Scientific Computing

Final Project Ideas

For this course, instead of a final exam, I'd like you to work on a final project, which will be due on the scheduled exam date for this class.

Your assignment for this project is to use MATLAB (or another programming language of your choice) to solve a non-trivial problem in scientific computing. I expect you to build a numerical model and then analyze its behavior, discussing special properties and unusual behavior.

You may consider any of the main categories of problem we've discussed in this class:

Zero-finding (what parameter values are needed to balance a set of equations?)
Optimization (what parameter values give the "best answer" in some sense?)
Ordinary differential equations (how do a set of variables evolve forward in time?)
Partial differential equations (How do a set of variables vary in both time and space, or in multiple spatial directions?)
Spectral analysis (detecting periodic signals in data — we haven't covered this yet)

You're welcome to come up with your own topics, perhaps using something from an internship or senior thesis, but on the next page are some sample topics you might try. For each of these, I'm just giving you a tiny glimpse of the problem here: come and talk to me for more details.

Some of these topics involve more difficult programming tasks and more complicated code than others. My expectations vary depending on the project: for the "harder" ones, simply building a working model is enough; for the "easier" ones, I expect you to do lots of extra work testing, analyzing, and improving the model beyond just the basics.

However, in all cases, you should:

1) Write some code to solve a physical problem
2) Consider several case studies for that physical problem
3) Complete a write-up that describes the problem, the numerical method you used to solve it, the case studies, and any pitfalls you encountered along the way.

I do not have any specific length requirements for the write-up or specific number of case studies you should consider, since some problems are easier than others; a good guideline is that your final project should require as much effort as two or three weekly homework assignments. If you dash it off in an hour, you're doing it wrong. If you turn it into a senior thesis, also doing it wrong.

Due Dates:

Monday, November 12: Submit proposed project topic

Wednesday, Nov 28: Submit preliminary status report (1-2 pages describing your approach, and hopefully some preliminary code you have written)

Friday, Dec 14: **All work must be completed by this date**.

Electronic Circuits

Write some code that represents the voltages in an electrical circuit containing linear time-dependent components like capacitors and inductors as ordinary differential equations, and solves the equations using in-class methods. Build the same circuit in the real world, and compare the results.

Ballistics

Computing the path of a projectile (like a cannonball) launched into the air is a pretty simple problem in introductory physics... assuming you neglect air resistance. But when frictional effects are included, it gets a lot more complicated. Build a model to accurately find the trajectory of a projectile, and test your model's predictions using a real-world apparatus.

In particular, you'll want to include the drag equation

$$F_D = \frac{1}{2}\rho v^2 C_D A$$

which depends on the density of air, the moving object's velocity, the "drag coefficient" which depends on the shape, and the cross-sectional area of the object.

Orbital dynamics

Build a numeric model to solve for the orbits of at least three interacting celestial objects. Example problems:

- How does the Moon's gravity affect a satellite in Earth orbit?
- Identify stable "Lagrange points" where a small satellite remains stationary with respect to Earth and Moon
- Demonstrate a "least energy" trajectory for launching a spacecraft from Earth to Mars with a minimum amount of fuel. (Look up "Hohmann transfer orbits".)
- Demonstrate how to increase a spacecraft's speed by using a "gravitational slingshot" maneuver.
- You can also include carefully chosen "rocket firings" which change the spacecraft's velocity.

In tackling this problem, you'll want to calculate the total force on each object caused by the gravitational influence of all the others combined, in both X and Y directions. It's very important to choose a stable, accurate timestepping scheme for this project. At minimum, read about "Verlet integration".

Highway traffic waves

People tend to drive more slowly when there are lots of cars closely packed around them. This can lead to "traffic waves", alternating sections of clear highway and traffic jams which occur spontaneously as a result of individual driving habits. Build a model to simulate these waves. (Talk to me for help setting up this problem.)

Climate "box" model

The Earth's ocean has an overturning circulation, in which cold water sinks near the poles, and warm water rises near the equator. But rainfall and evaporation change the density of the water, which some scientists speculate might lead to a "shutdown" of this overturning circulation, with important climate consequences. Build a simplified model of this climate system. See https://www.pik-potsdam.de/members/willeit/klimageschichte-ubungen/stommel-model/tutorial.pdf for more details. This will involve solving a 4-variable ordinary differential equation.

1-d ice sheet model

During the last ice age, giant ice sheets covered most of North America. How tall were these ice sheets? How rapidly did the ice in them flow? Build a simplified one-dimensional model of a continental ice sheet, taking into account accumulation, flow, and melting of ice. For more details, see the handout "Ice Sheet Modeling" on OnCourse.

Advanced ecosystem modeling

• Investigate the"NPZ" model, a 3-element model for ocean ecosystems which includes animals, plants, and nutrients.

Reconsider the Lotka-Volterra model, with one or more of the following additions:

• Consider an ecosystem with 3 or more species, which eat each other in a food chain.

• Consider an situation where there the number of animals in a species varies from place to place. Use a diffusion model to describe how they move and spread out over time.

2-d wave equation

Build a simple model to solve the "wave equation" in two dimensions. This model can describe the behavior of water waves in a harbor, sound waves in a musical instrument, or light waves in a prism or lens, or tsunami waves propagating across the ocean.

2-d electrostatics

Solve "Laplace's Equation" to find the electric field and potential within and near a parallel-plate capacitor. You will have a homework assignment to do this in a simple way on October 31: if you adopt this as a final project, I'd like you to find a more efficient way to solve the problem and/or solve a 3-dimensional problem instead of 2d.

Gravitational Sensing

Underground mass concentrations, such as ore deposits, can often be detected via their effect on the local gravitational field. Write code to solve for gravitational potential anomalies. (This is closely related to the Laplace equation assignment.)

Multigrid Method for Laplace Problem

The Gauss-Seidel method we used to solve the Laplace problem in Assignment 6 is pretty slow. The "Multigrid" scheme is an advanced technique that's much faster. Can you implement it?