Appendix A

-Screenshots from when Jupyter cut off the code output in the main PDF

Screenshot 0: Lagrangian

```
#------------------------#

KE1 = 0.5 * m1 * (x1d**2 + y1d**2)
KE2 = 0.5 * m2 * (x2d**2 + y2d**2)

U1 = m1 * g * y1
U2 = m2 * g * y2

lagrangian = (KE1 + KE2) - (U1 + U2)
print('Lagrangian:')
display(lagrangian)
```

```
Lagrangian:
```

$$R_1 g m_1 \cos(\theta_1(t)) - g m_2 (-R_1 \cos(\theta_1(t)) - R_2 \cos(\theta_1(t) + \theta_2(t))) + 0.5 m_1 \left(R_1^2 \sin^2(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1^2 \cos^2(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 \right)$$

$$+ 0.5 m_2 \left(\left(R_1 \sin(\theta_1(t)) \frac{d}{dt}\theta_1(t) + R_2 \left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right) \sin(\theta_1(t) + \theta_2(t)) \right)^2 + \left(R_1 \cos(\theta_1(t)) \frac{d}{dt}\theta_1(t) + R_2 \left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right) \cos(\theta \right. \right.$$

```
Lagrangian:
```

$$- g m_2 (-R_1 \cos(\theta_1(t)) - R_2 \cos(\theta_1(t) + \theta_2(t))) + 0.5 m_1 \left(R_1^2 \sin^2(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1^2 \cos^2(\theta_1(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 \right)$$

$$\mathrm{n}(\theta_1(t)) \frac{d}{dt}\theta_1(t) + R_2 \left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right) \sin(\theta_1(t) + \theta_2(t)) \Big)^2 + \left(R_1 \cos(\theta_1(t)) \frac{d}{dt}\theta_1(t) + R_2 \left(\frac{d}{dt}\theta_1(t) + \frac{d}{dt}\theta_2(t)\right) \cos(\theta_1(t) + \theta_2(t)) \right)^2 \Big)$$

Screenshot 1: Euler-Lagrange Equations

```
# You can start your implementation here :)

q = sym.Matrix([theta1, theta2])
qd = q.diff(t)
qdd = qd.diff(t)

eqn = compute_EL(lagrangian, q)
eqn = eqn.simplify()
print("Euler-Lagrange equations:")
display(eqn)
```

```
Euler-Lagrange equations:
```

$$\begin{bmatrix} -1.0 R_1^2 m_1 \frac{d^2}{dt^2}\theta_1(t) - R_1 g m_1 \sin(\theta_1(t)) - g m_2 (R_1 \sin(\theta_1(t)) + R_2 \sin(\theta_1(t) + \theta_2(t))) \\ - 1.0 m_2 \left(R_1^2 \frac{d^2}{dt^2}\theta_1(t) - 2 R_1 R_2 \sin(\theta_2(t)) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) - R_1 R_2 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + 2 R_1 R_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_1 R_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + R_2^2 \frac{d^2}{dt^2}\theta_1(t) + R_2^2 \frac{d^2}{dt^2}\theta_2(t) \right) \\ -1.0 R_2 m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_2 \frac{d^2}{dt^2}\theta_1(t) + R_2 \frac{d^2}{dt^2}\theta_2(t) + g \sin(\theta_1(t) + \theta_2(t)) \right) \end{bmatrix}$$

Screenshot 2: Solved Euler-Lagrange Equations

```
print("Solved:")
for eq in solved_eqns:
    eq_new = eq.simplify()
    simplified_eqns.append(eq_new)
    display(eq_new)
```

```
Solved:
```

$$\overset{2}{\underset{2}{-}}\theta_1(t) = \frac{0.5R_1m_2\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 + 1.0R_2m_2\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 + 2.0R_2m_2\sin(\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) + 1.0R_2m_2\sin(\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2 - 1.0gm_1\sin(\theta_1(t)) + 0.5gm_2\sin(\theta_1(t)+2\theta_2(t)) - 0.5gm_2\sin(\theta_1(t))}{R_1\left(m_1+m_2\sin^2(\theta_2(t))\right)}$$

$$t) = \frac{2\left(-1.0R_1^2m_1\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 1.0R_1^2m_2\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 1.0R_1R_2m_2\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 1.0R_1R_2m_2\sin(2\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) - 0.5R_1R_2m_2\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2 + 0.5R_1gm_1\sin(\theta_1(t)-\theta_2(t)) - 0.5R_1gm_1\sin(\theta_1(t)+\theta_2(t)) + 0.5R_1gm_2\sin(\theta_1(t)-\theta_2(t)) - 0.5R_1gm_2\sin(\theta_1(t)+\theta_2(t)) - 1.0R_2^2m_2\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 2.0R_2^2m_2\sin(\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) - 1.0R_2^2m_2\sin(\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2 + 1.0R_2gm_1\sin(\theta_1(t)) - 0.5R_2gm_2\sin(\theta_1(t)+2\theta_2(t)) + 0.5R_2gm_2\sin(\theta_1(t))\right)}{R_1R_2\cdot(2m_1 - m_2\cos(2\theta_2(t)) + m_2)}$$

Screenshot 3: Solved Euler-Lagrange Equations with substitutions

```
theta1dd_sy = theta1dd_sy.subs(consts_dict)
theta2dd_sy = theta2dd_sy.subs(consts_dict)

print("Theta1dd and Theta1dd with constants substituted in:")
display(theta1dd_sy)
display(theta2dd_sy)

q_ext = sym.Matrix([theta1, theta2, theta1d, theta2d])
theta1dd_np = sym.lambdify(q_ext, theta1dd_sy.rhs)
theta2dd_np = sym.lambdify(q_ext, theta2dd_sy.rhs)
```

```
Theta1dd and Theta1dd with constants substituted in:
```

$$\cdot\theta_1(t) = \frac{9.8\sin(\theta_1(t)+2\theta_2(t)) - 19.6\sin(\theta_1(t)) + 2.0\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 + 4.0\sin(\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) + 2.0\sin(\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2 + 2.0\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2}{2\cdot\left(2\sin^2(\theta_2(t))+1\right)}$$

$$) = \frac{29.4\sin(\theta_1(t)-\theta_2(t)) - 29.4\sin(\theta_1(t)+\theta_2(t)) - 9.8\sin(\theta_1(t)+2\theta_2(t)) + 19.6\sin(\theta_1(t)) - 14.0\sin(\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 4.0\sin(\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) - 2.0\sin(\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2 - 4.0\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_1(t)\right)^2 - 4.0\sin(2\theta_2(t))\frac{d}{dt}\theta_1(t)\frac{d}{dt}\theta_2(t) - 2.0\sin(2\theta_2(t))\left(\frac{d}{dt}\theta_2(t)\right)^2}{4 - 2\cos(2\theta_2(t))}$$