Trigonometry

$\tan x = \frac{opposite}{adjacent}$

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x - \tan^2 x = 1$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

Differentiation

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cot x$

$\frac{d}{dx}\left(a^{f(x)}\right) = f'(x)\,(\ln a)a^{f(x)}$

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} = -\frac{d}{dx}(\cos^{-1}x)$

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} = -\frac{d}{dx}(\cot^{-1}x)$

$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}} = -\frac{d}{dx}(\mathrm{cosec}^{-1}x)$

$\frac{d}{dx}(fg) = f'g + g'f$

$\frac{d^2y}{dt^2} = \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$

$\frac{dz}{dt} = \frac{dz}{dy}\left(\frac{dy}{dt}\right) + \frac{dz}{dx}\left(\frac{dx}{dt}\right)$

Integration

$\int \cot x\, dx = \ln|\sin x| + C$

$\int \tan x\, dx = \ln|\sec x| + C$

$\int \mathrm{cosec}\, x\, dx = -\ln|\mathrm{cosec}\, x + \cot x| + C$

$\int \sec x\, dx = \ln|\sec x + \tan x| + C$

$\int \ln x\, dx = x\ln x - x + C$

$\int \sin^{-1}x\, dx = x\sin^{-1}x + \sqrt{1-x^2} + C$

$\int \tan^{-1}x\, dx = x\tan^{-1}x + -\frac{1}{2}\ln(1+x^2) + C$

$\frac{d}{dx}\int_a^x f(t)dt = f(x)$

$\frac{d}{dx}\int_a^b f(t)dt = f(b)\cdot b' - f(a)\cdot a'$

Partial Fractions:

$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$

$\frac{f(x)}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$

$\frac{f(x)}{(ax+b)(cx^2+dx+e)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e}$

Vectors

$||\vec{a}|| = \sqrt{x^2 + y^2 + z^2}$

$\hat{a} = \frac{\vec{a}}{||\vec{a}||}\, c$

$\cos x = \frac{\vec{a}\cdot\vec{b}}{||\vec{a}||\,||\vec{b}||}$

$\nabla f = f_x i + f_y j + f_z k$

$D_u f = \nabla f \cdot u = f_x(u_1) + f_y(u_2)$

$\Delta f = D_u f(\Delta t)$

Partial Differentiation

$f_{xy} = \frac{d}{dy}\left(\frac{d}{dx}(f)\right)$

Critical Points: $f_x = 0\ OR\ f_y = 0$

$f_x = \pm\infty\ OR\ f_y = \pm\infty$

Min Critical Point: $D > 0\ AND\ f_{xx} > 0$

Max Critical Point: $D > 0\ AND\ f_{xx} < 0$

Saddle Critical Point: $D < 0$

$D = f_{xx}f_{yy} - (f_{xy})^2$

L' Hospital's Rule

$\lim_{n\to\infty}\left(\frac{f(x)}{g(x)}\right) = \lim_{n\to\infty}\left(\frac{f'(x)}{g'(x)}\right)$ *if both f and g* $\to 0$ *or* $\pm\infty$

Ratio Test

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |ax - b| < 1$

$\to \frac{b-1}{a} < x < \frac{b+1}{a}$

$\to$ *Centre* $= \frac{b}{a}$ *and Radius* $= \frac{1}{a}$

Half-Life

$k_u = \frac{\ln 2}{half-life}$

$U = U_0\, e^{-k_u(t)}$

$\frac{T}{U} = \frac{k_u}{k_t - k_u}\left[1 - e^{(k_u - k_t)t}\right]$

Differential Equations:

$y' + Py = Q \to y = \frac{1}{R}\int RQ\, dx$

$R = e^{\int P\, dx}$

$y' + Py = Qy^n \to z = y^{1-n}$

Solve for: $z + (1-n)Pz = Q(1-n)$

$y'' + ay' + b = 0 \to \lambda^2 + a\lambda + b = 0$

If $a^2 - 4b > 0$

$y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$

If $a^2 - 4b = 0$

$y = (A + Bx)e^{-\frac{ax}{2}}$

If $a^2 - 4b < 0$

$\lambda = \alpha \pm \beta i$

$y = Ae^{\alpha x}\cos\beta x + Be^{\alpha x}\sin\beta x$

$y' = (A\alpha + B\beta)e^{\alpha x}\cos\beta x + (B\alpha - A\beta)e^{\alpha x}\sin\beta x$

Partial Differential Equations

$U(x,y) = X(x)Y(y)$

$k = f(x)\left(\frac{X'}{X}\right) = g(y)\left(\frac{Y'}{Y}\right)$

$X = Ae^{k\int\frac{1}{f(x)}dx}$

$Y = Be^{k\int\frac{1}{g(y)}dy}$

Areas and Volumes

$V_{about\ x-axis} = \pi\int_a^b [f(x)]^2 dx$

$V_{about\ y-axis} = \pi\int_a^b [f(y)]^2 dx$

$A = \pi\int_a^b f(x) - g(x)dx$

$V_{cone} = \frac{1}{3}\pi r^2$ *and* $A_{cone} = \pi r\sqrt{r^2 + h^2} + \pi r^2$

$V_{cylinder} = \pi r^2 h$ *and* $A_{cylinder} = 2\pi rh + 2\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3$ *and* $A_{sphere} = 4\pi r^2$

Arithmetic and Geometric Progressions

$Sum_{AP} = \frac{n}{2}[2a + (n-1)d]$

$Sum_{GP} = \frac{a(1-r^n)}{(1-r)}$

$Sum_{GP\to\infty} = \frac{a}{(1-r)}$

Taylor Series:

$\sum_{n=0}^{\infty} a^n(b+x)^n = \frac{1}{1-a(x+b)}$

$\sum_{n=0}^{\infty}\frac{f^n(a)}{n!} = f(a) + f'(a)(x-a) + f''(a)(x-a)^2 + \cdots$

Maclaurin Series

$\frac{1}{1-x} = \sum x^n = 1 + x + x^2 + \cdots$

$\frac{1}{1+x} = \sum(-1)^n x^n = 1 - x + x^2 - \cdots$

$e^x = \sum\frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \cdots$

$e^{-x} = \sum\frac{(-1)^n x^n}{n!} = 1 - x + \frac{x^2}{2} - \cdots$

$e^{-x^2} = \sum\frac{(-1)^n x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \cdots$

$\sin x = \sum\frac{(-1)^n x^{2n+1}}{(2n+1)!} = 1 - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$

$\cos x = \sum\frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$

$\ln(1+x) = \sum\frac{(-1)^n x^{n+1}}{(n+1)} = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

$(1+x)^k = \sum\binom{k}{n}x^n = \binom{k}{0} + \binom{k}{1}x + \binom{k}{2}x^2 + \cdots$

$\tan^{-1}x = \sum\frac{(-1)^n x^{2n+1}}{(2n+1)} = 1 - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$

$\tan x = 1 - \frac{x^3}{3} + \frac{2x^5}{15} + \frac{7x^7}{315} + \cdots$

Malthus Model

$\frac{dN}{dt} = BN - DN$

$N = N_0 e^{(B-D)t}$

Logist Model

$\frac{dN}{dt} = BN - sN^2$

$\frac{d^2N}{dt^2} = (B - sN)(N)(B - 2sN)$

$N_\infty = Carrying\ Capacity = \frac{B}{s}$

$Equilibirum \to N = 0\ or\ N = \frac{B}{s}$

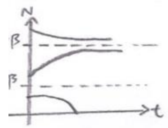
$N = \frac{N_\infty}{1 - \left(\frac{N_\infty}{N_0} - 1\right)e^{-Bt}}$

Harvesting Model:

$\frac{dN}{dt} = BN - sN^2 - E$

$\frac{1}{N} = \frac{s}{B} + Ce^{-Bt}$

If $0 < E < \frac{B^2}{4s}$: $\beta = \frac{B^2 \pm \sqrt{B^2 - 4s}}{2s}$



If $E > \frac{B^2}{4s}$: $\beta = \frac{B}{2s}$