ME314 HW0 Scan

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1. Derivative and directional derivative of $f(x) = \sin(x)$:

a) $f(x) = \sin(x)$

$Df(x) = \frac{d}{dx}(\sin(x))$

$\boxed{Df(x) = \cos(x)}$ $\longrightarrow$ derivative

b) Directional derivative:

$f(x) = \sin(x)$

$f(x + \varepsilon N) = \sin(x + \varepsilon N)$

$\frac{d}{d\varepsilon} f(x + \varepsilon N) = \frac{d}{dx}(\sin(x + \varepsilon N)) \cdot \frac{d}{d\varepsilon}(x + \varepsilon N)$

$= N\cos(x + \varepsilon N)$

$Df(x) \cdot N = \frac{d}{d\varepsilon} f(x + \varepsilon N)\Big|_{\varepsilon = 0}$

$= N\cos(x + \varepsilon N)\Big|_{\varepsilon = 0}$

$\boxed{Df(x) \cdot N = N\cos(x)}$ $\longrightarrow$ directional derivative

2. ~~Function of a trajectory:~~ $\cancel{J(x(t)) = \int_0^{\pi/2} \frac{1}{2} x(t)^2 \, dt}$

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2. Function of a trajectory: $J(x(t)) = \int_0^{\pi/2} \frac{1}{2}x(t)^2 dt$

Given $x(t) = \cos(t)$, $J(x(t)) = \int_0^{\pi/2} \frac{1}{2}\cos^2(t)dt$;
solve for $J(t)$.

$\cos^2(t)$ is related to $\cos(2t)$ via this double-angle formula:

$$\cos(2t) = \cos^2(t) - \sin^2(t)$$
$$= 2\cos^2(t) - 1$$
$$\cos^2(t) = \frac{\cos(2t)+1}{2}$$

Analytical solution of $J(x(t))$:

$$J(\cos(t)) = \int_0^{\pi/2} \frac{1}{2}\cos^2(t)dt$$
$$= \int_0^{\pi/2} \frac{1}{4}(\cos(2t)+1)dt$$
$$= \frac{1}{4}\left[\frac{1}{2}\sin(2t)+t\right]\Big|_0^{\pi/2}$$
$$= \frac{1}{4}\left[\cancel{\frac{1}{2}\sin(\pi)}^{0} + \frac{\pi}{2} - \cancel{\frac{1}{2}\sin(0)}^{0} - 0\right]$$

$$\boxed{J(\cos(t)) = \frac{\pi}{8}} \approx 0.392699$$

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3. Directional derivative of $J(x(t))$ at $x(t) = \cos(t)$ in the direction $n(t) = \sin(t)$:

$$J(x(t)) = \int_0^{\pi/2} \frac{1}{2} x(t)^2 \, dt$$

$$J(\cos(t)) = \int_0^{\pi/2} \frac{1}{2} \cos^2(t) \, dt$$

For $J(x + \varepsilon n)$, modify $x(t) = \cos(t)$ to give $x(t) + \varepsilon n(t) = \cos(t) + \varepsilon \sin(t)$

$$J(x(t) + \varepsilon n(t)) = \int_0^{\pi/2} \frac{1}{2} \left(\cos(t) + \varepsilon \sin(t)\right)^2 dt$$

$$\frac{d}{d\varepsilon} J(x(t) + \varepsilon n(t)) \Big|_{\varepsilon=0} = \frac{d}{d\varepsilon} \int_0^{\pi/2} \frac{1}{2} \left(\cos(t) + \varepsilon \sin(t)\right)^2 dt \Big|_{\varepsilon=0}$$

differentiation inside the integral

$$= \int_0^{\pi/2} \frac{d}{d\varepsilon} \frac{1}{2} \left(\cos(t) + \varepsilon \sin(t)\right)^2 \Big|_{\varepsilon=0} dt$$

$$= \int_0^{\pi/2} \frac{d}{dt} \left(\frac{1}{2} (\cos(t) + \varepsilon \sin(t))^2 \right) \cdot \frac{d}{d\varepsilon} \left(\cos(t) + \varepsilon \sin(t)\right) \Big|_{\varepsilon=0} dt$$

$$= \int_0^{\pi/2} \left[(\cos(t) + \varepsilon \sin(t)) \cdot \sin(t) \right]_{\varepsilon=0} dt$$

$$= \int_0^{\pi/2} \cos(t) \sin(t) \, dt$$

$$= \frac{1}{2} \sin^2(t) \Big|_0^{\pi/2}$$

$$= \frac{1}{2} \left[\sin^2(\pi/2) - \cancelto{0}{\sin^2(0)} \right]$$

$$\boxed{= \frac{1}{2}}$$