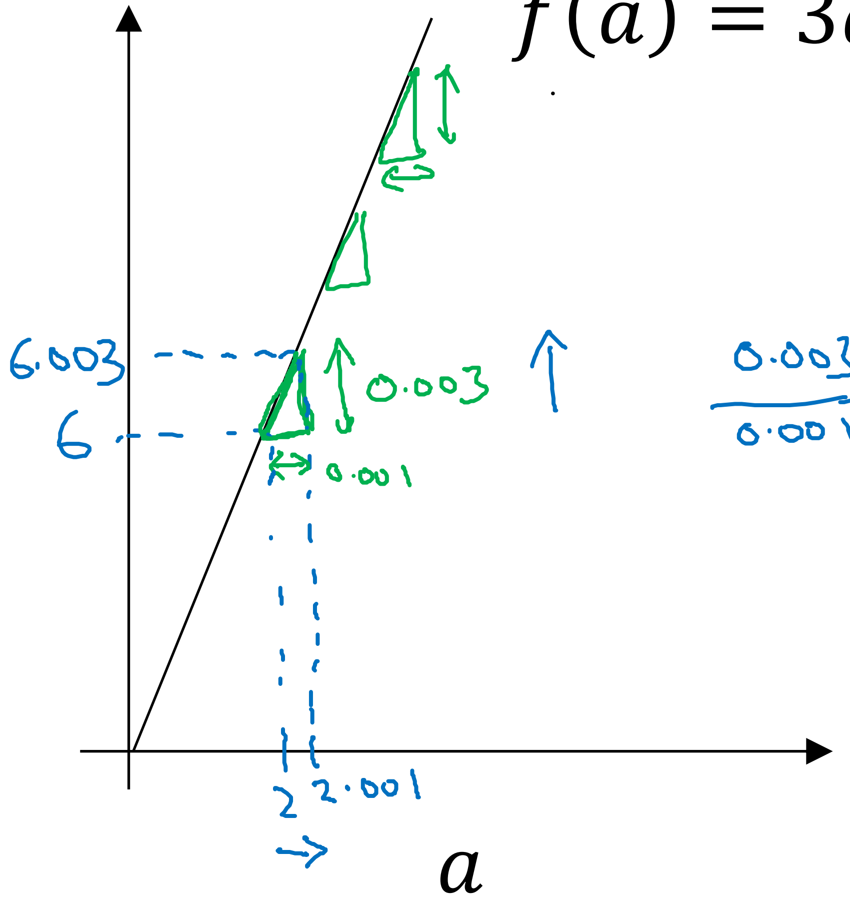
Intuition about derivatives

$f(a) = 3a$



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

$\rightarrow a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

slope (derivative) of $f(a)$ at $a = 2$ is 3

$\rightarrow a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a = 5$ is also 3

$$\frac{d\,f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001

0.00000001

0.000000000001

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More derivatives examples

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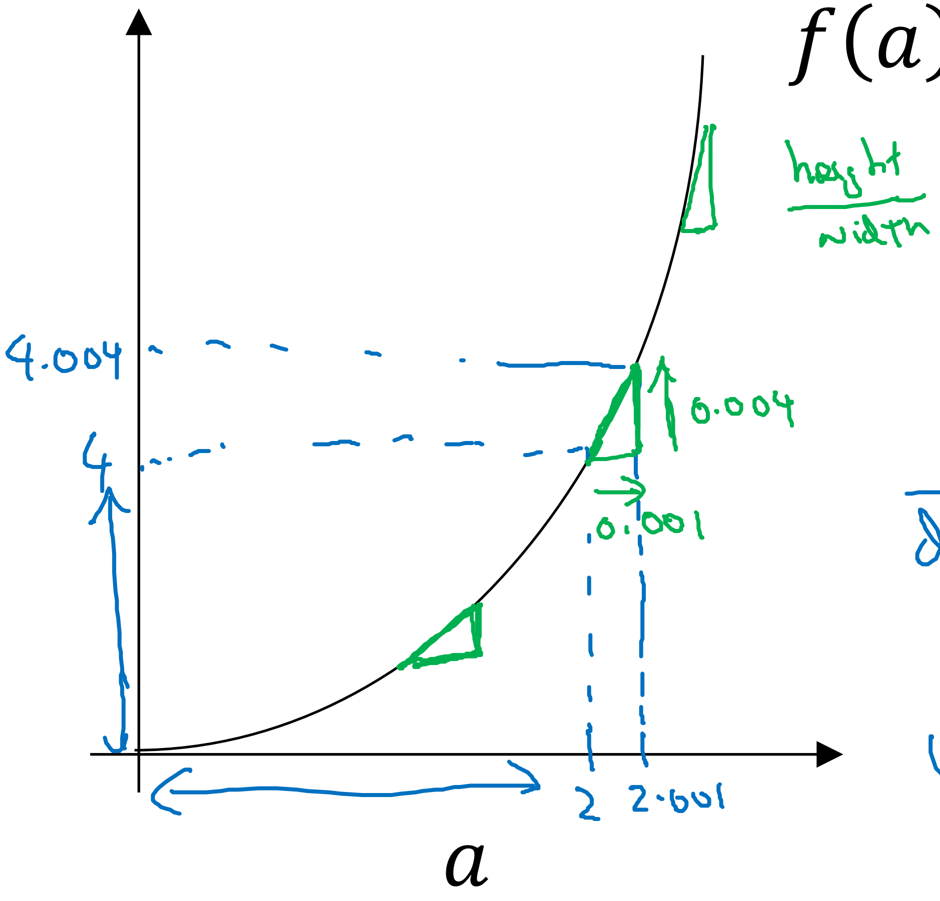
Intuition about derivatives

0.001 ←
$0.00000\ldots01$ ←

$$f(a) = a^2$$



$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of $f(a)$ at $a=2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a=2}$.

$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

$a = 5$ $\quad f(a) = 25$

$a = 5.001$ $\quad f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a=5}$

$$\frac{d}{da} f(a) = \underbrace{\frac{d}{da} a^2} = \boxed{2a}$$

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More derivative examples

$f(a) = a^2$ $\qquad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\qquad a = 2 \quad f(a) = 4$

$a = 2.001 \quad f(a) \approx 4.004$

$f(a) = a^3$ $\qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$ $\qquad a = 2 \quad f(a) = 8$

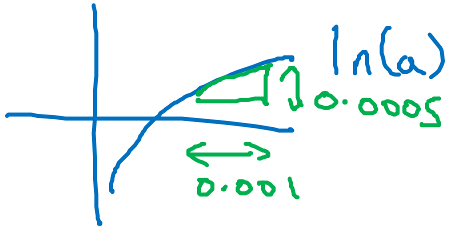
$a = \underline{2.001} \quad f(a) \approx 8.\underline{012}$

$f(a) = \log_e(a)$

$\ln(a)$

$\frac{d}{da} f(a) = \frac{1}{a}$

$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$



$a = 2 \quad f(a) \approx 0.69315$

$a = 2.\underline{001} \quad \underline{f(a)} \approx 0.69365$

$0.0005 \quad \underline{0.0005}$

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Basics of Neural Network Programming

Computation Graph

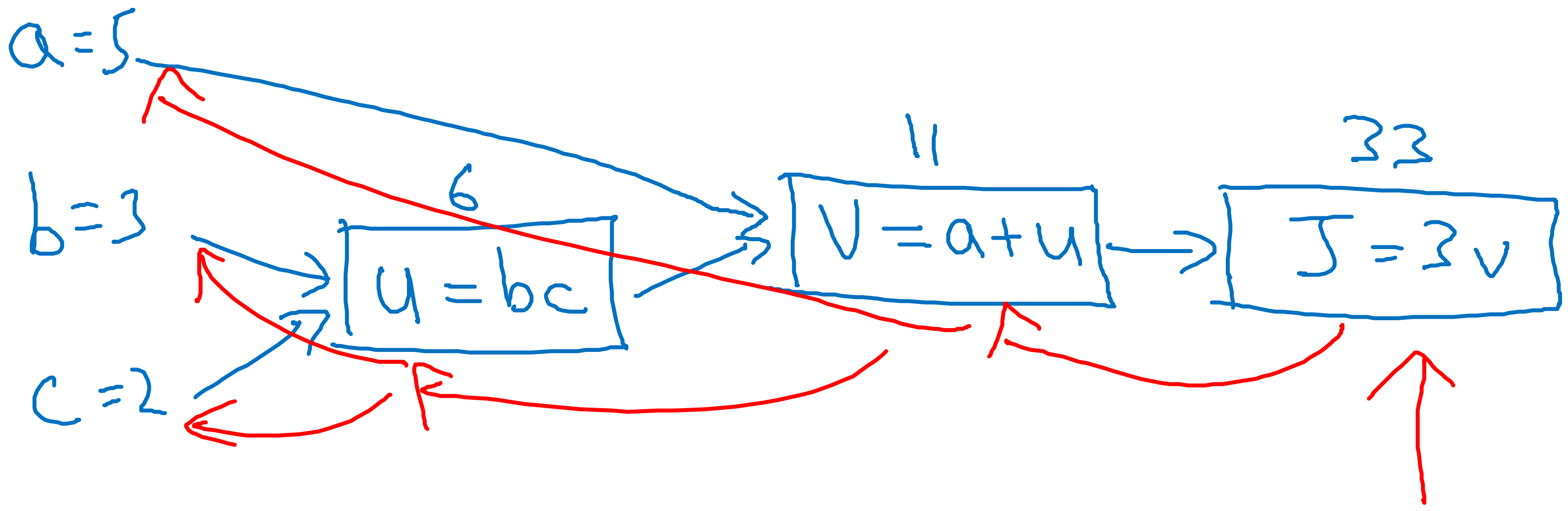
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a+\underbrace{bc}_{u}}_{v})}_{J} = 3(5+3\times 2) = 33$$

$u = bc$

$v = a+u$

$J = 3v$



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Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives

$\frac{dJ}{da}$ "da" $= 3$

$a = 5$

$b = 3$

$c = 2$

6

$u = bc$

11

$v = a + u$

33

$J = 3v$

$\frac{dJ}{dv}$ "dv" $= 3$

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$f(a) = 3a$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$J = 3v$

$\frac{dJ}{dv} = 3$

$\boxed{\frac{dJ}{dv} = ? = 3}$

$a \rightarrow v \rightarrow J$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \frac{dv}{da}$

3×1

$\boxed{\frac{dv}{da} = 1}$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{\frac{d \text{FinalOutputVar}}{d \text{var}}}$

dJdvar

"dvar"

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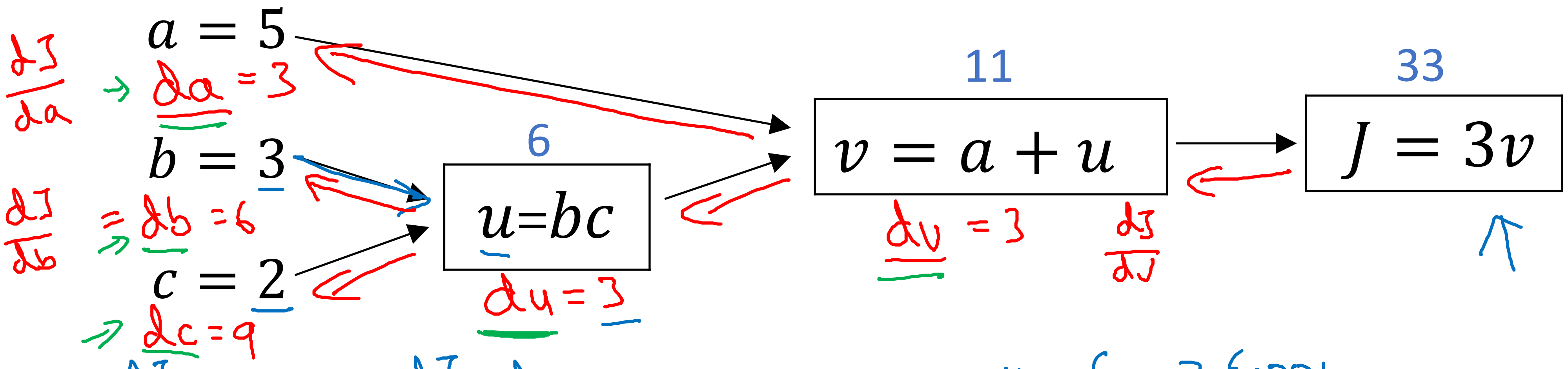
Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\rightarrow 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \boxed{\frac{dJ}{du}} \cdot \frac{du}{da} = 9$$

$$3 \times 3$$

$u = 6 \rightarrow 6.001$
$v = 11 \rightarrow 11.001$
$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$
$u = b \cdot c = 6 \rightarrow 6.002$ $\quad c = 2$
$J = 33.006$ $\quad .006$

$v = 11.002$
$J = 3v$

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Basics of Neural Network Programming

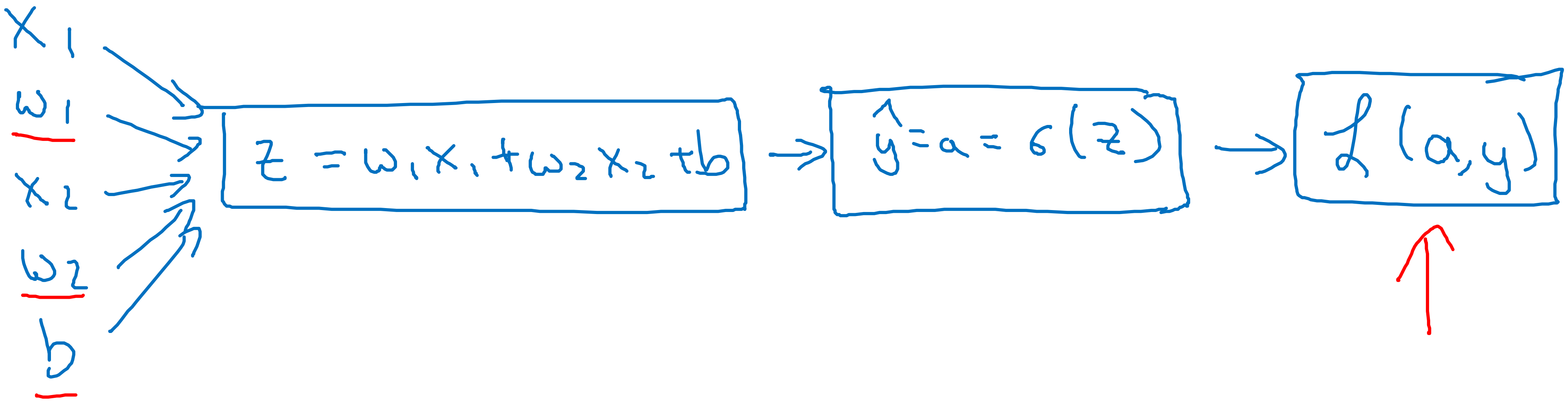
Logistic Regression Gradient descent

Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives

x_1
w_1
x_2
w_2
b

$$\boxed{z = w_1x_1 + w_2x_2 + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$$

$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz}$$

$$= a - y$$

$$= \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$$

$$a(1-a)$$

$$\text{"}da\text{"} = \frac{d\mathcal{L}(a,y)}{da}$$

$$= \boxed{-\frac{y}{a} + \frac{1-y}{1-a}}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$
$$w_2 := w_2 - \alpha\, dw_2$$
$$b := b - \alpha\, db.$$

Basics of Neural Network Programming

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Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$(x^{(i)}, y^{(i)})$$

$$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}}}$$

$$\underline{dw_1^{(i)}} - (x^{(i)}, y^{(i)})$$

Logistic regression on m examples

$J = 0; \; dw_1 = 0; \; dw_2 = 0; \; db = 0$

$\rightarrow$ For $i = 1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

$n = 2$

dw_3

$\vdots$

dw_n

$db \mathrel{+}= dz^{(i)}$

$J /= m \leftarrow$

$dw_1 /= m; \quad dw_2 /= m; \quad db /= m. \leftarrow$

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha \, dw_1$

$w_2 := w_2 - \alpha \, dw_2$

$b := b - \alpha \, db$

Vectorization