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Beyond the properties described above, a URDF can describe other properties of a robot, such as its visual appearance (including geometric models of the links) as well as simplified representations of link geometries that can be used for collision detection in motion planning algorithms.

4.3 Summary

- Given an open chain with a fixed reference frame $\{s\}$ and a reference frame $\{b\}$ attached to some point on its last link this frame is denoted the end-effector frame the forward kinematics is the mapping $T(\theta)$ from the joint values θ to the position and orientation of $\{b\}$ in $\{s\}$.
- In the Denavit–Hartenberg representation the forward kinematics of an open chain is described in terms of the relative displacements between reference frames attached to each link. If the link frames are sequentially labeled $\{0\}, \ldots, \{n+1\}$, where $\{0\}$ is the fixed frame $\{s\}, \{i\}$ is a frame attached to link i at joint i (with $i = 1, \ldots, n$), and $\{n+1\}$ is the endeffector frame $\{b\}$ then the forward kinematics is expressed as

$$T_{0,n+1}(\theta) = T_{01}(\theta_1) \cdots T_{n-1,n}(\theta_n) T_{n,n+1}$$

where θ_i denotes the joint *i* variable and $T_{n,n+1}$ indicates the (fixed) configuration of the end-effector frame in $\{n\}$. If the end-effector frame $\{b\}$ is chosen to be coincident with $\{n\}$ then we can dispense with the frame $\{n+1\}$.

• The Denavit–Hartenberg convention requires that reference frames assigned to each link obey a strict convention (see Appendix C). Following this convention, the link frame transformation $T_{i-1,i}$ between link frames $\{i-1\}$ and $\{i\}$ can be parametrized using only four parameters, the Denavit–Hartenberg parameters. Three of these parameters describe the kinematic structure, while the fourth is the joint value. Four numbers is the minimum needed to represent the displacement between two link frames.

• The forward kinematics can also be expressed as the following product of exponentials (the space form),

$$T(\theta) = e^{[S_1]\theta_1} \cdots e^{[S_n]\theta_n} M,$$

where $S_i = (\omega_i, v_i)$ denotes the screw axis associated with positive motion along joint i expressed in fixed-frame $\{s\}$ coordinates, θ_i is the joint-i variable, and $M \in SE(3)$ denotes the position and orientation of the end-effector frame $\{b\}$ when the robot is in its zero position. It is not necessary to define individual link frames; it is only necessary to define M and the screw axes S_1, \ldots, S_n .

• The product of exponentials formula can also be written in the equivalent body form,

$$T(\theta) = Me^{[\mathcal{B}_1]\theta_1} \cdots e^{[\mathcal{B}_n]\theta_n}$$

where $\mathcal{B}_i = [\mathrm{Ad}_{M^{-1}}]\mathcal{S}_i$, $i = 1, \ldots, n$; $\mathcal{B}_i = (\omega_i, v_i)$ is the screw axis corresponding to joint axis i, expressed in $\{b\}$, with the robot in its zero position.

• The Universal Robot Description Format (URDF) is a file format used by the Robot Operating System and other software for representing the kinematics, inertial properties, visual properties, and other information for general tree-like robot mechanisms, including serial chains. A URDF file includes descriptions of joints, which connect a parent link and a child link and fully specify the kinematics of the robot, as well as descriptions of links, which specify its inertial properties.

4.4 Software

Software functions associated with this chapter are listed in MATLAB format below.

T = FKinBody(M,Blist,thetalist)

Computes the end-effector frame given the zero position of the end-effector M, the list of joint screws Blist expressed in the end-effector frame, and the list of joint values thetalist.

T = FKinSpace(M,Slist,thetalist)

Computes the end-effector frame given the zero position of the end-effector M, the list of joint screws Slist expressed in the fixed-space frame, and the list of joint values thetalist.