

▼ ME314 Homework 7

Submission instructions

Deliverables that should be included with your submission are shown in **bold** at the end of each problem statement and the corresponding supplemental material. **Your homework will be graded IFF you submit a single PDF, .mp4 videos of animations when requested and a link to a Google colab file that meet all the requirements outlined below.**

- List the names of students you've collaborated with on this homework assignment.
- Include all of your code (and handwritten solutions when applicable) used to complete the problems.
- Highlight your answers (i.e. **bold** and outline the answers) and include simplified code outputs (e.g. `.simplify()`).
- Enable Google Colab permission for viewing
 - Click Share in the upper right corner
 - Under "Get Link" click "Share with..." or "Change"
 - Then make sure it says "Anyone with Link" and "Editor" under the dropdown menu
- Make sure all cells are run before submitting (i.e. check the permission by running your code in a private mode)
 - Please don't make changes to your file after submitting, so we can grade it!
- Submit a link to your Google Colab file that has been run (before the submission deadline) and don't edit it afterwards!

NOTE: This Jupyter Notebook file serves as a template for you to start homework. Make sure you first copy this template to your own Google driver (click "File" -> "Save a copy in Drive"), and then start to edit it.

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1 #####
2 # If you're using Google Colab, uncomment this section by selecting the whole section
3 # ctrl+'/' on your and keyboard. Run it before you start programming, this will enable
4 # LaTeX "display()" function for you. If you're using the local Jupyter environment
5 #####
6 import sympy as sym
7 def custom_latex_printer(exp,**options):
8     from google.colab.output._publish import javascript
9     url = "https://cdn.jsdelivr.net/npm/mathjax@3.1.1/latest.js?config=TeX-AMS-MML_HTMLorMML"
10     javascript(url=url)
11     return sym.printing.latex(exp,**options)
12 sym.init_printing(use_latex="mathjax",latex_printer=custom_latex_printer)

```

▼ Problem 1 (20pts)

Show that if $R \in SO(n)$, then the matrix $A = \frac{d}{dt}(R)R^{-1}$ is skew symmetric.

Turn in: A scanned (or photograph from your phone or webcam) copy of your hand written solution. Or you can use \LaTeX.

```
1 # handwritten solution attached to end of pdf
```

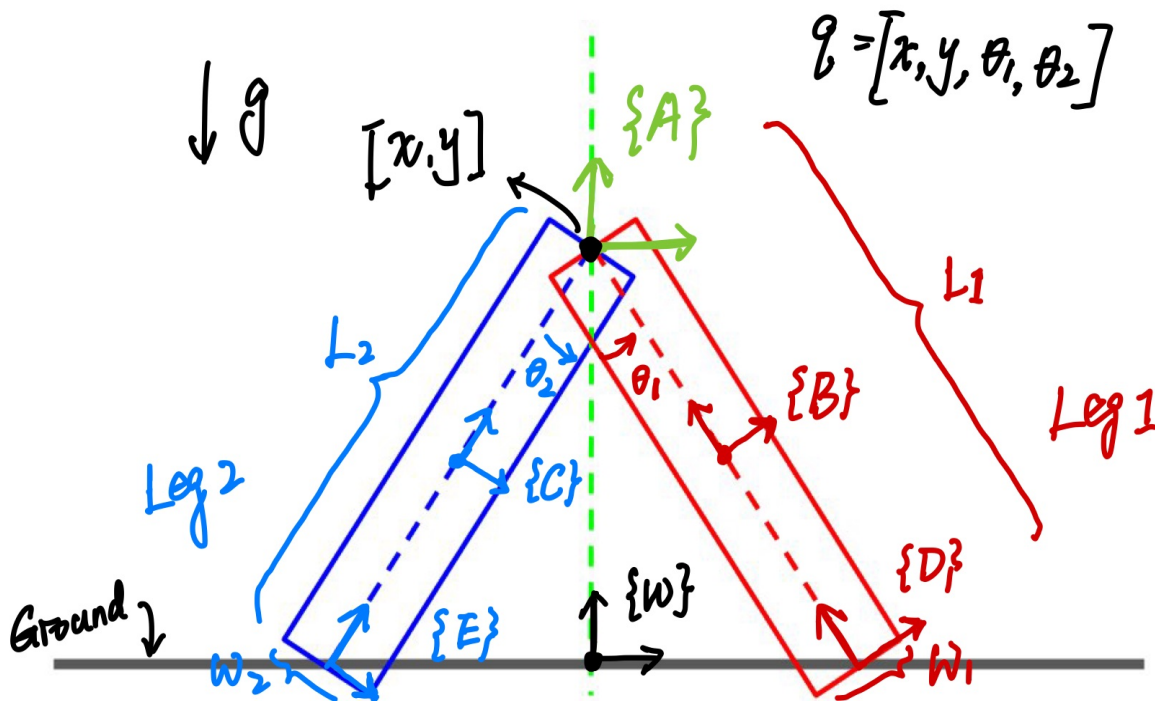
▼ Problem 2 (20pts)

Show that $\widehat{\underline{\omega}} \underline{r}_b = -\underline{\hat{r}}_b \underline{\omega}$.

Turn in: A scanned (or photograph from your phone or webcam) copy of your hand written solution. Or you can use \LaTeX.

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1 # handwritten solution attached to end of pdf
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```
1 from IPython.core.display import HTML
2 display(HTML("<table><tr><td><img src='https://github.com/MuchenSun/ME314pngs/raw/r
```



▼ Problem 3 (60pts)

Consider a person doing the splits (shown in the image above). To simplify the model, we ignore the upper body and assume the knees can not bend --- which means we only need four variables $q = [x, y, \theta_1, \theta_2]$ to configure the system. x and y are the position of the intersection point of the two legs, θ_1 and θ_2 are the angles between the legs and the green vertical dash line. The feet are constrained on the ground, and there is no friction between the feet and the ground.

Each leg is a rigid body with length $L = 1$, width $W = 0.2$, mass $m = 1$, and rotational inertia $J = 1$ (assuming the center of mass is at the center of geometry). Moreover, there are two torques applied on θ_1 and θ_2 to control the legs to track a desired trajectory:

$$\theta_1^d(t) = \frac{\pi}{15} + \frac{\pi}{3} \sin^2\left(\frac{t}{2}\right)$$

$$\theta_2^d(t) = -\frac{\pi}{15} - \frac{\pi}{3} \sin^2\left(\frac{t}{2}\right)$$

and the torques are:

$$F_{\theta_1} = -k_1(\theta_1 - \theta_1^d)$$

$$F_{\theta_2} = -k_1(\theta_2 - \theta_2^d)$$

In this problem we use $k_1 = 20$.

Given the model description above, define the frames that you need (several example frames are shown in the image as well), simulate the motion of the biped from rest for $t \in [0, 10]$, $dt = 0.01$, with initial condition $q = [0, L_1 \cos(\frac{\pi}{15}), \frac{\pi}{15}, -\frac{\pi}{15}]$. You will need to modify the animation function to display the leg as a rectangle, an example of the animation can be found at

<https://youtu.be/w8XHYrEoWTc>.

Hint 1: Even though this is a 2D system, in order to compute kinetic energy from both translation and rotation you will need to model the system in the 3D world --- the z coordinate is always zero and the rotation is around the z axis (based on these facts, what should the $SE(3)$ matrix and rotational inertia tensor look like?). This also means you need to represent transformations in $SE(3)$ and the body velocity $\mathcal{V}_b \in \mathbb{R}^6$.

Hint 2: It could be helpful to define several helper functions for all the matrix operations you will need to use. For example, a function that returns $SE(3)$ matrices given a rotation angle and 2D translation vector, functions for "hat" and "unhat" operations, a function for the matrix inverse of $SE(3)$ (which is definitely not the same as the SymPy matrix inverse function), and a function that returns the time derivative of $SO(3)$ or $SE(3)$.

Hint 3: In this problem the external force depends on time t . Therefore, in order to solve for the symbolic solution you need to substitute your configuration variables, which are defined as symbolic functions of time t (such as $\theta_1(t)$ and $\frac{d}{dt}\theta_1(t)$), with dummy symbolic variables. For the same reason (the dynamics now explicitly depend on time), you will need to do some tiny modifications on the "integrate" and "simulate" functions, a good reference can be found at

https://en.wikipedia.org/wiki/Runge-Kutta_methods.

Hint 4: Symbolically solving this system should be fast, but if you encountered some problem when solving the dynamics symbolically, an alternative method is to solve the system numerically --- substitute in the system state at each time step during simulation and solve for the numerical solution --- but based on my experience, this would cost more than one hour for 500 time steps, so it's not recommended.

Hint 5: The animation of this problem is similar to the one in last homework --- the coordinates of the vertices in the body frame are constant, you just need to transfer them back to the world frame using the the transformation matrices you already have in the simulation.

Hint 6: Be careful to consider the relationship between the frames and to not build in any implicit assumptions (such as assuming some variables are fixed).

Hint 7: The rotation, by convention, is assumed to follow the right hand rule, which means the z-axis is out of the screen and the positive rotation orientation is counter-clockwise. Make sure you follow a consistent set of positive directions for all the computation.

Hint 8: This problem is designed as a "mini-project", it could help you estimate the complexity of your final project, and you could adjust your proposal based on your experience with this problem.

Turn in: A copy of the code used to simulate and animate the system. Also, include a plot of the trajectory and upload a video of the animation separately through Canvas. The video should be in ".mp4" format, you can use screen capture or record the screen directly with your phone.

```
1 ## Your code goes here
```

```
1 import numpy as np
2 import sympy as sym
3 from sympy import Function, symbols, lambdify, solve, Eq, Matrix
4 import matplotlib.pyplot as plt
5
6
7 def integrate(f, xt, dt, tt):
8     ## slightly modified as referenced in hint 3
9     """
10     This function takes in an initial condition x(t) and a timestep dt,
11     as well as a dynamical system f(x) that outputs a vector of the
12     same dimension as x(t). It outputs a vector x(t+dt) at the future
13     time step.
14
15     Parameters
16     =====
17     dyn: Python function
18         derivate of the system at a given step x(t),
19         it can considered as  $\dot{x}(t) = \text{func}(x(t))$ 
20     xt: NumPy array
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21         current step x(t)
22     dt:
23         step size for integration
24
25     Return
26     =====
27     new_xt:
28         value of x(t+dt) integrated from x(t)
29     """
30     k1 = dt * f(xt, tt)
31     k2 = dt * f(xt+k1/2., tt+dt/2.)
32     k3 = dt * f(xt+k2/2., tt+dt/2.)
33     k4 = dt * f(xt+k3, tt+dt)
34     new_xt = xt + (1/6.) * (k1+2.0*k2+2.0*k3+k4)
35     return new_xt
36
37 def simulate(f, x0, tspan, dt, integrate):
38     ## slightly modified as referenced in hint 3
39
40     """
41     This function takes in an initial condition x0, a timestep dt,
42     a time span tspan consisting of a list [min_time, max_time],
43     as well as a dynamical system f(x) that outputs a vector of the
44     same dimension as x0. It outputs a full trajectory simulated
45     over the time span of dimensions (xvec_size, time_vec_size).
46
47     Parameters
48     =====
49     f: Python function
50         derivate of the system at a given step x(t),
51         it can considered as  $\dot{x}(t) = \text{func}(x(t))$ 
52     x0: NumPy array
53         initial conditions
54     tspan: Python list
55         tspan = [min_time, max_time], it defines the start and end
56         time of simulation
57     dt:
58         time step for numerical integration
59     integrate: Python function
60         numerical integration method used in this simulation
61
62     Return
63     =====
64     x_traj:
65         simulated trajectory of x(t) from t=0 to tf
66     """
67     N = int((max(tspan)-min(tspan))/dt)
68     x = np.copy(x0)
69     tvec = np.linspace(min(tspan),max(tspan),N)
70     xtraj = np.zeros((len(x0),N))
71     for i in range(N):

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72     xtraj[:,i]=integrate(f,[*x],dt, tvec[i])
73     x = np.copy(xtraj[:,i])
74     return xtraj

```

```

1 ### helper functions ###
2
3 # helper functions for getting SE3 with given theta, x, y(hint 2)
4 def SE3_sym(theta, x, y):
5     SE3_sym = sym.Matrix([[sym.cos(theta), -sym.sin(theta), 0.0, x],
6                           [sym.sin(theta), sym.cos(theta), 0.0, y],
7                           [0.0, 0.0, 1.0, 0.0],
8                           [0.0, 0.0, 0.0, 1.0]])
9     return SE3_sym
10
11 def SE3_np(theta, x, y):
12     SE3_np = sym.Matrix([[np.cos(theta), -np.sin(theta), 0.0, x],
13                          [np.sin(theta), np.cos(theta), 0.0, y],
14                          [0.0, 0.0, 1.0, 0.0],
15                          [0.0, 0.0, 0.0, 1.0]])
16     return SE3_np
17
18 # helper function for getting the inverse of the input matrix (hint 2)
19 def inverse_matrix(mat):
20     R = sym.Matrix([[mat[0,0], mat[0,1], mat[0,2]],
21                    [mat[1,0], mat[1,1], mat[1,2]],
22                    [mat[2,0], mat[2,1], mat[2,2]]])
23
24     p = sym.Matrix([mat[0,3], mat[1,3], mat[2,3]])
25
26     R_inverse = R.T
27     p_inverse = -R_inverse * p
28
29     inverse_mat = sym.Matrix([[R_inverse[0,0], R_inverse[0,1], R_inverse[0,2], p_inve
30                              [R_inverse[1,0], R_inverse[1,1], R_inverse[1,2], p_inve
31                              [R_inverse[2,0], R_inverse[2,1], R_inverse[2,2], p_inve
32                              [0.0, 0.0, 0.0, 1.0]])
33
34     return inverse_mat
35
36
37 # helper function for unhatting input matrix (hint 2)
38 def unhat(mat):
39     unhat_vec = sym.Matrix([mat[0,3], mat[1,3], mat[2,3], mat[2,1], mat[0,2], mat[1,0]
40     return unhat_vec
41
42
43

```

```

1 ## constants ##

```

```

2 L1 = 1
3 L2 = 1
4 W1 = 0.2
5 W2 = 0.2
6 m1 = 1
7 m2 = 1
8 J1 = 1
9 J2 = 1
10 g = 9.8
11 k1 = 20
12
13
14 ## variables ##
15 t = sym.symbols('t')
16 lam1 = sym.symbols(r'\lambda_1')
17 lam2 = sym.symbols(r'\lambda_2')
18 lam = sym.Matrix([lam1, lam2])
19
20 x = sym.Function(r'x')(t)
21 y = sym.Function(r'y')(t)
22
23 theta1 = sym.Function(r'\theta_1')(t)
24 theta2 = sym.Function(r'\theta_2')(t)
25
26 q = sym.Matrix([x, y, theta1, theta2])
27 qdot = q.diff(t)
28 qddot = qdot.diff(t)
29
30
31

1 ## transformations ##
2 ## referencing lecture
3 g_wa = SE3_sym(0, q[0], q[1])
4 #display(g_wa)
5
6 g_wb = g_wa * SE3_sym(theta1, 0, 0) * SE3_sym(0, 0, -L1/2.0)
7 #display(g_wb)
8
9 g_wc = g_wa * SE3_sym(theta2, 0, 0) * SE3_sym(0, 0, -L2/2.0)
10 #display(g_wc)
11
12 # constraint transformations
13 g_wb_base = g_wa * SE3_sym(theta1, 0, 0) * SE3_sym(0, 0, -L1)
14 g_wc_base = g_wa * SE3_sym(theta2, 0, 0) * SE3_sym(0, 0, -L2)
15
16
17
18
19
20 ## body velocities ##
21 v_b = unhat(inverse_matrix(g_wb) * q_wb.diff(t)) # body velocity for b

```

```

21 Vb_ab = unhat(inverse_matrix(g_wb) * g_wb.diff(t)) # body velocity for b
22 #display(Vb_ab)
23 Vb_ac = unhat(inverse_matrix(g_wc) * g_wc.diff(t)) # body velocity for c
24 #display(Vb_ac)
25
26
27 ## body inertia matrices (lecture notes)##
28 I_b = sym.Matrix([[m1, 0.0, 0.0, 0.0, 0.0, 0.0],
29                   [0.0, m1, 0.0, 0.0, 0.0, 0.0],
30                   [0.0, 0.0, m1, 0.0, 0.0, 0.0],
31                   [0.0, 0.0, 0.0, 0.0, 0.0, 0.0],
32                   [0.0, 0.0, 0.0, 0.0, 0.0, 0.0],
33                   [0.0, 0.0, 0.0, 0.0, 0.0, J1]])
34
35 I_c = sym.Matrix([[m2, 0.0, 0.0, 0.0, 0.0, 0.0],
36                   [0.0, m2, 0.0, 0.0, 0.0, 0.0],
37                   [0.0, 0.0, m2, 0.0, 0.0, 0.0],
38                   [0.0, 0.0, 0.0, 0.0, 0.0, 0.0],
39                   [0.0, 0.0, 0.0, 0.0, 0.0, 0.0],
40                   [0.0, 0.0, 0.0, 0.0, 0.0, J2]])
41
42
43 ## Potential Energies ##
44 PE_b = m1 * g * (g_wb * sym.Matrix([0.0, 0.0, 0.0, 1.0]))[1]
45 PE_c = m2 * g * (g_wc * sym.Matrix([0.0, 0.0, 0.0, 1.0]))[1]
46
47 ## Kinetic Energies ##
48 KE_b = 0.5 * (Vb_ab.T * I_b * Vb_ab)[0]
49 KE_c = 0.5 * (Vb_ac.T * I_c * Vb_ac)[0]
50
51 ## Lagrangian ##
52 L = KE_b + KE_c - (PE_b + PE_c)
53
54
55 ## constraint ##
56 phi1 = (g_wb_base * sym.Matrix([0.0, 0.0, 0.0, 1.0]))[1]
57 #display(phi1)
58 phi2 = (g_wc_base * sym.Matrix([0.0, 0.0, 0.0, 1.0]))[1]
59 #display(phi2)
60 phi = sym.Matrix([phi1, phi2])
61 #display(phi)
62
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```

6 ## desired thetas ##
7 theta1_d = sym.pi/15.0 + sym.pi/3.0 * sym.sin(t/2.0)**2
8 theta2_d = -sym.pi/15.0 - sym.pi/3.0 * sym.sin(t/2.0)**2
9
10 ## torques ##
11 F_theta1 = -k1 * (theta1 - theta1_d)
12 F_theta2 = -k1 * (theta2 - theta2_d)
13 F_ext = sym.Matrix([0.0, 0.0, F_theta1, F_theta2])
14
15
16 EL_eqns_LHS1 = sym.simplify(d_dLdqdot_dt - dLdq)
17 EL_eqns_LHS2 = phi.diff(t,t)
18 EL_eqns_LHS = sym.Matrix([EL_eqns_LHS1, EL_eqns_LHS2])
19
20 EL_eqns_RHS1 = phi1.diff(q) * lam1 + phi2.diff(q) * lam2 + F_ext
21 EL_eqns_RHS = sym.Matrix([EL_eqns_RHS1, 0.0, 0.0])
22
23 EL_eqns = sym.Eq(EL_eqns_LHS, EL_eqns_RHS)
24 display(EL_eqns)
25
26 EL_solns = sym.solve(EL_eqns, [*qddot, *lam])
27 #display(EL_solns)
28
29 xddot = sym.lambdify([*q, *qdot, t], EL_solns[qddot[0]])
30 yddot = sym.lambdify([*q, *qdot, t], EL_solns[qddot[1]])
31 theta1ddot = sym.lambdify([*q, *qdot, t], EL_solns[qddot[2]])
32 theta2ddot = sym.lambdify([*q, *qdot, t], EL_solns[qddot[3]])
33
34

```

$$\begin{bmatrix}
 -0.5 \sin(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 - 0.5 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + 0.5 \cos(\theta_1(t)) \frac{d^2}{dt^2}\theta_1(t) + 0.5 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + 0.5 \sin(\theta_1(t)) \frac{d^2}{dt^2}\theta_1(t) + 0.5 \sin(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + 0.5 \cos(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + 0.5 \cos(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 \\
 0.5 \sin(\theta_1(t)) \frac{d^2}{dt^2}y(t) + 4.9 \sin(\theta_1(t)) + 0.5 \cos(\theta_1(t)) \frac{d^2}{dt^2}x(t) + 1.25 \frac{d^2}{dt^2}x(t) \\
 0.5 \sin(\theta_2(t)) \frac{d^2}{dt^2}y(t) + 4.9 \sin(\theta_2(t)) + 0.5 \cos(\theta_2(t)) \frac{d^2}{dt^2}x(t) + 1.25 \frac{d^2}{dt^2}x(t) \\
 1.0 \left(\sin(\theta_1(t)) \frac{d^2}{dt^2}\theta_1(t) + \cos(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + \frac{d^2}{dt^2}y(t) \right) \\
 1.0 \left(\sin(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + \cos(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + \frac{d^2}{dt^2}y(t) \right) \\
 0 \\
 1.0\lambda_1 + 1.0\lambda_2 \\
 1.0\lambda_1 \sin(\theta_1(t)) - 20\theta_1(t) + 6.666666666666667\pi \sin^2(0.5t) + 1.333333333333333\pi \\
 1.0\lambda_2 \sin(\theta_2(t)) - 20\theta_2(t) - 6.666666666666667\pi \sin^2(0.5t) - 1.333333333333333\pi \\
 0.0
 \end{bmatrix}$$

```

1 #sym.simplify(EL_solns[qddot[0]])
2 #sym.simplify(EL_solns[qddot[1]])
3 #sym.simplify(EL_solns[qddot[2]])
4 sym.simplify(EL_solns[qddot[3]])

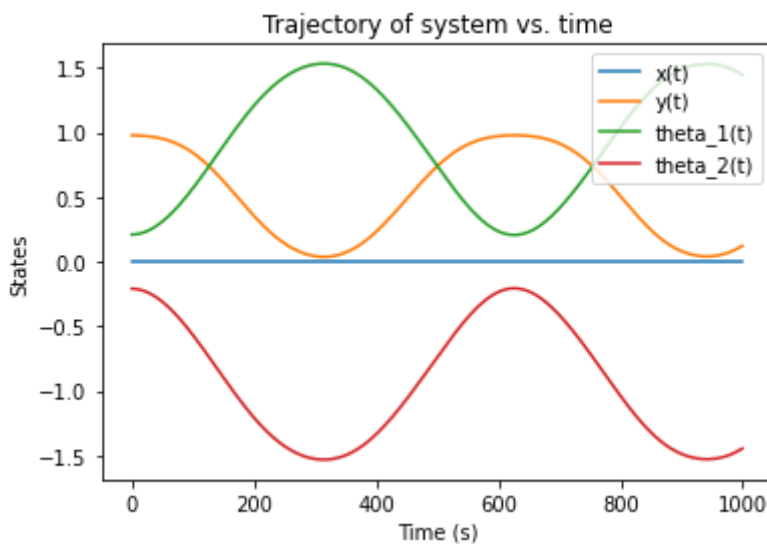
```

$$2400.0\theta_1(t) \sin(\theta_1(t)) \sin(\theta_2(t)) + 2400.0\theta_2(t) \sin^2(\theta_1(t)) + 2513.27412287183 \sin^2(0.5t) \sin^2$$

```

1 def dyn(s, t):
2     return np.array([s[4], s[5], s[6], s[7], xddot(*s, t), yddot(*s, t), thetaldot('
3
4
5 IC = [0.0, L1*np.cos(np.pi/15.0), np.pi/15.0, -np.pi/15.0, 0.0, 0.0, 0.0, 0.0]
6 traj = simulate(dyn, IC, [0,10], 0.01, integrate)
7 traj.shape
8
9 plt.figure()
10 plt.plot(traj[0:4].T)
11 plt.title('Trajectory of system vs. time')
12 plt.legend(['x(t)', 'y(t)', 'theta_1(t)', 'theta_2(t)'], loc='upper right')
13 plt.xlabel('Time (s)')
14 plt.ylabel('States')
15 plt.show()

```



```

1 def animate_legs(config_array,L1=1.0,L2=1.0,W1=0.2,W2=0.2,T=10):
2     """
3     Function to generate web-based animation of double-pendulum system
4
5     Parameters:
6     =====
7     config_array:
8         trajectory of x, y, theta1 and theta2, should be a NumPy array with
9         shape of (4,N)
10    L1:
11        length of the first leg
12    L2:
13        length of the second leg
14    W1:
15        width of first leg

```

```

16 W2:
17     width of second leg
18 T:
19     length/seconds of animation duration
20
21 Returns: None
22 """
23
24 #####
25 # Imports required for animation.
26 from plotly.offline import init_notebook_mode, iplot
27 from IPython.display import display, HTML
28 import plotly.graph_objects as go
29
30 #####
31 # Browser configuration.
32 def configure_plotly_browser_state():
33     import IPython
34     display(IPython.core.display.HTML(''
35         <script src="/static/components/requirejs/require.js"></script>
36         <script>
37             requirejs.config({
38                 paths: {
39                     base: '/static/base',
40                     plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
41                 },
42             });
43         </script>
44         ''))
45 configure_plotly_browser_state()
46 init_notebook_mode(connected=False)
47
48 #####
49 # Getting data from pendulum angle trajectories.
50 # xx1=L1*np.sin(config_array[0])
51 # yy1=-L1*np.cos(_array[0])
52 # xx2=xx1+L2*np.sin(theta_array[0]+theta_array[1])
53 # yy2=yy1-L2*np.cos(theta_array[0]+theta_array[1])
54
55 # data from config array
56 N = len(config_array[0]) # Need this for specifying length of simulation
57 x_array = config_array[0]
58 y_array = config_array[1]
59 theta1_array = config_array[2]
60 theta2_array = config_array[3]
61
62
63
64 #####
65 # Define arrays containing data for frame axes
66 # In each frame, the x and y axis are always fixed
67 # x axis = np.array([0, 2, 0, 0])

```

```

67 # x_axis = np.array([0.5, 0.0])
68 # y_axis = np.array([0.0, 0.3])
69 # # Use homogeneous tranformation to transfer these two axes/points
70 # # back to the fixed frame
71
72 # need to add b, c, and d frames
73 # frame_a_x_axis = np.zeros((2,N))
74 # frame_a_y_axis = np.zeros((2,N))
75 # frame_b_x_axis = np.zeros((2,N))
76 # frame_b_y_axis = np.zeros((2,N))
77 # frame_c_x_axis = np.zeros((2,N))
78 # frame_c_y_axis = np.zeros((2,N))
79 # frame_d_x_axis = np.zeros((2,N))
80 # frame_d_y_axis = np.zeros((2,N))
81
82
83 # 4 points per leg (x and y for each), therefore 16 arrays needed
84 leg1_point1_xarray = np.zeros(N, dtype=np.float32)
85 leg1_point1_yarray = np.zeros(N, dtype=np.float32)
86 leg1_point2_xarray = np.zeros(N, dtype=np.float32)
87 leg1_point2_yarray = np.zeros(N, dtype=np.float32)
88 leg1_point3_xarray = np.zeros(N, dtype=np.float32)
89 leg1_point3_yarray = np.zeros(N, dtype=np.float32)
90 leg1_point4_xarray = np.zeros(N, dtype=np.float32)
91 leg1_point4_yarray = np.zeros(N, dtype=np.float32)
92
93 leg2_point1_xarray = np.zeros(N, dtype=np.float32)
94 leg2_point1_yarray = np.zeros(N, dtype=np.float32)
95 leg2_point2_xarray = np.zeros(N, dtype=np.float32)
96 leg2_point2_yarray = np.zeros(N, dtype=np.float32)
97 leg2_point3_xarray = np.zeros(N, dtype=np.float32)
98 leg2_point3_yarray = np.zeros(N, dtype=np.float32)
99 leg2_point4_xarray = np.zeros(N, dtype=np.float32)
100 leg2_point4_yarray = np.zeros(N, dtype=np.float32)
101
102
103
104 for i in range(N): # iteration through each time step
105     # evaluate homogeneous transformation
106     # t_wa = np.array([[np.cos(theta_array[0][i]), -np.sin(theta_array[0][i]),
107     #                  [np.sin(theta_array[0][i]), np.cos(theta_array[0][i]),
108     #                  [0, 0, 0,
109     # # transfer the x and y axes in body frame back to fixed frame at
110     # # the current time step
111     # frame_a_x_axis[:,i] = t_wa.dot([x_axis[0], x_axis[1], 1])[0:2]
112     # frame_a_y_axis[:,i] = t_wa.dot([y_axis[0], y_axis[1], 1])[0:2]
113
114     # # now have to transfer the x and y axes from a to b, b to c, and c to d
115     # t_ab = np.array([[1,0,0], [0,1,-L1], [0,0,1]])
116     # frame_b_x_axis[:,i] = (t_wa.dot(t_ab)).dot([x_axis[0], x_axis[1], 1])[0:2]
117     # frame_b_y_axis[:,i] = (t_wa.dot(t_ab)).dot([y_axis[0], y_axis[1], 1])[0:2]
118

```

```

119 # t_bc = np.array([np.cos(theta_array[1][i]), -np.sin(theta_array[1][i]),
120 #                  [np.sin(theta_array[1][i]), np.cos(theta_array[1][i]),
121 #                  [0, 0, 0,
122 # frame_c_x_axis[:,i] = (t_wa.dot(t_ab.dot(t_bc))).dot([x_axis[0], x_axis[1]
123 # frame_c_y_axis[:,i] = (t_wa.dot(t_ab.dot(t_bc))).dot([y_axis[0], y_axis[1]
124
125 # t_cd = t_ab = np.array([[1,0,0], [0,1,-L2], [0,0,1]])
126 # frame_d_x_axis[:,i] = (t_wa.dot(t_ab.dot(t_bc.dot(t_cd))).dot([x_axis[0]
127 # frame_d_y_axis[:,i] = (t_wa.dot(t_ab.dot(t_bc.dot(t_cd))).dot([y_axis[0]
128
129 ## transformations
130 np_g_wa = SE3_np(0, x_array[i], y_array[i])
131 np_g_wb = np_g_wa * (SE3_np(theta1_array[i], 0, 0) * (SE3_np(0, 0, -L1/2.0)
132 np_g_wc = np_g_wa * (SE3_np(theta2_array[i], 0.0, 0.0) * (SE3_np(0.0, 0.0,
133
134 leg1_point1 = np_g_wb.dot(np.array([W1/2.0, L1/2.0, 0.0, 1.0]))
135 leg1_point1_xarray[i] = leg1_point1[0]
136 leg1_point1_yarray[i] = leg1_point1[1]
137
138 leg1_point2 = np_g_wb.dot(np.array([-W1/2.0, L1/2.0, 0.0, 1.0]))
139 leg1_point2_xarray[i] = leg1_point2[0]
140 leg1_point2_yarray[i] = leg1_point2[1]
141
142 leg1_point3 = np_g_wb.dot(np.array([-W1/2.0, -L1/2.0, 0.0, 1.0]))
143 leg1_point3_xarray[i] = leg1_point3[0]
144 leg1_point3_yarray[i] = leg1_point3[1]
145
146 leg1_point4 = np_g_wb.dot(np.array([W1/2.0, -L1/2.0, 0.0, 1.0]))
147 leg1_point4_xarray[i] = leg1_point4[0]
148 leg1_point4_yarray[i] = leg1_point4[1]
149
150 leg2_point1 = np_g_wc.dot(np.array([W2/2.0, L2/2.0, 0.0, 1.0]))
151 leg2_point1_xarray[i] = leg2_point1[0]
152 leg2_point1_yarray[i] = leg2_point1[1]
153
154 leg2_point2 = np_g_wc.dot(np.array([-W2/2.0, L2/2.0, 0.0, 1.0]))
155 leg2_point2_xarray[i] = leg2_point2[0]
156 leg2_point2_yarray[i] = leg2_point2[1]
157
158 leg2_point3 = np_g_wc.dot(np.array([-W2/2.0, -L2/2.0, 0.0, 1.0]))
159 leg2_point3_xarray[i] = leg2_point3[0]
160 leg2_point3_yarray[i] = leg2_point3[1]
161
162 leg2_point4 = np_g_wc.dot(np.array([W2/2.0, -L2/2.0, 0.0, 1.0]))
163 leg2_point4_xarray[i] = leg2_point4[0]
164 leg2_point4_yarray[i] = leg2_point4[1]
165
166
167
168
169 #####
170 # Using these to specify axis limits.

```

```

171     xm = -1.5 #np.min(xx1)-0.5
172     xM = 1.5 #np.max(xx1)+0.5
173     ym = -1.5 #np.min(yy1)-2.5
174     yM = 1.5 #np.max(yy1)+1.5
175
176     #####
177     # Defining data dictionary.
178     # Trajectories are here.
179     data=[
180         # note that except for the trajectory (which you don't need this time),
181         # you don't need to define entries other than "name". The items defined
182         # in this list will be related to the items defined in the "frames" list
183         # later in the same order. Therefore, these entries can be considered as
184         # labels for the components in each animation frame
185
186         # have to add B, C, and D frames
187         # dict(name='Arm'),
188         # dict(name='Mass 1'),
189         # dict(name='Mass 2'),
190         # dict(name='World Frame X'),
191         # dict(name='World Frame Y'),
192         # dict(name='A Frame X Axis'),
193         # dict(name='A Frame Y Axis'),
194         # dict(name='B Frame X Axis'),
195         # dict(name='B Frame Y Axis'),
196         # dict(name='C Frame X Axis'),
197         # dict(name='C Frame Y Axis'),
198         # dict(name='D Frame X Axis'),
199         # dict(name='D Frame Y Axis'),
200
201         dict(name='Leg 1'),
202         dict(name='Leg 2'),
203         dict(x=[xm, xM], y=[0,0],
204             mode='lines', name='Ground',
205             line=dict(color="brown",width=3),
206             ),
207
208
209         # You don't need to show trajectory this time,
210         # but if you want to show the whole trajectory in the animation (like what
211         # you did in previous homeworks), you will need to define entries other than
212         # "name", such as "x", "y". and "mode".
213
214         # dict(x=xx1, y=yy1,
215             # mode='markers', name='Pendulum 1 Traj',
216             # marker=dict(color="fuchsia", size=2)
217             # ),
218         # dict(x=xx2, y=yy2,
219             # mode='markers', name='Pendulum 2 Traj',
220             # marker=dict(color="purple", size=2)
221             # ),
222         #

```

```

222     ]
223
224     #####
225     # Preparing simulation layout.
226     # Title and axis ranges are here.
227     layout=dict(autosize=False, width=1000, height=1000,
228                 xaxis=dict(range=[xm, xM], autorange=False, zeroline=False,dtick=1),
229                 yaxis=dict(range=[ym, yM], autorange=False, zeroline=False,scaleanc
230                 title='Simulation of Legs of Body',
231                 hovermode='closest',
232                 updatemenus= [{ 'type': 'buttons',
233                                'buttons': [{ 'label': 'Play', 'method': 'animate',
234                                              'args': [None, { 'frame': { 'duration': 1
235                                              { 'args': [[None], { 'frame': { 'duration':
236                                              'transition': { 'duration': 0} }], 'label'
237                                ]
238                                } }
239                                )
240
241     #####
242     # Defining the frames of the simulation.
243     # This is what draws the lines from
244     # joint to joint of the pendulum.
245     frames=[dict(data=[# first three objects correspond to the arms and two masses,
246                       # same order as in the "data" variable defined above (thus
247                       # they will be labeled in the same order)
248                       # dict(x=[0,xx1[k],xx2[k]],
249                       #      y=[0,yy1[k],yy2[k]],
250                       #      mode='lines',
251                       #      line=dict(color='orange', width=3),
252                       #      ),
253                       # go.Scatter(
254                       #      x=[xx1[k]],
255                       #      y=[yy1[k]],
256                       #      mode="markers",
257                       #      marker=dict(color="blue", size=12)),
258                       # go.Scatter(
259                       #      x=[xx2[k]],
260                       #      y=[yy2[k]],
261                       #      mode="markers",
262                       #      marker=dict(color="blue", size=12)),
263                       # # display x and y axes of the fixed frame in each animatic
264                       # dict(x=[0,x_axis[0]],
265                       #      y=[0,x_axis[1]],
266                       #      mode='lines',
267                       #      line=dict(color='green', width=3),
268                       #      ),
269                       # dict(x=[0,y_axis[0]],
270                       #      y=[0,y_axis[1]],
271                       #      mode='lines',
272                       #      line=dict(color='red', width=3),
273                       #      ),

```

```

274 # # display x and y axes of the {A} frame in each animation
275 # dict(x=[0, frame_a_x_axis[0][k]],
276 #      y=[0, frame_a_x_axis[1][k]],
277 #      mode='lines',
278 #      line=dict(color='green', width=3),
279 #      ),
280 # dict(x=[0, frame_a_y_axis[0][k]],
281 #      y=[0, frame_a_y_axis[1][k]],
282 #      mode='lines',
283 #      line=dict(color='red', width=3),
284 #      ),
285
286 # # display x and y axes of the {B} frame in each animation
287 # dict(x=[xx1[k], frame_b_x_axis[0][k]],
288 #      y=[yy1[k], frame_b_x_axis[1][k]],
289 #      mode='lines',
290 #      line=dict(color='green', width=3),
291 #      ),
292 # dict(x=[xx1[k], frame_b_y_axis[0][k]],
293 #      y=[yy1[k], frame_b_y_axis[1][k]],
294 #      mode='lines',
295 #      line=dict(color='red', width=3),
296 #      ),
297
298 # # display x and y axes of the {C} frame in each animation
299 # dict(x=[xx1[k], frame_c_x_axis[0][k]],
300 #      y=[yy1[k], frame_c_x_axis[1][k]],
301 #      mode='lines',
302 #      line=dict(color='green', width=3),
303 #      ),
304 # dict(x=[xx1[k], frame_c_y_axis[0][k]],
305 #      y=[yy1[k], frame_c_y_axis[1][k]],
306 #      mode='lines',
307 #      line=dict(color='red', width=3),
308 #      ),
309
310 # # display x and y axes of the {D} frame in each animation
311 # # now use xx2 and yy2 for D frame
312 # dict(x=[xx2[k], frame_d_x_axis[0][k]],
313 #      y=[yy2[k], frame_d_x_axis[1][k]],
314 #      mode='lines',
315 #      line=dict(color='green', width=3),
316 #      ),
317 # dict(x=[xx2[k], frame_d_y_axis[0][k]],
318 #      y=[yy2[k], frame_d_y_axis[1][k]],
319 #      mode='lines',
320 #      line=dict(color='red', width=3),
321 #      ),
322
323 ## from point1 --> point2 --> point3 --> point4 --> point 1
324 dict(x=[leg1_point1_xarray[k],
325        leg1_point2_xarray[k],

```



```

326         leg1_point3_xarray[k],
327         leg1_point4_xarray[k],
328         leg1_point1_xarray[k]],
329     y=[leg1_point1_yarray[k],
330        leg1_point2_yarray[k],
331        leg1_point3_yarray[k],
332        leg1_point4_yarray[k],
333        leg1_point1_yarray[k]],
334     mode='lines',
335     line=dict(color='blue',width=3)),
336
337     dict(x=[leg2_point1_xarray[k],
338            leg2_point2_xarray[k],
339            leg2_point3_xarray[k],
340            leg2_point4_xarray[k],
341            leg2_point1_xarray[k]],
342         y=[leg2_point1_yarray[k],
343            leg2_point2_yarray[k],
344            leg2_point3_yarray[k],
345            leg2_point4_yarray[k],
346            leg2_point1_yarray[k]],
347         mode='lines',
348         line=dict(color='green',width=3)),
349
350
351
352     ]) for k in range(N)]
353
354     #####
355     # Putting it all together and plotting.
356     figure1=dict(data=data, layout=layout, frames=frames)
357     iplot(figure1)

```

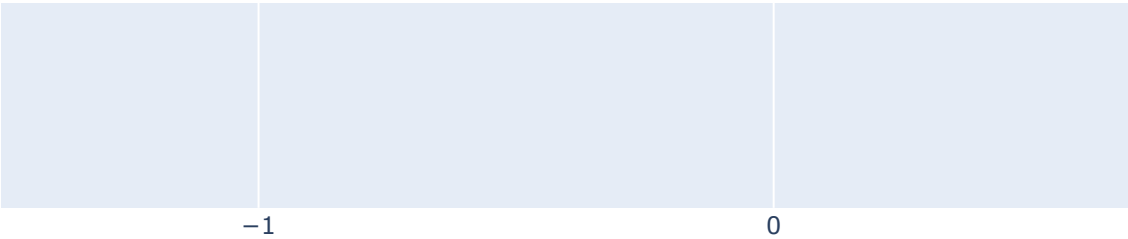
```
1 animate_legs(traj)
```

```
/usr/local/lib/python3.7/dist-packages/sympy/matrices/matrices.py:1357: SymPyDeprecationWarning: 
```

Dot product of non row/column vectors has been deprecated since SymPy 1.2. Use `*` to take matrix products instead. See <https://github.com/sympy/sympy/issues/13815> for more info.

Simulation of Legs of Body





✓ 26s completed at 6:24 PM ● ✕

MECH-ENG 314 Homework 7

1. $R \in SO(n)$, $A = \frac{d}{dt}(R)R^{-1}$ is skew symmetric matrix

$$R = e^{\hat{\omega}\theta} \quad (\text{lecture notes})$$

$$\frac{d}{dt}(R) = \frac{d}{dt} e^{\hat{\omega}\theta} = \frac{\partial}{\partial \theta} (e^{\hat{\omega}\theta}) \cdot \frac{d\theta}{dt}$$

$$= \hat{\omega} e^{\hat{\omega}\theta} \cdot \dot{\theta} = R \hat{\omega} \dot{\theta}$$

$\hat{\omega}$ is skew-symmetric matrix

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$A = \frac{d}{dt}(R)R^{-1} = R \hat{\omega} \dot{\theta} R^{-1} = \hat{\omega} \dot{\theta} \underbrace{R R^{-1}}_I$$

$$\boxed{A = \hat{\omega} \dot{\theta}}$$

A skew-symmetric matrix
because of $\hat{\omega}$

2. $\hat{\omega} r_b = -\hat{r}_b \omega$

$$\hat{\omega} r_b = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} -\omega_3 r_2 + \omega_2 r_3 \\ \omega_3 r_1 - \omega_1 r_3 \\ -\omega_2 r_1 + \omega_1 r_2 \end{bmatrix}$$

$$-\hat{r}_b \omega = \begin{bmatrix} 0 & r_3 & -r_2 \\ -r_3 & 0 & r_1 \\ r_2 & -r_1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} r_3 \omega_2 - r_2 \omega_3 \\ -r_3 \omega_1 + r_1 \omega_3 \\ r_2 \omega_1 - r_1 \omega_2 \end{bmatrix}$$

equal!

$$\boxed{\therefore \hat{\omega} r_b = -\hat{r}_b \omega}$$