Unique Binary Trees

Low Jun Kai, Sean

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Preface

Given an integer n, return the number of structurally unique BSTs (binary search trees) which has exactly n nodes of unique values from 1 to n

Theorem

Let the function $f(n)$ denote the function where given n nodes, we get the # of structurally unique BSTs.

$$f(n) = \left\{ \begin{array}{ll} 0 & \textbf{for } n = 0 \vee n = 1 \\ \sum_{i=0}^{n-1} f(i) \cdot f(n-i-1) & \textbf{for } n > 1 \end{array} \right\}$$

Proof of Correctness

1. To prove the two base cases are true is **trivial**
   (a) There is only 1 way to arrange a BST with 0 nodes
   (b) There is only 1 way to arrange a BST with 1 node
2. Assuming $f(n)$ is true, we have to prove that $f(n+1)$ holds true as well for some $n > 1$
3. To prove the correctness of this is equivalent to **Proving the correctness of the closed formula for Catalan Numbers**. As a result, we will skip this proof for this exercise.