1. Show that $\frac{d}{dt}(R)R^{-1}$ is skew-symmetric:

$$RR^T = I$$

$$\frac{d}{dt}(RR^T) = \frac{d}{dt}(I) = 0$$

$$\frac{d}{dt}(R)(R^T) + (R)\frac{d}{dt}(R^T) = 0$$

$$\dot{R}R^T = -R\dot{R}^T$$

$$= -(\dot{R}R^T)^T$$

$\hookrightarrow$ $R^T = R^{-1}$ for $SO(n)$

$$\frac{d}{dt}(R)R^{-1} = -\left(\frac{d}{dt}(R)R^{-1}\right)^T$$

$$A = -A^T \quad \checkmark$$

fulfilling the condition for a matrix to be skew-symmetric.

2. Show that $\underline{\hat{\omega}}\,\underline{r_B} = -\underline{\hat{r}_B}\,\underline{\omega}$:

let $\omega \in \mathbb{R}^3 = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$, $\quad \hat{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$

$r_B \in \mathbb{R}^3 = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$, $\quad \hat{r}_B = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$

$$\hat{\omega} r_B = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} 0 \quad -r_y \omega_z + r_z \omega_y \\ r_x \omega_z + 0 \quad -r_z \omega_x \\ -r_x \omega_y + r_y \omega_x + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -r_y \omega_z + r_z \omega_y \\ r_x \omega_z - r_z \omega_x \\ -r_x \omega_y + r_y \omega_x \end{bmatrix}$$

continued →

2.

$$\hat{r}_y w = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 + -w_y r_z + w_z r_y \\ w_x r_z + 0 + -w_z r_x \\ -w_x r_y + w_y r_x + 0 \end{bmatrix}$$

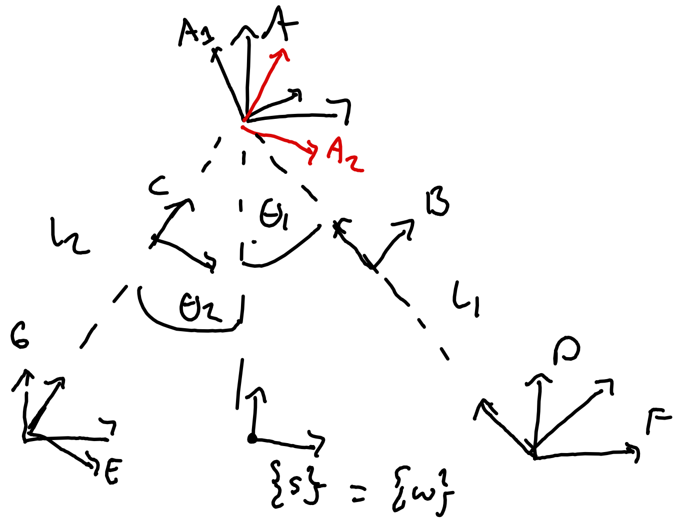
$$= \begin{bmatrix} r_y w_z - r_z w_y \\ -r_x w_z + r_z w_x \\ r_x w_y - w_x r_y \end{bmatrix}$$

$$= - \begin{bmatrix} -r_y w_z + r_z w_y \\ r_x w_z - r_z w_x \\ -r_x w_y + r_y w_x \end{bmatrix}$$

Therefore

$$\boxed{\hat{w} r_y = -\hat{r}_y w.}$$

3. Additional frames I defined for the biped:



- A_1 : a rotation of A in the direction of leg 1. $G_{AA1} = SE3(SO3\_planar(\theta_1), [0,0])$
  $G_{A1B} = SE3(I, [0, -\frac{1}{2}L_1])$
- A_2 : a rotation of A in the direction of leg 2. $G_{AA2} = SE3(SO3\_planar(\theta_2), [0,0])$
  $G_{A2B} = SE3(I, [0, -\frac{1}{2}L_2])$
- F : a rotation of D by $-\theta_1$, used for calculating the constraint EQs.
  $G_{DF} = SE3(SO3\_planar(-\theta_1), [0,0])$
- G : a rotation of E by $-\theta_2$, used for calculating the constraint EQs.
  $G_{EG} = SE3(SO3\_planar(-\theta_2), [0,0])$
- S = space frame = world frame = W.