1) Linear System:

$\textbf{No solution} \leftrightarrow \textbf{Last col is pivot col}$

$\textbf{One solution} \leftrightarrow \textbf{All cols except last is pivo}$

$\boldsymbol{\infty}\ \textbf{solution} \leftrightarrow \geq \textbf{1 non} - \textbf{pivot column}$

$\textbf{\# parameters} \leftrightarrow \textbf{\# non} - \textbf{pivot cols in LHS}$

2) Matrices:

Diagonal Matrix: $\mathbf{A} = (a_{ij})_{m*n} = \begin{cases} = \mathbf{0 \text{ if } i \neq j} \\ \neq \mathbf{0 \text{ if } i = j} \end{cases}$

Triangular Matrix: $\textit{Lower}: \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ or $\textit{Upper}: \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Symmetrical Matrix: $\forall i, j, a_{ij} = a_{ji}$ or $A^T = A$

Multiplication of Matrices:

Pre-multiply: $AB = A \text{ onto } B$

Post-multiply: $BA = A \text{ onto } B$

Formulae: $\textit{If } A = (a_{ij})_{m*p} \textit{ and } B = (b_{ij})_{p*n}, \textit{ then } AB = \sum_{k=1}^{p} a_{ik} b_{ki}$

Matrix Notation:

$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$ where a_i is the i^{th} row $= (a_{i1} \quad a_{i2} \quad \dots \quad a_{ip})$

$B = (b_1 \quad b_2 \quad \dots \quad b_n)$ where b_i is the i^{th} col $= \begin{pmatrix} b_{1i} \\ \vdots \\ b_{pi} \end{pmatrix}$

$A(B_1\ B_2) = (AB_1\ AB_2);\ A\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} AB_1 \\ AB_2 \end{pmatrix}$

Transpose Matrices:

$A = (a_{ij})_{m*n}$ and $A^T = (a_{ji})_{n*m}$

$(A^T)^T = A$ and $(A+B)^T = A^T + B^T$ and $(AB)^T = B^T A^T$

Inverse Matrices:

$B \textit{ is left inverse of } A \textit{ if } BA = I;\ B \textit{ is right inverse of } A \textit{ if } AB = I$

$AA^{-1} = A^{-1}A = I; (cA)^{-1} = \frac{1}{c} A^{-1}; (AB)^{-1} = B^{-1}A^{-1}$

$det(A) \neq 0$

$\textit{Singular matrix} \leftrightarrow 0 \textit{ row or } 0 \textit{ col} \leftrightarrow det(A) = 0$

Elementary Row Operations:

$E_k E_{k-1} \dots E_2 E_1 A = B$ and $A = E_1^{-1} E_2^{-1} \dots E_{k-1}^{-1} E_k^{-1} B$

$A \textit{ and } B \textit{ are row} - \textit{equivalent} \leftrightarrow A \textit{ and } B \textit{ have the same RREF}$

$(A \quad I) \xrightarrow{Gaussian} (I \quad A)$ refers $A^{-1} = E_k \dots E_2 E_1;\ A = E_1^{-1} \dots E_{k-1}^{-1} E_k^{-1}$

Determinants:

$det(A) = \begin{cases} a_{11} & \textit{if } N = 1 \\ \sum_{k=1}^{N} a_{1k} A_{1k} & \textit{if } N > 1 \end{cases}$ where

$A_{ij} = (-1)^{i+j} \det(M_{ij})$ or Δ matrix, $det(A) = \prod_{i=1}^{N} a_{ii}$

$det(A) = ad - bc \leftrightarrow A \textit{ is } 2*2 \textit{ matrix}$

$det(A) = aei - afh \mp bfg - bdi + cdh - ceg \leftrightarrow A \textit{is } 3*3 \textit{ matrix}$

Determinant Formulae:

$det(E_{add}) = 1;\ det(E_{mult}) = c;\ det(E_{swap}) = -1;$

$det(A^{-1}) = (det(A))^{-1};\ det(cA) = c^n det(A);\ det(AB) = det(A) * det(B);$

Adjoints:

$adj(A) = (A_{ij})^T_{m*n}; A\ adj(A) = det(A)\ I;\ A^{-1} = (det(A))^{-1} adj(A)$

Cramer's Rule:

$\textit{If } A \textit{ is invertible}, Ax = b \leftrightarrow \frac{1}{\det(A)} \begin{pmatrix} det(A_1) \\ \vdots \\ det(A_n) \end{pmatrix}$

$\textit{where } A_i = (a_1 \dots a_{i-1}\ b\ a_{i+1} \dots a_n)$

$\textit{For example}: x = \frac{|b \quad a_2 \quad a_3|}{|a_1 \quad a_2 \quad a_3|}, y = \frac{|a_1 \quad b \quad a_3|}{|a_1 \quad a_2 \quad a_3|}$

3) Vector Spaces:

Vectors:

$\textit{row vector}: (u_1\ u_2 \dots u_n); \textit{col vector}: \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} = \textit{row vector}^T$

Solutions Sets:

$\textit{Implicit}: \{(x, y, z) \mid x + y + z = 0\ \&\ x - y + 2z = 1\}$

$\textit{Explicit}: \left\{\left(\frac{1}{2} - \frac{3}{2}t, -\frac{1}{2} + \frac{1}{2}t, t\right) \middle| t \in \mathbb{R}\right\}$

Linear Spans:

$\textit{Linear spans} \leftrightarrow \{\sum_{i=1}^{k} c_i u_i \mid c_1 \dots c_k \in \mathbb{R}$

$span(k) \leftrightarrow span\ \{u_1, u_2 \dots u_k\}$

Theorem 3.2.10:

$\textit{Let } S_1 = \{u_1 \dots u_k\} \textit{ and } S_2 = \{v_1 \dots v_m\} \subseteq \mathbb{R}^n$

$span(S_1) \subseteq span(S_2) \leftrightarrow \forall v_i \textit{ is a l.c of } \{u_1 \dots u_k\}$

$\leftrightarrow (v_1\ v_2 \dots v_m\ | u_1 | u_2 | .. u_k) \textit{ is a consistent linear system}$

Redundant Vectors:

$\textit{If } u_k = c_1 u_1 + \dots + c_{k-1} u_{k-1}, u_k \textit{ is known as a redundant vector}$

Solution Spaces:

$\textit{Homogeneous System has trivial sol} \leftrightarrow \textit{solution set is } \{0\}$

$\textit{Homogeneous System has } \infty \textit{ sol} \leftrightarrow \textit{solution set is general sol}$

Subspaces:

$V \subseteq \mathbb{R}^n \textit{ and } V \textit{ is a subspace} \leftrightarrow V = span(S) \textit{for some } S \subseteq \mathbb{R}^n$

$\forall \alpha, \beta \in \mathbb{R}, u, v \in V, \alpha u + \beta v \in V \textit{ and thus}, S \textit{ spans } V$

$0 \in V \textit{ given } V \textit{ is non} - \textit{empty}$

Linear Independence:

$S \textit{ is called a l.i set if } \forall u_1 \textit{ to } u_k \in \mathbb{R}\ s.t. \textit{ from } c_1 u_1 \textit{ to } c_k u_k, c_1 \textit{ to } c_k = 0$

$\textit{If vector set is not l.i}, \exists \geq 1 \textit{ redundant vector}$

$\textit{If vector set is l.i}, \exists\ 0 \textit{ redundant vectors}$

Basis:

$V \textit{ is a vector space} \leftrightarrow V = \mathbb{R}^n$ **OR V is a subspace of** $\mathbb{R}^n$

$V \textit{ is a subspace of } W \leftrightarrow V \textit{ is a vector space AND } V \subseteq W$

$\textit{Let } S \textit{ be a subset of vector space } V, S \textit{ is a basis for } V$

$\leftrightarrow S \textit{ is l.i AND } (S \textit{ spans } V \textit{ OR } V = span(S))$

$\textit{If } A \textit{ is invertible}:$

$\textit{The rows of } A \textit{ OR The cols of } A \textit{ form a bsis for } \mathbb{R}^n$

Standard Basis:

$E = \{e_1, \dots e_k\} \textit{ is the standard basis for } \mathbb{R}^n, e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \dots e_k = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

Co-ordinate Systems:

$\textit{Let } S = \{u_1, \dots, u_k\} \textit{be a basis for a vector space } V \textit{ and } v \in V$

$v = c_1 u_1 + \dots + c_k u_k \textit{ and } (v)_S = (c_1 \dots c_k) \in \mathbb{R}^k$

$\forall u, v \in V, u = v \leftrightarrow (u)_S = (v)_S$

$\forall u, v \in V. c, d \in \mathbb{R}: (cu + dv)_S = c(u)_S + d(v)_S$

$u_1, \dots, u_k \textit{ are l.i.} \leftrightarrow (u_1)_S \dots (u_k)_S \textit{ are also l.i}$

Dimensions:

$\textit{Let } S \subseteq V,$

$S > k \textit{ vectors}: S \neq \textit{l.i. AND } S < k \textit{ vecotrs}: V \neq span(S)$

$dim(V) = \textit{ of vectors for basis of } V$

$f\ U \textit{ is a subspace of vector space } V, dim(U) \leq \dim(V)$

Theorem 3.6.7:

$\textit{Let } V \textit{ be a vector space where } dim(V) = k, S \subseteq V,$

$S \textit{ is a basis for } V; \quad S \textit{ is l.i. and } |S| = k; V = span(S) \textit{and } |S| = k;$

Transition Matrices:

$(v)_S = (c_1 \dots c_k) \textit{ AND } [v]_S = \begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix}$

$P_{S,T} \textit{ is the transition matrix} = ([s_1]_T, \dots [s_k]_T)$

$P^T \textit{ is the transition matrix from } T \textit{ to } S$

$[v]_T = P_{S,T}\ [v]_S \textit{ AND } [v]_S = P_{S,T}^{-1}\ [v]_T \textit{ where } [v_i]_T = \begin{pmatrix} v_i \cdot u_1 \\ \vdots \\ v_i \cdot u_n \end{pmatrix}$

4) Spaces:

Row Space:

$row(A) \textit{is the subspaces spanned by the rows of } A$

$\textit{The non} - \textit{zero rows of } rref(A) \textit{ form a basis for } row(A)$

$A \textit{ and } B \textit{ are row} - \textit{equivalent}, row(A) \textit{and } row(B) \textit{are identical}$

Col Space:

$col(A) \textit{is the subspace spanned by the cols of } A$

$\textit{Pivot cols of } rref(A) \textit{from a basis for } col(A)$

$\textit{If } A \textit{ and } B \textit{ are row} - \textit{equivalent},$

$\textit{cols of } A \textit{ that are l.i} \leftrightarrow \textit{corresponding cols of } B \textit{ that are l.i.}$

Ranks:

$rank(A) = dim(row(A)) = dim(col(A))$

$rank(A) = rank(A^T) = min\{m, n\} \textit{ if } A = (a_{ij})_{m*n}$

$rank(AB) = min\{rank(A), rank(B)\}$

$\textit{Square matrices are of full rank where } m = n$

$\textit{Linear system is consistent} \leftrightarrow A \textit{ and } (A|B) \textit{ have same rank}$

Nullspaces:

$\textit{Nullspace} = \textit{Sol space of } Ax = 0$

$dim(nullspace) = nullity(A) \leq dim(\mathbb{R}^n) = n$

$rank(A) + nullity(A) = n$

Solution Set:

$\textit{If } v \textit{ is a sol to } Ax = b, \textit{sol set is } M =$

$\{u + v \mid u \in \textit{nullspace of } A\}$

5) Orthogonality:

Vectors:

$d(u, v) = \|u - v\|;$

$cos(u, v) = \frac{u \cdot v}{\|u\|\ \|v\|};$

$\|u + v\| \leq \|u\| + \|v\|$

$\textit{If } u \textit{ and } v \textit{ are col vectors}, u \cdot v = u^T v = \textit{dot product}$

Orthogonality:

$u \textit{ and } v \textit{ are orthogonal vectors} \leftrightarrow u \cdot v = 0 \textit{ AND } u, v \in \mathbb{R}^n \leftrightarrow u \perp v$

$\textit{Set of Vectors in } \mathbb{R}^n \textit{ is orthogonal} \leftrightarrow \textit{every pair is orthogonal}$

$\textit{Set is only orthonormal} \leftrightarrow \forall \textit{ vectors are unit vectors}$

Bases:

$S \textit{ is an orthogonal} - \textit{orthonormal basis}:$

$\leftrightarrow S \textit{ is orthogonal} - \textit{orthonormal}$

$\leftrightarrow |S| = dim(V) \textit{ OR } span(S) = V$

Projections:

$\textit{Let } V \textit{ be a subspace of } \mathbb{R}^n. \textit{Every vector } u \in \mathbb{R}^n$

$\textit{can be written uniquely as } u = n + p$

$s.t.\ n \textit{ is a vector orthogonal to } V \textit{ and } p \textit{ is a vector in } V$

$\textit{If } \{u_1, \dots, u_k\} \textit{is an orthogonal basis for } V, \textit{then } \forall\ w \in \mathbb{R}^n$

$p = \frac{w \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{w \cdot u_2}{u_2 \cdot u_2} u_2 + \dots + \frac{w \cdot u_k}{u_k \cdot u_k} u_k \textit{ is the projection of } w \textit{ onto } V$

Gram-Schmidt Process:

$\textit{Let } \{u_1, \dots, u_k\} \textit{ be a basis for a vector space } V:$

$v_i = u_i \textit{ only if } i = 1$

$$v_i = u_i - \sum_{j=1}^{i-1} \frac{u_i \cdot v_j}{v_j \cdot v_j} v_j$$

$\{v_1, \dots, v_k\} \textit{ form an orthogonal basis}$

Least Square Solutions:

$u \textit{ is a sol to } Ax = b \leftrightarrow u \textit{ is a soln to } A^T Ax = A^T b$

Orthogonal Matrices:

$\textit{Square matrix is orthogonal} \leftrightarrow A^T = A^{-1}$

$\textit{Rows and cols form orthonormal basis for } \mathbb{R}^n$

$P_{T,S} = (P_{S,T})^{-1} = (P_{S,T})^T, \textit{similar to transpose matrices}$

6) Diagonalization:

Eigenvalues:

$\textit{For a square matix } A, u \in \mathbb{R}^n \textit{ is an eigenvector of } A$

$\textit{if } Au = \lambda u \rightarrow \lambda \textit{ is an eigenvalue of } A$

$det(\lambda I - A) = 0 \textit{ is the characteristic equation}$

$det(\lambda I - A) \textit{ is the characteristic polynomial}$

$\textit{For } \Delta \textit{ matrix}, \textit{diagonal entries are the eigenvalues}$

$\textit{NOTE}: 0 \textit{ is not an eigenvalue of } A$

$(\lambda I - A)x = 0 \leftrightarrow \textit{eigenspace of the associated eigenvalue}$

$\textit{To find characteristic polynomial}, \textit{work backwards}:$

$1)\ \textit{Find basis for the eigenspaces of their eigenvalues}$

$2)\ \#\ \textit{of vectors for basis for eigne space} = \textit{degree in polynomial}$

Diagonalizable Matrices:

$\textit{Let } A \textit{ be a square matrix}, A \textit{ is diagonalizable}$

$\leftrightarrow \exists \textit{ invertible matrix } P\ s.t.\ P^{-1}AP \textit{ is a diagonal matrix}$

$\leftrightarrow A \textit{ has } n \textit{ l.i. eigenvectors}$

$\leftrightarrow dim(Eig(A)) = \textit{multiplicity of associated eigenvalue}$

$\leftrightarrow A \textit{ has } n \textit{ distinct eigenvalues}$

Orthogonal Diagonalization:

$\textit{Let } A \textit{ be a square matrix}, A \textit{ is orthogonally diagonalizable}$

$\leftrightarrow \exists \textit{ orthogonal matrix } P\ s.t.\ P^{-1}AP \textit{ is a diagonal matrix}$

$\leftrightarrow A \textit{ is a symmetric matrix}$

$P^T AP = D \leftrightarrow A = PDP^T$

Steps:

$1)\ \textit{Find all distinct eigenvalues}$

$2)\ \textit{Find an orthonormal basis for each unique eigenvalue}$

$3)\ A = PDP^T \textit{ where } P = \textit{all basis vectors and}$

$\quad D = \textit{all eigenvalues placed diagonally}$

7) Linear Transformations:

Basic Properties:

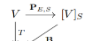
$T: \mathbb{R}^n \rightarrow \mathbb{R}^m \textit{ is a linear transformation}$

$\leftrightarrow T(0) = 0 \leftrightarrow T(c_1 u_1, \dots, c_k u_k) = (c_1 T u_1, \dots, c_k T u_k)$

$T(u) = Au \rightarrow A \textit{ is the standard matrix}$

$(T \cdot S)(u) = T(S(u)) \rightarrow BA \textit{ is the standard matrix for } T \cdot S$

$(S \cdot T)(u) = S(T(u)) \rightarrow AB \textit{ is the standard matrix for } S \cdot T$



Standard Matrices:

$\textit{Suppose } T: V \rightarrow W \textit{ is a linear transformation},$

$S = \{u_1, \dots, u_n\} \textit{is a basis for } V$

$T(e_i) = Ae_i \rightarrow A = [T(e_1) \quad \dots \quad T(e_n)]$

$\textit{Able to find strandard matrix using}$

$\textit{images of basic vectors of standard basis}$

Ranger and Kernels:

$R(T) = \textit{set of images of } T$

$R(T) = span\{T(u_1) \dots T(u_k)\} = \textit{col space}$

$rank(T) = \dim(R(T))$

$Ker(T) = \textit{set of vectors where image is the } 0 \textit{ vector}$

$Ker(T) = \textit{nullspace}$

$nullity(T) = \dim(Ker(T))$

$\textit{Find nullity of } T, \textit{find the basis for kernel of } T \textit{ using } Tx = 0$

$\textit{If } A \textit{ is the standard matrix for } T: nullity(A) = nullity(T)$

$\textit{If } T: \mathbb{R}^n \rightarrow \mathbb{R}^m, rank(T) + nullity(T) = n$

Invertible Matrix Theorem:

Let A be an n × n matrix. The following statements are equivalent:

1. A is invertible.

2. The linear system Ax = 0 has only the trivial solution.

3. The reduced row-echelon form of A is an identity matrix.

4. A can be expressed as a product of elementary matrices.

5. det(A) ≠ 0.

6. The rows of A form a basis for n.

7. The columns of A form a basis for n.

8. rank(A) = n.

9. 0 is not an eigenvalue of A.

Tutorial 1:
An inconsistent linear system with more unknows than equations exists.

Tutorial 2:
$Suppose\ B\ commutes\ with\ A,$
$B\ must\ be\ in\ the\ form \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}\ given\ A\ is \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$
$\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} A = \begin{pmatrix} D_1A \\ D_2A \end{pmatrix}\ and\ A(D_1 \quad D_2) = (AD_1 \quad AD_2)$
$(A+B)^2 = A^2 + B^2 + AB \leftrightarrow AB = BA$
$If\ AA^T = 0, A = 0$

Tutorial 3:
$If\ A^n = 0\ for\ some\ n \geq 2, then\ (I-A)^{-1} = I + \sum_{i=1}^{n-1} A$
$Let\ A = m * n\ matrix\ and\ B = n * m\ matrix,$
$if\ A, equivalent\ to \begin{pmatrix} R \\ 0 \cdots 0 \end{pmatrix}, AB\ is\ singular \leftrightarrow m < n$

Tutorial 4:
$If\ A\ and\ B\ are\ \blacksquare\ matrices\ of\ same\ size, det(AB)$
$= \det(BA)\ f\ A\ is\ a\ \blacksquare\ matrix\ w\ integer\ entries,$
$det(A) = 1, A^{-1} also\ has\ integer\ entries$
$If\ A\ is\ invertible, adj(A) is\ invertible$
1) $det(adj(A)) = det(A)^{n-1}\ and\ adj(A)^{-1} = \frac{A}{det(A)}$
2) $adj(adj(A)) = det(adj(A)) * adj(A)^{-1} = A$

Tutorial 5:
$span\{u_1, \cdots, u_k\} = R^n \leftrightarrow (A|v)\ is\ consistent\ for\ all\ v \in R^n$

Tutorial 6:
$To\ prove\ that\ S\ is\ a\ basis\ for\ V, (s_1 \cdots s_k|v_1| \cdots |v_j)\ is\ consistent$
$To\ find\ coordinate\ vector\ for\ u, solve\ for\ (s_1 \cdots s_k|u)$
$Let\ W\ be\ a\ subspace\ of\ R^n\ amd\ v \in R^n, if\ W' = \{u + v\ | u \in W\} \subseteq R^n$
1) $if\ v\ NOT \in W, W' is\ not\ a\ subspace\ of\ R^n$
2) $if\ v \in W, W = W' is\ a\ subspace\ of\ R^n$

$If\ Au, Av, Aw\ are\ l.i;\ u, v\ and\ w\ must\ be\ l.i.$
$If\ u, v, w\ are\ l.i. vectors\ in\ R^4, Au, Av\ and\ Aw\ are\ l.i.$

Tutorial 7:
Let V be a vector space:
$Suppose\ S \subseteq V\ s.t. span(S) = V, \exists\ S' \subseteq S\ s.t. S' is\ a\ basis\ for\ V$
$uppose\ T \subseteq V\ s.t. T\ is\ l.i., \exists\ a\ basis\ T' for\ V\ s.t. T \subseteq T'$

$Let\ A\ be\ a\ \blacksquare\ matrix, Suppose\ A^m = 0\ and\ A^{m-1} \neq 0\ where\ m \geq 2,$
$Since\ A^{m-1} \neq 0,\ A^{m-1}\ must\ have \geq 1\ non-zero\ col$
$c_k = 0, \forall k = 1, \cdots, m-1\ Suppose\ c_1u + \cdots + c_kA^{m-1}u = 0$

Tutorial 8:
$Let\ S = \{u_1, \cdots, u_k\} = be\ an\ orthnormal\ basis\ for\ subspace\ V\ of\ R^n$
$\forall v, w \in V, v \cdot w = (v)_s \cdot (w)_s$

Tutorial 9:
$Let\ P\ be\ a\ n * n\ orthogonal\ matrix$
$\forall x, y \in R^n, (Px) \cdot (Py) = x \cdot y$
$If\ \{u_1, \cdots, u_k\}\ is\ an\ orthonormal\ basis\ for\ R^n,$
$\{Pu_1, \cdots, Pu_k\}\ is\ also\ an\ orthonormal\ basis\ for\ R^n.$

$Let\ V = span\{u_1, \cdots, u_k\}\ be\ a\ vector\ space\ s.t. u_i\ are\ all\ unit\ vectors$
$if\ u_i \cdot u_j < 0, if\ i \neq j, 90^\circ \leq \theta_{ij} \leq 180^\circ$
$no\ two\ vectors\ among\ \{u_1, \cdots, u_k\}\ are\ l.i. and\ dim(V) \geq 3$

Tutorial 10:
$If\ A\ is\ orthogonally\ diagonalizable, eigenspaves\ are\ orthogonal$
$\lambda\ is\ eigenvalue\ of\ A, \lambda^n\ is\ eigenvalueof\ A^n, \lambda\ is\ eigenvalue\ of\ A^T$

Tutorial 11:
$Let\ T: R^n \to R^m\ and\ S: R^m \to R^n\ such\ that\ S \cdot T = I,$
$R(S) = R^n\ and\ Ker(T) = \{0\}$

Homework 1:
$A\ \blacksquare\ matrix\ C\ is\ anti-symmetric \leftrightarrow C^T = -C$
$if\ B = \frac{A+A^T}{2}\ and\ C = \frac{A-A^T}{2}, B\ is\ symmetric\ and\ C\ is\ anti-symmetric$

$If\ A\ and\ B\ are\ symmetric, A+B, rA\ and\ rB\ are\ all\ symmetric$
$if\ A\ and\ B\ are\ symmetric, AB = BA, AB\ is\ symmetric$

Homework 2:
$Vandermorde\ matrix: V = (a_i^{j-1})_{n*n}\ and\ det(V) = \prod_{1 \leq i \leq j \leq n}^{n} (a_j - a_i)$

$If\ A^3 = A, det(A) = -1, 0, 1$

$If\ sum\ of\ entries\ in\ each\ column\ is\ 0, det(A) = 0$
$If\ n > 2, a_{ij} = i + j, det(A) = 0$

Homework 3:
$V \subseteq R^n\ is\ a\ subspace\ of\ R^n \leftrightarrow V \neq \emptyset\ and\ cv + du \in V$
$If\ U\ and\ V\ are\ subspaces\ of R^n, U + V\ is\ also\ a\ subspace\ of\ R^n$

$if\ S = \{u, \ldots, v\} = \subseteq R^n, if\ x\ NOT \in span(S), S\ is\ not\ l.i.$
$if\ S = \{u, \ldots, v\} = \subseteq R^n, uB + \cdots + vB = 0, S\ is\ not\ l.i$
$if\ S\ and\ T\ are\ both\ l.i. set\ of\ vectors,$
$\{s_1 \cdots s_k, t_1 \cdots t_k\} is\ l.i. \leftrightarrow span(S) \cap span(T) = \{0\}$

Homework 4:
$If\ V \subseteq R^n\ be\ a\ subspace.\ V^\perp\ is\ a\ subspace\ of\ R^n\ and\ (V^\perp)^\perp = V$
$Let\ v \in R^n, p_1, p_2 \in V\ and\ n_1, n_2 \in V^\perp\ s.t. v = p_1 + n_1 = p_2 + n_2,$
$thus\ p_1 = p_2\ and\ n_1 + n_2$

$Assuming\ u \cdot v = 0, ||u+v||^2 = ||u||^2 + ||v||^2$
$(u \cdot v)^2 = ||u||^2 ||v||^2 - ||u - p||^2 ||v||^2 and\ ||u \cdot v|| \leq ||u|| ||v||$

$If\ \lambda\ be\ an\ eigenvalue\ of\ AB, \lambda\ is\ also\ an\ eigenvalue\ of\ BA$

Past Midterms:
$Let\ B = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 1 & 2 \\ 6 & 6 & 12 \end{pmatrix}, if\ AX = B\ has\ no\ soln, A\ must\ be\ 0$

$If\ A\ is\ row\ equivalent\ to\ B\ and\ B\ is\ invertibel, then\ A\ is\ invertible$
$\forall r \in R, there\ is\ a\ 2x2\ matrix\ A\ with\ det(A) = r$
$\forall \blacksquare\ matrices\ A, if\ A^TA = A, then\ A^T = A$

$If\ Ax = b\ and\ Ax = c\ are\ consistent, Ax = b + c\ is\ also\ consistent$
$If\ AB = I_m\ and\ BA = I_n, m = n$

$If\ Ax = b\ has\ only\ one\ solution, then\ Ax = 0\ ha\ only\ the\ trivial\ soln$
$det(A) = det(B^{-1}AB)$
$If\ A\ is\ a\ \blacksquare\ matrix\ w\ integer\ entries, A^{-1}\ also\ has\ integer\ entries$
$A\ \blacksquare matrix\ is\ invertible \leftrightarrow A^T\ is\ also\ invertible$

Past Finals:
$\forall u, w \in R^n, (Au) \cdot w = u \cdot (A^Tw)\ and\ given\ v_1\ to\ v_n\ are\ orthonormal$
$Given\ b_i = (Av_i) \cdot w\ and\ q = \sum_{i=1}^n b_iv_i, \to v_i \cdot q = b_i$

Adjoint Proofs:
$adj(B)adj(A) = det(B)B^{-1}det(A)A^{-1} = det(B)det(A)(AB)^{-1} = adj(AB)$

To prove adj(A) = 0 if rank(A) < n-1,
$Every\ (n-1) submatrix, every\ A_{ij}\ is\ singular, thus\ adj(A) = 0$

Bases Proofs:
If V and W are subspaces of R^n,
$V \cap W\ is\ a\ subspace\ of\ R^n$
$V \cup W\ is\ a\ subspace\ of\ R^n \leftrightarrow V \subseteq W\ or\ W \subseteq V$
$\exists\ a\ basis\ S_1\ for\ V\ and\ S_2\ for\ W\ s.t. S_1 \cap S_2\ is\ a\ basis\ for\ V \cap W$
$\exists\ a\ basis\ S_1\ for\ V\ and\ S_2\ for\ W\ s.t. S_1 \cap S_2\ is\ a\ basis\ for\ V + W$

Row space and Col Space Proofs:
$If\ A\ and\ B\ are\ row-equivalent\ matrices,$
$cols\ of\ A\ are\ l.i. \leftrightarrow corresponding\ cols\ of\ B\ are\ l.i.$
$cols\ of\ A\ form\ a\ basis\ for\ col(A) \leftrightarrow$
$corresponsing\ cols\ of\ B\ form\ basis\ for\ col(B)$

$If\ P\ is\ invertible, rank(PA) = rank(A)$

$nullspace\ of\ A = nullspace\ of\ A^TA$
$nullity(A) = nullity(A^TA) and\ rank(A) = rank(A^TA) = rank(AA^T)$

Vector Proofs:
$||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$
$u \cdot v = \frac{1}{4}||u+v||^2 - \frac{1}{4}||u-v||^2$

Diagonalization Proofs:
$\begin{pmatrix} a & 1 \\ 0 & b \end{pmatrix}\ is\ diagonalizable \leftrightarrow a \neq b$
$\blacksquare matrix\ is\ stochastic\ if\ \forall entries \geq 0\ and\ \forall j, \sum_{i=1}^n a_{ij} = 1$

$If\ A\ and\ B\ are\ same\ size\ diagonalizable\ matrices:$
$A + B\ and\ AB\ may\ not\ all\ be\ diagonalizable$
$Howeveer, if\ A\ and\ B\ are\ same\ size\ matrices\ and$
$orthogonally\ diagonalizable\ matrices:$
$A + B\ is\ also\ orthogonally\ diagonalizable$

Linear Transformation Proofs:
$Let\ T: R^n \to R^n\ s.t. T \cdot T = T:$
$Standard\ matrix\ is: \begin{pmatrix} r & s \\ t & 1-r \end{pmatrix}$
$If\ T\ is\ not\ the\ zero\ transformation, \exists u \neq 0 \in R^n\ s.t. T(u) = u$
$If\ T\ is\ not\ the\ identity\ transformation, \exists u \neq 0 \in R^n\ s.t. T(u) = u$

$Let\ S: R^n \to R^m\ and\ T: R^m \to R^k\ be\ linear\ transformations:$
$Ker(S) \subseteq Ker(T \cdot S)\ and\ R(T \cdot S) \subseteq R(T)$

2x2 matrices:

$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ lower triangular	$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ upper triangular
$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ zero	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ diagonal	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ one

Computations:
Inverse Matrices:
$(A|I) \xrightarrow{Gausssian} (I|A^{-1})$

Projection of w onto V, given a basis S = {u₁ … uₙ}
$\sum_{i=1}^{n} (v_i \cdot u_i) u_i$
$Or\ solve\ for\ Least\ Square\ solutions \to Ax = w$

Rank and Nullity:
$rank(A) \to \#\ of\ pivot\ cols\ in\ rref(A)$
$nullity(A) \to \#\ of\ non-pivot\ cols\ in\ rref(A)$

PDP⁻¹ = A
$P = (v\ for\ E_\lambda \quad \ldots \quad v\ for\ E_\lambda)$
$D = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda \end{pmatrix}$

Standard Matrix:
$T(x) \to Ax, A\ is\ standard\ matrix$