Practice Problems for Exam II

Below are exercises to aid in your studying. If you are able to do all of these problems, then you are in a good position walking into the exam. This list of problems is longer than the exam will be and contains questions much harder the exam will ask. Furthermore, this is a list of practice problems, and only contains exercises for solving ODEs & IVPs as well as approximations. On the exam you will not only be asked to show that you can solve such things; you will also be probed for understanding, and as such you should also study your notes and read the associated sections. I highly suggest asking in the review about any problems you struggle on.

1. Find general solution to $\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} - 18y = 0$.

2. Find a differential operator that annihilates $x^3(1-5x)$.

3. Find general solution to $2x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + y + x = x^2$.

4. Find general solution to $6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$ given $y_1 = e^{x/3}$.

5. Find general solution to $\frac{1}{4}\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2 - 2x$ via undetermined coefficients.

6. Find general solution to $\frac{d^4y}{dx^4} - 4\frac{d^2y}{dx^2} = x + e^x$ given $y_c = c_1 + c_2x + c_3e^{2x} + c_4e^{-2x}$.

7. Consider the IVP $\frac{dy}{dx} = y^2 + x^2$, $y(0) = 1$. For the following, write the expressions, and then use a calculator to compute.
   (a) Approximate $y(0.2)$ via Euler's method with step size 0.1.
   (b) Approximate $y(0.2)$ via improved Euler's method with step size 0.1.
   (c) The true solution is $y(0.2) = 1.25302$. What are the absolute and relative errors?

8. Solve the system given by the equations $\frac{dx}{dt} - 4y = 1$ and $\frac{dy}{dt} + x = 2$.

9. Find general solution to $y^2\frac{d^2y}{dx^2} = \frac{dy}{dx}$

10. Find general solution to $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 8x^2$ via annihilators.

11. Find a differential operator that annihilates $13x + 9x^2 - \sin(4x)$.

12. Find general solution to $\frac{d^2y}{dx^2} + y = \sec^2(x)$ via variation of parameters given $y_c = c_1\cos(x) + c_2\sin(x)$.

13. Find general solution to $x^3\frac{d^3y}{dx^3} + x\frac{dy}{dx} - y = 0$.

14. Find general solution to $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x\sin(x)$ via annihilators.

15. Find general solution to $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} - 18y = 0$.

16. Find general solution to $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - y = e^x - x + 16$.

17. Solve the system given by the equations $\dfrac{dx}{dt} + \dfrac{dy}{dt} = e^t$ and $\dfrac{d^2x}{dt^2} - \dfrac{dx}{dt} - x - y = 0$.

18. Find general solution to $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - y = x - 4e^x$ via undetermined coefficients.

19. Find general solution to $\dfrac{d^3y}{dx^3} + 4\dfrac{dy}{dx} = \sec(2x)$ via variation of parameters.

20. Find general solution to $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 2y = 0$ given $y_1 = x\sin(\ln(x))$.

21. Use a degree five Taylor polynomial to approximate the solution to $\dfrac{d^2y}{dx^2} + y^2 = 1$, $y(0) = 2$, $\dfrac{dy}{dx}(0) = 3$.