Methods for First Order Equations

Name	Normal Form	Solution Method	Special cases/Remarks
Autonomous	$\frac{dy}{dx} = h(y)$	$\int \frac{1}{h(y)} dy = \int 1\ dx$	Common in physical laws
Separable	$\frac{dy}{dx} = g(x)h(y)$	$\int \frac{1}{h(y)} dy = \int g(x)$ dx	Autonomous when g(x) = 1
Linear	$\frac{dy}{dx} + P(x)y = g(x)$ OR $(P(x)y - g(x))\ dx + dy = 0$	Integrating factor: $\mu(x) = e^{\int P(x)dx}$ $\Longrightarrow \frac{d}{dx}\Big(y\mu(x)\Big) = g(x)\mu(x)$	Autonomous when P(x) = 0 Separable when $g(x) = 0$ (homogeneous)
Bernoulli	$\frac{dy}{dx} + P(x)y = g(x)y^n$ OR $(P(x)y - g(x)y^n x)\ dx + dy = 0$	Substitution: $u = y^{1-n}$ Reduce to linear	Linear when $n = 0$
Exact	$M(x, y)\ dx + N(x, y)\ dy = 0$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	$f(x, y) = \int M(x, y)dx + h(y)$ OR $f(x, y) = \int N(x, y)dy + g(x)$ $\Longrightarrow d(f(x, y)) = 0$	
α-Homogeneous	$M(x, y)\ dx + N(x, y)\ dy = 0$ $M(tx, ty) = t^{\alpha} M(x, y)$ $N(tx, ty) = t^{\alpha} N(x, y)$	Substitution: $u = y/x, \quad dy = udx + xdu$ or $v = x/y, \quad dx = vdy + ydv$ Reduce to separable	Intuition: M and N have the same 'degree'
'Almost' Exact (But not anything else)	$M(x, y)\ dx + N(x, y)\ dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$	Integrating factor is either: I. $\mu(x) = e^{\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx}$ II. $\mu(y) = e^{\int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dx}$ Reduce to Exact	If neither integration factor works then check again if it is Homogeneous/Bernoulli/Linear/etc
Other Nonseparable (special)	$\frac{dy}{dx} = f(Ax + By + C)$ $B \neq 0$	Substitution: $u = Ax + By + C \quad$ Separable	A and C can be zero.