as $\mathbb{R}^3 \times S^2$. The workspace could be the reachable points in $\mathbb{R}^3 \times S^2$, or, to simplify visualization, the user could define the workspace to be the subset of $\mathbb{R}^3$ corresponding to the reachable Cartesian positions of the nozzle.

2.6 Summary

- A robot is mechanically constructed from links that are connected by various types of joint. The links are usually modeled as rigid bodies. An end-effector such as a gripper may be attached to some link of the robot. Actuators deliver forces and torques to the joints, thereby causing motion of the robot.

- The most widely used one-dof joints are the revolute joint, which allows rotation about the joint axis, and the prismatic joint, which allows translation in the direction of the joint axis. Some common two-dof joints include the cylindrical joint, which is constructed by serially connecting a revolute and prismatic joint, and the universal joint, which is constructed by orthogonally connecting two revolute joints. The spherical joint, also known as the ball-and-socket joint, is a three-dof joint whose function is similar to the human shoulder joint.

- The configuration of a rigid body is a specification of the location of all its points. For a rigid body moving in the plane, three independent parameters are needed to specify the configuration. For a rigid body moving in three-dimensional space, six independent parameters are needed to specify the configuration.

- The configuration of a robot is a specification of the configuration of all its links. The robot's configuration space is the set of all possible robot configurations. The dimension of the C-space is the number of degrees of freedom of a robot.

- The number of degrees of freedom of a robot can be calculated using Grübler's formula,

$$\text{dof} = m(N - 1 - J) + \sum_{i=1}^{J} f_i,$$

  where $m = 3$ for planar mechanisms and $m = 6$ for spatial mechanisms, N is the number of links (including the ground link), J is the number of joints, and f_i is the number of degrees of freedom of joint i.

- A robot's C-space can be parametrized explicitly or represented implicitly. For a robot with n degrees of freedom, an explicit parametrization uses n coordinates, the minimum necessary. An implicit representation involves m coordinates with $m \geq n$, with the m coordinates subject to $m - n$ constraint equations. With an implicit parametrization, a robot's C-space can be viewed as a surface of dimension n embedded in a space of higher dimension m.

- The C-space of an n-dof robot whose structure contains one or more closed loops can be implicitly represented using k loop-closure equations of the form $g(\theta) = 0$, where $\theta \in \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$. Such constraint equations are called holonomic constraints. Assuming that θ varies with time t, the holonomic constraints $g(\theta(t)) = 0$ can be differentiated with respect to t to yield
$$\frac{\partial g}{\partial \theta}(\theta)\dot{\theta} = 0,$$
where $\partial g(\theta)/\partial \theta$ is a $k \times m$ matrix.

- A robot's motion can also be subject to velocity constraints of the form
$$A(\theta)\dot{\theta} = 0,$$
where $A(\theta)$ is a $k \times m$ matrix that cannot be expressed as the differential of some function $g(\theta)$. In other words, there does not exist any $g(\theta), g : \mathbb{R}^m \to \mathbb{R}^k$, such that
$$A(\theta) = \frac{\partial g}{\partial \theta}(\theta).$$
Such constraints are said to be nonholonomic constraints, or nonintegrable constraints. These constraints reduce the dimension of feasible velocities of the system but do not reduce the dimension of the reachable C-space. Nonholonomic constraints arise in robot systems subject to conservation of momentum or rolling without slipping.

- A robot's task space is a space in which the robot's task can be naturally expressed. A robot's workspace is a specification of the configurations that the end-effector of the robot can reach.

2.7 Notes and References

In the kinematics literature, structures that consist of links connected by joints are also called mechanisms or linkages. The number of degrees of freedom of a