

A new table of percentage points of the chi-square distribution

By H. LEON HARTER

Aerospace Research Laboratories, Wright-Patterson Air Force Base

1. INTRODUCTION

Greenwood & Hartley (1962, pp. 140–3) have given a comprehensive list of published tables of the percentage points of the chi-square distribution. The present author has, however, recently encountered applications (see Harter, 1963*a* and *b*) for which more extensive or more accurate tables are required and as a result he set out to compute a six-significant-figure table, accurate to within a unit in the last digit, of percentage points of chi-square corresponding to cumulative probabilities of $P = 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.025, 0.05, 0.1 (0.1) 0.9, 0.95, 0.975, 0.99, 0.995, 0.999, 0.9995, 0.9999$, for

$$\nu = 1 (1) 150 (2) 330$$

degrees of freedom. The table printed on pp. 234–9 below covers only the case of degrees of freedom $\nu = 1 (1) 100$; the complete table with other material relating to the gamma and beta distributions will be published shortly in a separate volume, through the U.S. Government Printing Office (Harter, 1964).

Three different methods of computation were used which, by overlapping, provided checks on accuracy. These methods are described in the following section.

2. METHODS OF COMPUTATION

Method 1. The cumulative distribution function of χ^2 with ν degrees of freedom may be written in the form

$$P(\chi^2|\nu) = I(u, p), \quad (1)$$

where
$$I(u, p) = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{(p+1)}} v^p e^{-v} dv, \quad (2)$$

with
$$u = \chi^2/\sqrt{(2\nu)}, \quad p = \frac{1}{2}\nu - 1.$$

The incomplete Γ -function ratio $I(u, p)$ was tabulated by Karl Pearson (1922, Tables I and II) to seven decimal places for $p = -1.0 (0.05) 0.0 (0.1) 5.0 (0.2) 50.0$ and u at intervals of 0.1. However, these Tables do not permit sufficiently accurate inverse interpolation for our purpose, and therefore a new table of $I(u, p)$ was computed by a method to be described elsewhere (Harter, 1964), accurate to within a unit in the tenth decimal place, for

$$p = -0.5 (0.5) 74 (1) 164$$

and u at intervals of 0.1. The percentage points of u were found by inverse interpolation (after subtabulation if necessary) in the new table, by Aitken's method, and these were multiplied by $\sqrt{(2\nu)}$ to find the percentage points of χ^2 corresponding to the required values of the cumulative probability P . Most of the resulting tabular values are accurate to within a unit in the sixth significant digit. A serious exception occurs when P and ν are both small

REFERENCES

- GREENWOOD, J. ARTHUR & HARTLEY, H. O. (1962). *Guide to Tables in Mathematical Statistics*. Princeton, New Jersey: Princeton University Press.
- HARTER, H. LEON (1963*a*). The use of quasi-ranges in setting exact confidence bounds for the standard deviation of a normal population. (Submitted for publication in a technical journal.)
- HARTER, H. LEON (1963*b*). Exact confidence bounds, based on one order statistic, for the parameter of an exponential population. (Submitted for publication in a technical journal.)
- HARTER, H. LEON (1964). *New Tables of the Incomplete Gamma Function Ratio and of Percentage Points of the Chi-Square and Beta Distributions*. Washington: U.S. Government Printing Office.
- PEARSON, E. S. & HARTLEY, H. O. (1954). *Biometrika Tables for Statisticians*, 1. Cambridge University Press.
- PEARSON, KARL (1922). *Tables of the Incomplete Γ -Function*. Cambridge University Press.

Table 1. *Percentage points of the chi-square distribution*

ν P	1	2	3	4	5	6	7
0.0001	0.07157080	0.0200010	0.0521483	0.0284185	0.0821774	0.172352	0.299967
.0005	0.0392699	0.0100025	0.0152790	0.0639220	0.158137	0.299408	0.484875
.0010	0.0157080	0.0200100	0.0242976	0.0908040	0.210213	0.381067	0.598494
.0050	0.0392704	0.0100251	0.0717218	0.206989	0.411742	0.675727	0.989256
.0100	0.0157088	0.0201007	0.114832	0.297109	0.554298	0.872090	1.23904
.0250	0.03982069	0.0506356	0.215795	0.484418	0.831212	1.23734	1.68987
.0500	0.0393214	0.102587	0.351846	0.710723	1.14548	1.63538	2.16735
0.1000	0.0157908	0.210721	0.584374	1.06362	1.61031	2.20413	2.83311
.2000	0.0641847	0.446287	1.00517	1.64878	2.34253	3.07009	3.82232
.3000	0.148472	0.713350	1.42365	2.19470	2.99991	3.82755	4.67133
.4000	0.274996	1.02165	1.86917	2.75284	3.65550	4.57015	5.49324
.5000	0.454936	1.38629	2.36597	3.35669	4.35146	5.34812	6.34581
.6000	0.78326	1.83258	2.94617	4.04463	5.13187	6.21076	7.28321
.7000	1.07419	2.40795	3.66487	4.87843	6.06443	7.23114	8.38343
.8000	1.64237	3.21888	4.64163	5.98862	7.28928	8.55806	9.80325
.9000	2.70554	4.60517	6.25139	7.77944	9.23636	10.6446	12.0170
0.9500	3.84146	5.99146	7.81473	9.48773	11.0705	12.5916	14.0671
.9750	5.02389	7.37776	9.34840	11.1433	12.8325	14.4494	16.0128
.9900	6.63490	9.21034	11.3449	13.2767	15.0863	16.8119	18.4753
.9950	7.87944	10.5966	12.8382	14.8603	16.7496	18.5476	20.2777
.9990	10.8276	13.8155	16.2662	18.4668	20.5150	22.4577	24.3219
.9995	12.1157	15.2018	17.7300	19.9974	22.1063	24.1028	26.0178
.9999	15.1367	18.4207	21.1075	23.5127	25.7448	27.8563	29.8775

ν P	8	9	10	11	12	13	14
0.0001	0.463594	0.660809	0.888920	1.14529	1.42749	1.73332	2.06080
.0005	0.710379	0.971699	1.26498	1.58685	1.93438	2.30506	2.69673
.0010	0.857105	1.15195	1.47874	1.83385	2.21421	2.61722	3.04067
.0050	1.34441	1.73493	2.15586	2.60322	3.07382	3.56503	4.07467
.0100	1.64650	2.08790	2.55821	3.05348	3.57057	4.10692	4.66042
.0250	2.17973	2.70039	3.24697	3.81575	4.40379	5.00875	5.62873
.0500	2.73264	3.32511	3.94030	4.57481	5.22603	5.89186	6.57063
0.1000	3.48954	4.16816	4.86518	5.57778	6.30380	7.04150	7.78953
.2000	4.59357	5.38005	6.17908	6.98867	7.80733	8.63386	9.46733
.3000	5.52742	6.39331	7.26722	8.14787	9.03428	9.92568	10.8215
.4000	6.42264	7.35703	8.29547	9.23729	10.1820	11.1291	12.0785
.5000	7.34412	8.34283	9.34182	10.3410	11.3403	12.3398	13.3393
.6000	8.35053	9.41364	10.4732	11.5298	12.5838	13.6356	14.6853
.7000	9.52446	10.6564	11.7807	12.8987	14.0111	15.1187	16.2221
.8000	11.0301	12.2421	13.4420	14.6314	15.8120	16.9848	18.1508
.9000	13.3616	14.6837	15.9872	17.2750	18.5493	19.8119	21.0641
0.9500	15.5073	16.9190	18.3070	19.6751	21.0261	22.3620	23.6848
.9750	17.5345	19.0228	20.4832	21.9201	23.3367	24.7356	26.1189
.9900	20.0902	21.6660	23.2093	24.7250	26.2170	27.6882	29.1412
.9950	21.9550	23.5893	25.1882	26.7568	28.2995	29.8195	31.3193
.9990	26.1245	27.8772	29.5883	31.2641	32.9095	34.5282	36.1233
.9995	27.8680	29.6658	31.4198	33.1366	34.8213	36.4778	38.1094
.9999	31.8276	33.7199	35.5640	37.3670	39.1344	40.8706	42.5793

Table 1 (cont.)

ν P	15	16	17	18	19	20	21	22
0.0001	2.40819	2.77394	3.15667	3.55516	3.96832	4.39516	4.83481	5.28646
.0005	3.10752	3.53581	3.98018	4.43939	4.91234	5.39807	5.89570	6.40447
.0010	3.48268	3.94163	4.41609	4.90485	5.40682	5.92104	6.44668	6.98297
.0050	4.60092	5.14221	5.69722	6.26480	6.84397	7.43384	8.03365	8.64272
.0100	5.22935	5.81221	6.40776	7.01491	7.63273	8.26040	8.89720	9.54249
.0250	6.26214	6.90766	7.56419	8.23075	8.90652	9.59078	10.2829	10.9823
.0500	7.26094	7.96165	8.67176	9.39045	10.1170	10.8508	11.5913	12.3380
0.1000	8.54876	9.31224	10.0852	10.8649	11.6509	12.4426	13.2396	14.0415
.2000	10.3070	11.1521	12.0023	12.8569	13.7158	14.5784	15.4446	16.3140
.3000	11.7212	12.6244	13.5307	14.4399	15.3517	16.2659	17.1823	18.1007
.4000	13.0297	13.9827	14.9373	15.8932	16.8504	17.8088	18.7683	19.7288
.5000	14.3389	15.3385	16.3382	17.3379	18.3376	19.3374	20.3372	21.3370
.6000	15.7332	16.7795	17.8244	18.8679	19.9102	20.9514	21.9915	23.0307
.7000	17.3217	18.4179	19.5110	20.6014	21.6891	22.7745	23.8578	24.9390
.8000	19.3107	20.4651	21.6146	22.7595	23.9004	25.0375	26.1711	27.3015
.9000	22.3071	23.5418	24.7690	25.9894	27.2036	28.4120	29.6151	30.8133
0.9500	24.9958	26.2962	27.5871	28.8693	30.1435	31.4104	32.6706	33.9244
.9750	27.4884	28.8453	30.1910	31.5264	32.8523	34.1696	35.4789	36.7807
.9900	30.5779	31.9999	33.4087	34.8053	36.1909	37.5662	38.9322	40.2894
.9950	32.8013	34.2672	35.7185	37.1565	38.5822	39.9968	41.4011	42.7956
.9990	37.6973	39.2524	40.7902	42.3124	43.8202	45.3147	46.7970	48.2679
.9995	39.7188	41.3081	42.8792	44.4338	45.9731	47.4984	49.0108	50.5111
.9999	44.2632	45.9249	47.5663	49.1894	50.7955	52.3860	53.9620	55.5246

ν P	23	24	25	26	27	28	29	30
0.0001	5.74941	6.22300	6.70663	7.19978	7.70193	8.21264	8.73149	9.25809
.0005	6.92367	7.45269	7.99096	8.53795	9.09320	9.65627	10.2268	10.8044
.0010	7.52924	8.08488	8.64934	9.22213	9.80278	10.3909	10.9861	11.5880
.0050	9.26042	9.88623	10.5197	11.1602	11.8076	12.4613	13.1211	13.7867
.0100	10.1957	10.8564	11.5240	12.1981	12.8785	13.5647	14.2565	14.9535
.0250	11.6886	12.4012	13.1197	13.8439	14.5734	15.3079	16.0471	16.7908
.0500	13.0905	13.8484	14.6114	15.3792	16.1514	16.9279	17.7084	18.4927
0.1000	14.8480	15.6587	16.4734	17.2919	18.1139	18.9392	19.7677	20.5992
.2000	17.1865	18.0618	18.9398	19.8202	20.7030	21.5880	22.4750	23.3641
.3000	19.0211	19.9432	20.8670	21.7924	22.7192	23.6475	24.5770	25.5078
.4000	20.6902	21.6525	22.6156	23.5794	24.5440	25.5092	26.4751	27.4416
.5000	22.3369	23.3367	24.3366	25.3365	26.3362	27.3362	28.3361	29.3360
.6000	24.0689	25.1063	26.1430	27.1789	28.2141	29.2486	30.2825	31.3159
.7000	26.0184	27.0960	28.1719	29.2463	30.3193	31.3909	32.4612	33.5302
.8000	28.4288	29.5533	30.6752	31.7946	32.9117	34.0266	35.1394	36.2502
.9000	32.0069	33.1962	34.3816	35.5632	36.7412	37.9159	39.0875	40.2560
0.9500	35.1725	36.4150	37.6525	38.8851	40.1133	41.3371	42.5570	43.7730
.9750	38.0756	39.3641	40.6465	41.9232	43.1945	44.4608	45.7223	46.9792
.9900	41.6384	42.9798	44.3141	45.6417	46.9630	48.2782	49.5879	50.8922
.9950	44.1813	45.5585	46.9279	48.2899	49.6449	50.9934	52.3356	53.6720
.9990	49.7282	51.1786	52.6179	54.0520	55.4760	56.8923	58.3012	59.7030
.9995	52.0002	53.4788	54.9475	56.4069	57.8576	59.3000	60.7346	62.1619
.9999	57.0746	58.6130	60.1403	61.6573	63.1645	64.6624	66.1517	67.6326

Table 1. *Percentage points of the chi-square distribution (cont.)*

ν P	31	32	33	34	35	36	37	38
0.0001	9.79209	10.3331	10.8810	11.4352	11.9957	12.5622	13.1343	13.7120
.0005	11.3887	11.9794	12.5763	13.1791	13.7875	14.4012	15.0202	15.6441
.0010	12.1963	12.8107	13.4309	14.0567	14.6878	15.3241	15.9653	16.6112
.0050	14.4578	15.1340	15.8153	16.5013	17.1918	17.8867	18.5858	19.2889
.0100	15.6555	16.3622	17.0735	17.7891	18.5089	19.2327	19.9602	20.6914
.0250	17.5387	18.2908	19.0467	19.8063	20.5694	21.3359	22.1056	22.8785
.0500	19.2806	20.0719	20.8665	21.6643	22.4650	23.2686	24.0749	24.8839
0.1000	21.4336	22.2706	23.1102	23.9523	24.7967	25.6433	26.4921	27.3429
.2000	24.2551	25.1478	26.0422	26.9383	27.8359	28.7350	29.6355	30.5373
.3000	26.4397	27.3728	28.3069	29.2421	30.1782	31.1152	32.0532	32.9919
.4000	28.4087	29.3763	30.3444	31.3130	32.2821	33.2517	34.2216	35.1920
.5000	30.3359	31.3359	32.3358	33.3357	34.3356	35.3356	36.3355	37.3354
.6000	32.3486	33.3809	34.4126	35.4438	36.4746	37.5049	38.5348	39.5644
.7000	34.5981	35.6649	36.7307	37.7954	38.8591	39.9220	40.9839	42.0451
.8000	37.3591	38.4663	39.5718	40.6757	41.7780	42.8788	43.9782	45.0763
.9000	41.4217	42.5848	43.7452	44.9032	46.0588	47.2122	48.3634	49.5126
0.9500	44.9853	46.1943	47.3999	48.6024	49.8018	50.9985	52.1923	53.3835
.9750	48.2319	49.4804	50.7251	51.9660	53.2033	54.4373	55.6680	56.8955
.9900	52.1914	53.4858	54.7755	56.0609	57.3421	58.6192	59.8925	61.1621
.9950	55.0027	56.3281	57.6485	58.9639	60.2748	61.5812	62.8823	64.1814
.9990	61.0983	62.4872	63.8701	65.2472	66.6188	67.9851	69.3465	70.7029
.9995	63.6820	64.9955	66.4025	67.8035	69.1986	70.5881	71.9722	73.3512
.9999	69.1057	70.5712	72.0296	73.4812	74.9262	76.3650	77.7977	79.2247

ν P	39	40	41	42	43	44	45	46
0.0001	14.2950	14.8831	15.4761	16.0738	16.6762	17.2829	17.8940	18.5093
.0005	16.2729	16.9062	17.5440	18.1861	18.8323	19.4825	20.1366	20.7945
.0010	17.2616	17.9164	18.5754	19.2385	19.9055	20.5763	21.2507	21.9287
.0050	19.9959	20.7065	21.4208	22.1385	22.8595	23.5837	24.3110	25.0413
.0100	21.4262	22.1643	22.9056	23.6501	24.3976	25.1480	25.9013	26.6572
.0250	23.6543	24.4330	25.2145	25.9987	26.7854	27.5746	28.3662	29.1601
.0500	25.6954	26.5093	27.3256	28.1440	28.9647	29.7875	30.6123	31.4390
0.1000	28.1958	29.0505	29.9071	30.7654	31.6254	32.4871	33.3504	34.2152
.2000	31.4405	32.3450	33.2506	34.1574	35.0653	35.9744	36.8844	37.7955
.3000	33.9315	34.8719	35.8131	36.7550	37.6975	38.6408	39.5847	40.5292
.4000	36.1628	37.1340	38.1055	39.0774	40.0496	41.0222	41.9950	42.9682
.5000	38.3354	39.3353	40.3353	41.3352	42.3352	43.3352	44.3351	45.3351
.6000	40.5935	41.6222	42.6506	43.6788	44.7063	45.7336	46.7607	47.7874
.7000	43.1054	44.1649	45.2236	46.2817	47.3390	48.3957	49.4517	50.5071
.8000	46.1730	47.2685	48.3628	49.4560	50.5480	51.6389	52.7288	53.8177
.9000	50.6598	51.8051	52.9485	54.0902	55.2302	56.3685	57.5053	58.6405
0.9500	54.5722	55.7585	56.9424	58.1240	59.3035	60.4809	61.6562	62.8296
.9750	58.1201	59.3417	60.5606	61.7767	62.9904	64.2014	65.4102	66.6165
.9900	62.4281	63.6907	64.9501	66.2062	67.4593	68.7095	69.9568	71.2014
.9950	65.4756	66.7660	68.0527	69.3360	70.6159	71.8925	73.1661	74.4365
.9990	72.0547	73.4019	74.7449	76.0838	77.4186	78.7495	80.0767	81.4003
.9995	74.7253	76.0946	77.4593	78.8197	80.1757	81.5277	82.8757	84.2198
.9999	80.6462	82.0623	83.4733	84.8793	86.2806	87.6773	89.0694	90.4574

Table 1 (cont.)

ν P	47	48	49	50	51	52	53	54	55
0.0001	19.1285	19.7517	20.3787	21.0093	21.6435	22.2812	22.9223	23.5666	24.2141
.0005	21.4559	22.1209	22.7893	23.4610	24.1359	24.8139	25.4949	26.1789	26.8658
.0010	22.6101	23.2949	23.9828	24.6739	25.3680	26.0651	26.7650	27.4677	28.1731
.0050	25.7746	26.5106	27.2493	27.9907	28.7347	29.4812	30.2300	30.9813	31.7348
.0100	27.4158	28.1770	28.9406	29.7067	30.4750	31.2457	32.0185	32.7934	33.5705
.0250	29.9562	30.7545	31.5549	32.3574	33.1618	33.9681	34.7763	35.5863	36.3981
.0500	32.2876	33.0981	33.9303	34.7643	35.5999	36.4371	37.2759	38.1162	38.9580
0.1000	35.0814	35.9491	36.8182	37.6886	38.5604	39.4334	40.3076	41.1830	42.0596
.2000	38.7075	39.6205	40.5344	41.4492	42.3649	43.2814	44.1987	45.1167	46.0356
.3000	41.4744	42.4201	43.3664	44.3133	45.2607	46.2086	47.1571	48.1060	49.0554
.4000	43.9417	44.9154	45.8895	46.8638	47.8383	48.8132	49.7882	50.7635	51.7391
.5000	46.3350	47.3350	48.3350	49.3349	50.3349	51.3349	52.3348	53.3348	54.3348
.6000	48.8139	49.8401	50.8660	51.8916	52.9170	53.9421	54.9670	55.9916	57.0160
.7000	51.5619	52.6161	53.6697	54.7228	55.7753	56.8274	57.8789	58.9299	59.9805
.8000	54.9056	55.9926	57.0786	58.1638	59.2481	60.3316	61.4142	62.4961	63.5772
.9000	59.7743	60.9066	62.0375	63.1671	64.2954	65.4224	66.5482	67.6728	68.7962
0.9500	64.0011	65.1708	66.3387	67.5048	68.6693	69.8321	70.9935	72.1532	73.3115
.9750	67.8207	69.0226	70.2224	71.4202	72.6160	73.8098	75.0019	76.1920	77.3805
.9900	72.4433	73.8826	74.9195	76.1539	77.3860	78.6157	79.8433	81.0688	82.2921
.9950	75.7041	76.9688	78.2307	79.4900	80.7467	82.0008	83.2525	84.5019	85.7489
.9990	82.7204	84.0371	85.3506	86.6608	87.9679	89.2721	90.5734	91.8718	93.1675
.9995	85.5603	86.8971	88.2305	89.5605	90.8872	92.2108	93.5312	94.8487	96.1632
.9999	91.8412	93.2209	94.5967	95.9688	97.3371	98.7019	100.063	101.421	102.776

ν P	56	57	58	59	60	61	62	63	64
0.0001	24.8648	25.5185	26.1751	26.8346	27.4969	28.1619	28.8297	29.5000	30.1729
.0005	27.5554	28.2478	28.9428	29.6404	30.3405	31.0430	31.7480	32.4553	33.1648
.0010	28.8812	29.5918	30.3049	31.0204	31.7383	32.4586	33.1811	33.9058	34.6326
.0050	32.4905	33.2484	34.0084	34.7704	35.5345	36.3005	37.0684	37.8382	38.6098
.0100	34.3495	35.1305	35.9135	36.6982	37.4849	38.2732	39.0633	39.8551	40.6486
.0250	37.2116	38.0267	38.8435	39.6619	40.4817	41.3031	42.1260	42.9503	43.7760
.0500	39.8013	40.6459	41.4920	42.3393	43.1880	44.0379	44.8890	45.7414	46.5949
0.1000	42.9373	43.8162	44.6960	45.5770	46.4589	47.3418	48.2257	49.1105	49.9963
.2000	46.9552	47.8755	48.7965	49.7182	50.6406	51.5637	52.4873	53.4116	54.3365
.3000	50.0053	50.9556	51.9063	52.8575	53.8091	54.7611	55.7135	56.6663	57.6195
.4000	52.7148	53.6908	54.6670	55.6434	56.6200	57.5968	58.5738	59.5510	60.5283
.5000	55.3348	56.3347	57.3347	58.3347	59.3347	60.3346	61.3346	62.3346	63.3346
.6000	58.0402	59.0642	60.0879	61.1115	62.1348	63.1580	64.1810	65.2037	66.2263
.7000	61.0305	62.0802	63.1294	64.1782	65.2265	66.2745	67.3220	68.3692	69.4160
.8000	64.6576	65.7373	66.8162	67.8945	68.9721	70.0490	71.1253	72.2010	73.2761
.9000	69.9185	71.0397	72.1598	73.2789	74.3970	75.5141	76.6302	77.7454	78.8597
0.9500	74.4683	75.6238	76.7778	77.9305	79.0819	80.2321	81.3810	82.5287	83.6753
.9750	78.5671	79.7522	80.9356	82.1174	83.2977	84.4764	85.6537	86.8296	88.0040
.9900	83.5134	84.7328	85.9502	87.1657	88.3794	89.5913	90.8015	92.0100	93.2168
.9950	86.9937	88.2363	89.4768	90.7153	91.9517	93.1861	94.4186	95.6493	96.8781
.9990	94.4605	95.7510	97.0388	98.3242	99.6073	100.888	102.166	103.442	104.716
.9995	97.4749	98.7838	100.090	101.394	102.695	103.993	105.289	106.583	107.875
.9999	104.127	105.476	106.821	108.163	109.503	110.840	112.174	113.505	114.834

Table 1. *Percentage points of the chi-square distribution (cont.)*

ν P	65	66	67	68	69	70	71	72	73
0.0001	30.8483	31.5261	32.2063	32.8888	33.5736	34.2607	34.9499	35.6413	36.3348
.0005	33.8787	34.5906	35.3068	36.0250	36.7452	37.4674	38.1916	38.9177	39.6457
.0010	35.3616	36.0926	36.8257	37.5606	38.2976	39.0364	39.7770	40.5195	41.2637
.0050	39.3831	40.1582	40.9350	41.7135	42.4935	43.2752	44.0584	44.8431	45.6293
.0100	41.4436	42.2402	43.0384	43.8380	44.6392	45.4417	46.2457	47.0510	47.8577
.0250	44.6030	45.4314	46.2610	47.0920	47.9242	48.7576	49.5922	50.4279	51.2648
.0500	47.4496	48.3054	49.1623	50.0202	50.8792	51.7393	52.6003	53.4623	54.3253
0.1000	50.8829	51.7705	52.6588	53.5480	54.4381	55.3289	56.2205	57.1129	58.0061
.2000	55.2620	56.1881	57.1147	58.0419	58.9696	59.8978	60.8266	61.7558	62.6856
.3000	58.5731	59.5270	60.4812	61.4358	62.3908	63.3460	64.3016	65.2575	66.2137
.4000	61.5059	62.4836	63.4615	64.4395	65.4177	66.3961	67.3746	68.3533	69.3322
.5000	64.3346	65.3345	66.3345	67.3345	68.3345	69.3345	70.3345	71.3344	72.3344
.6000	67.2488	68.2710	69.2931	70.3150	71.3368	72.3583	73.3798	74.4011	75.4222
.7000	70.4624	71.5085	72.5542	73.5995	74.6446	75.6893	76.7337	77.7777	78.8215
.8000	74.3506	75.4245	76.4979	77.5707	78.6429	79.7147	80.7859	81.8566	82.9268
.9000	79.9730	81.0855	82.1971	83.3079	84.4179	85.5270	86.6354	87.7431	88.8499
0.9500	84.8206	85.9649	87.1081	88.2502	89.3912	90.5312	91.6702	92.8083	93.9453
.9750	89.1771	90.3489	91.5194	92.6885	93.8565	95.0232	96.1887	97.3531	98.5162
.9900	94.4221	95.6257	96.8278	98.0284	99.2275	100.425	101.621	102.816	104.010
.9950	98.1052	99.3304	100.554	101.776	102.996	104.215	105.432	106.648	107.862
.9990	105.988	107.258	108.526	109.791	111.055	112.317	113.577	114.835	116.092
.9995	109.164	110.451	111.736	113.018	114.299	115.578	116.854	118.129	119.402
.9999	116.160	117.484	118.805	120.124	121.440	122.755	124.067	125.377	126.684

ν P	74	75	76	77	78	79	80	81	82
0.0001	37.0303	37.7279	38.4274	39.1288	39.8322	40.5374	41.2445	41.9533	42.6639
.0005	40.3755	41.1072	41.8405	42.5757	43.3125	44.0509	44.7910	45.5328	46.2760
.0010	42.0097	42.7573	43.5066	44.2576	45.0101	45.7642	46.5199	47.2770	48.0357
.0050	46.4170	47.2060	47.9965	48.7884	49.5816	50.3761	51.1719	51.9690	52.7673
.0100	48.6657	49.4750	50.2856	51.0974	51.9104	52.7247	53.5401	54.3566	55.1743
.0250	52.1028	52.9419	53.7821	54.6234	55.4656	56.3089	57.1532	57.9984	58.8446
.0500	55.1892	56.0541	56.9198	57.7864	58.6539	59.5223	60.3915	61.2615	62.1323
0.1000	58.9000	59.7946	60.6898	61.5858	62.4825	63.3799	64.2778	65.1765	66.0757
.2000	63.6158	64.5466	65.4777	66.4094	67.3415	68.2740	69.2069	70.1403	71.0741
.3000	67.1702	68.1271	69.0841	70.0415	70.9992	71.9571	72.9153	73.8738	74.8325
.4000	70.3111	71.2903	72.2695	73.2489	74.2285	75.2081	76.1879	77.1679	78.1479
.5000	73.3344	74.3344	75.3344	76.3344	77.3344	78.3343	79.3343	80.3343	81.3343
.6000	76.4432	77.4640	78.4848	79.5053	80.5258	81.5461	82.5663	83.5863	84.6062
.7000	79.8650	80.9081	81.9510	82.9936	84.0359	85.0779	86.1197	87.1612	88.2025
.8000	82.9965	85.0658	86.1346	87.2030	88.2709	89.3383	90.4054	91.4720	92.5382
.9000	89.9561	91.0615	92.1662	93.2702	94.3735	95.4761	96.5782	97.6796	98.7803
0.9500	95.0815	96.2167	97.3510	98.4844	99.6169	100.749	101.879	103.010	104.139
.9750	99.6784	100.839	101.999	103.158	104.316	105.473	106.629	107.783	108.937
.9900	105.202	106.393	107.582	108.771	109.958	111.144	112.329	113.512	114.695
.9950	109.074	110.286	111.495	112.704	113.911	115.117	116.321	117.524	118.726
.9990	117.346	118.599	119.850	121.100	122.348	123.594	124.839	126.082	127.324
.9995	120.673	121.942	123.209	124.475	125.739	127.001	128.261	129.520	130.778
.9999	127.990	129.294	130.595	131.895	133.193	134.489	135.783	137.075	138.365

Table 1 (cont.)

ν P	83	84	85	86	87	88	89	90	91
0.0001	43.3763	44.0903	44.8060	45.5234	46.2424	46.9629	47.6851	48.4087	49.1339
.0005	47.0209	47.7672	48.5151	49.2644	50.0151	50.7673	51.5209	52.2758	53.0321
.0010	48.7958	49.5573	50.3203	51.0846	51.8503	52.6173	53.3856	54.1552	54.9261
.0050	53.5669	54.3677	55.1696	55.9727	56.7769	57.5823	58.3888	59.1963	60.0049
.0100	55.9931	56.8130	57.6339	58.4559	59.2790	60.1030	60.9281	61.7541	62.5811
.0250	59.6918	60.5398	61.3888	62.2386	63.0894	63.9409	64.7934	65.6466	66.5007
.0500	63.0039	63.8763	64.7494	65.6233	66.4979	67.3732	68.2493	69.1260	70.0035
0.1000	66.9756	67.8761	68.7772	69.6788	70.5810	71.4838	72.3872	73.2911	74.1955
.2000	72.0083	72.9429	73.8779	74.8132	75.7490	76.6851	77.6216	78.5584	79.4956
.3000	75.7915	76.7507	77.7102	78.6699	79.6299	80.5901	81.5505	82.5111	83.4719
.4000	79.1281	80.1084	81.0888	82.0693	83.0500	84.0307	85.0116	85.9925	86.9736
.5000	82.3343	83.3343	84.3343	85.3343	86.3342	87.3342	88.3342	89.3342	90.3342
.6000	85.6260	86.6457	87.6653	88.6847	89.7041	90.7233	91.7424	92.7614	93.7803
.7000	89.2435	90.2842	91.3247	92.3650	93.4050	94.4448	95.4844	96.5238	97.5629
.8000	93.6040	94.6693	95.7343	96.7990	97.8632	98.9271	99.9906	101.054	102.117
.9000	99.8805	100.980	102.079	103.177	104.275	105.372	106.469	107.565	108.661
0.9500	105.267	106.395	107.522	108.648	109.773	110.898	112.022	113.145	114.268
.9750	110.090	111.242	112.393	113.544	114.693	115.841	116.989	118.136	119.282
.9900	115.876	117.057	118.236	119.414	120.591	121.767	122.942	124.116	125.289
.9950	119.927	121.126	122.325	123.522	124.718	125.913	127.106	128.299	129.491
.9990	128.565	129.804	131.041	132.277	133.512	134.745	135.978	137.208	138.438
.9995	132.033	133.288	134.540	135.792	137.041	138.290	139.537	140.782	142.027
.9999	139.654	140.941	142.226	143.509	144.791	146.071	147.350	148.627	149.903

ν P	92	93	94	95	96	97	98	99	100
0.0001	49.8606	50.5887	51.3182	52.0492	52.7816	53.5153	54.2504	54.9869	55.7246
.0005	53.7896	54.5485	55.3087	56.0702	56.8329	57.5968	58.3619	59.1282	59.8957
.0010	55.6983	56.4716	57.2462	58.0219	58.7989	59.5770	60.3562	61.1365	61.9179
.0050	60.8146	61.6253	62.4370	63.2496	64.0633	64.8780	65.6936	66.5101	67.3276
.0100	63.4090	64.2379	65.0677	65.8984	66.7299	67.5624	68.3957	69.2290	70.0649
.0250	67.3556	68.2113	69.0677	69.9249	70.7828	71.6415	72.5009	73.3611	74.2219
.0500	70.8816	71.7603	72.6398	73.5198	74.4005	75.2819	76.1638	77.0463	77.9295
0.1000	75.1005	76.0060	76.9120	77.8184	78.7254	79.6329	80.5408	81.4492	82.3581
.2000	80.4332	81.3711	82.3093	83.2478	84.1867	85.1259	86.0654	87.0052	87.9453
.3000	84.4330	85.3943	86.3558	87.3175	88.2794	89.2415	90.2038	91.1663	92.1289
.4000	87.9548	88.9361	89.9175	90.8990	91.8806	92.8622	93.8440	94.8259	95.8078
.5000	91.3342	92.3342	93.3342	94.3342	95.3342	96.3341	97.3341	98.3341	99.3341
.6000	94.7991	95.8178	96.8364	97.8549	98.8733	99.8916	100.910	101.928	102.946
.7000	98.6018	99.6405	100.679	101.717	102.755	103.793	104.831	105.868	106.906
.8000	103.179	104.241	105.303	106.364	107.425	108.486	109.547	110.607	111.667
.9000	109.756	110.850	111.944	113.038	114.131	115.223	116.315	117.407	118.498
0.9500	115.390	116.511	117.632	118.752	119.871	120.990	122.108	123.225	124.342
.9750	120.427	121.571	122.715	123.858	125.000	126.141	127.282	128.422	129.561
.9900	126.462	127.633	128.803	129.973	131.141	132.309	133.476	134.642	135.807
.9950	130.681	131.871	133.059	134.247	135.433	136.619	137.803	138.987	140.169
.9990	139.666	140.893	142.119	143.344	144.567	145.789	147.010	148.230	149.449
.9995	143.269	144.511	145.751	146.990	148.228	149.465	150.700	151.934	153.167
.9999	151.177	152.450	153.721	154.991	156.259	157.526	158.791	160.056	161.319

