# Practice Problems for Exam I

Below are exercises to aid in your studying. If you are able to do all of these problems, then you are in a good position walking into the exam. This list of problems is longer than the exam will be and contains questions much harder the exam will ask. Furthermore, this is a list of practice problems, and only contains exercises for classifying/solving ODEs & IVPs as well as describing solutions by phase-portraits. On the exam you will not only be asked to show that you can solve such things; you will also be probed for understanding, and as such you should also study your notes and read the associated sections. I highly suggest asking in the review about any problems you struggle on.

The most important and most difficult part of solving *any* of these prolems is properly classifying the ODE as one of the types we can solve.

Solutions will be posted after the review.

For exercises 1-14, do the following:

(a) State the order of the ODE, and classify it
(autonomous, separable, linear, exact, homogeneous, exact, Bernoulli)

(b) Solve the ODE. If you simplify it to a different type of ODE by substitution or integrating factor, classify the new ODE you have reduced the original to.

(c) State whether your solution is explicit or implicit.

1. $x\dfrac{dy}{dx} + 4y = x^3 - x$
2. $(x)\ dx + (x^2y + 4y)\ dy = 0$
3. $(xe^x - 2y)\ dx = x\ dy$
4. $\dfrac{dy}{dx} = 1 + e^{y-x+5}$
5. $\dfrac{dy}{dx} = y(xy^3 - 1)$
6. $\dfrac{dy}{dx} + 2xy^2 = 0$
7. $(\tan(x) - \sin(x)\sin(y))\ dx + (\cos(x)\cos(y))\ dy = 0$
8. $x^2\dfrac{dy}{dx} + y^2 = ty$
9. $\dfrac{dy}{dx} = x\sqrt{1 - y^2}$
10. $(5y - 2x)\ dy - (2y)\ dx = 0$
11. $(x)\ dx + (y - 2x)\ dy = 0$
12. $\dfrac{dy}{dx} = \sqrt{y}$
13. $(y^2 + yx)\ dx + (x^2)\ dy = 0$
14. $(3 + 3x^2)\dfrac{dy}{dx} = 2xy(y^3 - 1)$

For exercises 15-17 , do the following:

(a) State the order of the ODE, and classify it
(autonomous, separable, linear, exact, homogeneous, exact, Bernoulli)

(b) Find the critical points of the ODE.

(c) Construct a one-dimensional phase portrait.

(d) Classify the critical points,

15. $\dfrac{dy}{dx} = y\ln(y+2)$

16. $\dfrac{dz}{dx} = \dfrac{ze^z - 9z}{e^z}$

17. $\dfrac{dy}{dx} = 10 + 3y - y^2$

For exercises 18-24, do the following:

(a) State the order of the ODE, and classify it
(autonomous, separable, linear, exact, homogeneous, exact, Bernoulli)

(b) Solve the ODE. If you simplify it to a different type of ODE by substitution or integrating factor, classify the new ODE you have reduced the original to.

(c) State whether your solution is explicit or implicit.

(d) Use the initial condition(s) to solve the IVP.

(e) Determine if the solution you found was unique (if possible).

18. $x^2\dfrac{dy}{dx} + xy = y \qquad y(-1) = -1$

19. $x\dfrac{dy}{dx} + y = 4x + 1 \qquad y(1) = 8$

20. Removed due to difficulty

21. $\sin(x)\,dx + y\,dy = 0 \qquad y(0) = -1$

22. $(x^2 + y^2 - 5)\,dx + (y + xy)\,dy = 0 \qquad y(0) = 1$

23. $xy^2\dfrac{dy}{dx} = y^3 - x^3 \qquad y(1) = 2$

24. $\sqrt{y}\dfrac{dy}{dx} + y^{3/2} = 1 \qquad y(1) = 1/2$