

ME 449 Midterm  
October 27, 2021

Name: *Solutions*

Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of a page. No electronics (phone, watch, calculator, computer, tablet, etc.) allowed.

pain points:

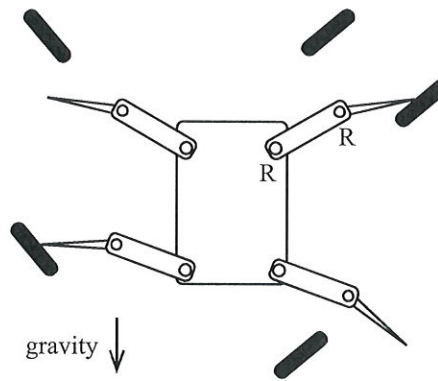
- express torques in terms

of applied wrench

- multiple transformations when

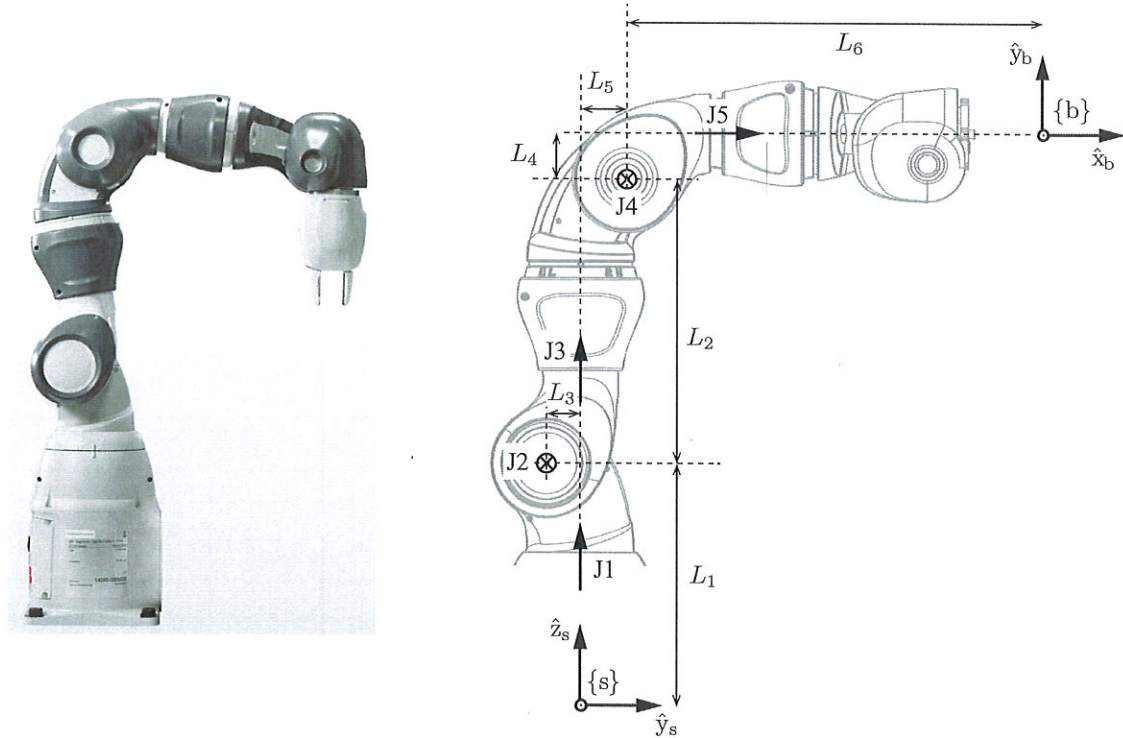
last one is in A frame

1. (8 pts) The figure below shows a climbing robot that climbs in a vertical plane. Its motion is confined to that vertical plane. The climber consists of a body and four legs. Each leg has two revolute joints, and when a "foot" is in point contact with a foothold, it forms a revolute joint with the foothold in parts (b) and (c) (but not in (d); see below).



- (a) When the robot has no feet in contact with a foothold, how many degrees of freedom does the robot have? Show your reasoning.
- (b) When the robot has one foot in contact with a foothold, forming a revolute joint at the contact, how many degrees of freedom does the robot have? Show your reasoning.
- (c) When the robot has four feet in contact with footholds, forming revolute joints at the contacts, how many degrees of freedom does the robot have? Show your reasoning.
- (d) Now assume the robot has two feet in contact with a foothold, but each foot can slide along the foothold in addition to rotate about the contact point. How many degrees of freedom does the robot have? Show your reasoning.

2. (12 pts) On the left is the 7R ABB Yumi collaborative robot arm, and on the right is a kinematic model that considers the first five joints, J1 to J5. (The last two joints near the end-effector are omitted.) Assume the Yumi is shown at its home configuration in the right image. Joint axes 1 and 3 are aligned and point up on the page, joint axes 2 and 4 point into the page, and joint axis 5 points to the right on the page. The various offsets are indicated by the lengths  $L_1 \dots L_6$ . The  $\{s\}$  frame at the base of the robot and the  $\{b\}$  frame at the end-effector are shown.



- (a) Write the matrix  $M = T_{sb}(0)$  describing the configuration of  $\{b\}$  relative to  $\{s\}$  at this home configuration.

- (b) Write the  $6 \times 5$  space Jacobian  $J_s(0)$ .

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3. (4 pts) You are sitting in an aircraft simulator cockpit, and the frame of your seat is initially aligned with a frame  $\{a\}$  fixed to the room. Your seat first follows a twist  $\mathcal{V}_{a1}$  (represented in the  $\{a\}$  frame) for  $t_1$  seconds, achieving a new frame location  $\{b\}$ . Your seat then follows the screw axis  $\mathcal{S}_b$  (represented in the  $\{b\}$  frame) a rotational distance  $\alpha$ , achieving a new frame location  $\{c\}$ . Finally, your seat follows a twist  $\mathcal{V}_{a2}$  (represented in the  $\{a\}$  frame) for a time  $t_2$ , achieving a new frame location  $\{d\}$ .

Your final configuration relative to the  $\{a\}$  frame can be represented by exponential coordinates  $\mathcal{S}_a\theta$ , where  $\mathcal{S}_a$  is a screw axis, represented in the  $\{a\}$  frame, that you would have had to follow a distance  $\theta$  to move to the frame  $\{d\}$ . Give the  $se(3)$  representation  $[\mathcal{S}_a\theta]$  of these exponential coordinates, expressed in terms of the functions  $\log$  and  $\exp$  and the information given above.

4. (4 pts) A human-collaborative robot arm is mounted to the floor, and a frame  $\{s\}$  is located at the mounting location. A frame  $\{b\}$  is at the robot's end-effector. The robot's joint vector  $\theta$ , end-effector configuration  $T_{sb}(\theta)$ , and body Jacobian  $J_b(\theta)$  are known. A human grabs the end-effector and applies a wrench to it, expressed in the  $\{s\}$  frame as  $\mathcal{F}_s$ . Give the expression for the joint torques  $\tau$  the robot must apply to resist motion in terms of the quantities defined above.

in 9:40

5. (2 pts) The exponential coordinates  $\mathcal{S}_c\theta$  representing a frame  $\{d\}$  relative to a frame  $\{c\}$  are  $(0, 1, 1, 10, 5, 0)$ . Give the corresponding screw axis  $\mathcal{S}_c$  and the angle of travel  $\theta$  along it.

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