Scientific Computing

Assignment 6: Thermal Conduction

In this assignment, we'll consider a problem in which a variable changes in both space and time. Specifically, we'll look at the problem of thermal conductivity. The conduction of heat through a material is described by the differential equation

$$\frac{\partial T}{\partial t} = \nu \frac{\partial^2 T}{\partial x^2}$$

where ν depends on the material which is conducting the heat. This is identical to the equation for molecular diffusion, and can be solved using the techniques discussed in class. That is, define *t* and *x* at regularly spaced intervals as follows:

$$t_i = i\Delta t \quad x_j = j\Delta x$$

$$T_j^i = T(t_i, x_j)$$

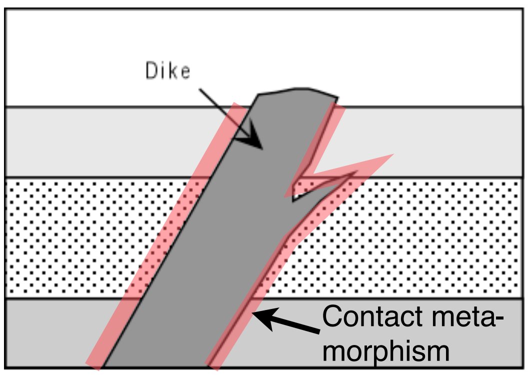
Then approximate the derivatives by using partial differences:

$$\frac{T_j^{i+1} - T_j^i}{\Delta t} = \nu \frac{T_{j-1}^i - 2T_j^i + T_{j+1}^i}{\Delta x^2}$$

Then, by looping over all *j*, one can solve this equation to find T_j^{i+1} one timestep into the future, given the current temperature values T_j^i. Repeating this process advances the solution forward in time.

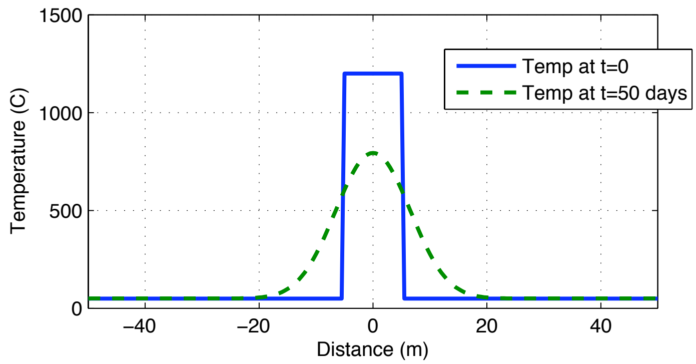
A Cooling Dike

Let's look at a specific conduction problem, which arises in geology. Often, molten magma from deep within the earth forces its way toward the surface by making a long vertical crack in the surrounding rock: the magma flows up through the crack. The magma quickly freezes inside the crack to form a "dike". Dikes are typically anywhere from a few cm in width to 20-100 meters.



The dike starts off at a temperature near the melting point of rock, about 1200 °C: the surrounding "country rock" might be only about 50 °C. But thermal conduction causes the dike to cool over time: the dike's heat is transferred into the surrounding rock. If conditions are right, the surrounding rock can be "cooked" by this heat, causing chemical changes which lead to the formation of new minerals in a zone near the dike. This is called "contact metamorphism". This happens in a narrow band: far from the dike, temperatures may never get hot enough to cause new minerals to form.

Let's model a simple dike intrusion. Let *x* be distance away from the centerline of the dike, and *t* be time. Here are the key parameters:



Width of dike: 10 meters

Temperature of dike at $t = 0$: 1200 °C

Temperature of surrounding rock at $t = 0$: 50 °C

$\nu = 3.7 \cdot 10^{-6}$ m^2/s for rock

Total simulation time: 100 days

It's up to you to pick a good Δx and Δt for this problem. Keep in mind the numerical stability issues discussed in class. For boundary conditions, just let Temp = 50 °C at the side walls, for all *t*.

The key question you should answer is this: <u>Hornblende</u> is a mineral that can form via contact metamorphism, but only in areas where the surrounding rock reaches a temperature of at least 500 °C. Given the 10-meter-wide dike mentioned above, how wide is the zone where hornblende can form? Make sure your model has fine enough resolution to estimate this width to +/- 10%.

Other questions:

At what time does the surrounding rock 2 meters away from the dike reach its peak temperature? 4 meters away? 6?

What happens if the dike is twice as wide? Does the "hornblende zone" become wider in direct proportion, or is it more complicated than that?