- Ways to represent transformations:
  - Screw axis $S \times \theta$ distance followed
  - Twist $V \times t$ time followed
  - Transformation matrix

- Matrix exponential integrates a twist; log differentiates → finds the constant twist needed to achieve a given displacement
- To differentiate screw axis + twist representations:
  - S is unit vector of velocity
  - $S \cdot \theta =$ displacement along unit axis
  - $S \cdot v \cdot t =$ displacement along unit axis $= V \cdot t$

Exponential: $[S]\theta \xrightarrow{\text{becomes}} se(3)$

Log: $se(3) \longrightarrow [S]\theta$

for no angular displacement,

$$T_{sb}' = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$$

Format of matrix exponential:

$$T_{sb}' = \underset{\substack{\uparrow \\ \text{element of} \\ se(3)}}{e^{[S]\theta}} = \begin{bmatrix} e^{[\omega]\theta} & (\text{Rodrigues}(\omega)) \cdot v \\ 0 & 1 \end{bmatrix}$$

where $S = \begin{bmatrix} \omega \\ v \end{bmatrix}$.

- Wrench represents both moments + forces
- product of effort • flow gives power;

$W = \begin{bmatrix} M \\ F \end{bmatrix}$; $V^T W$ = Power

- Power has to be independent of frame in which we represent it
  - energy conserved, etc. etc.

$$V_b^T F_b = V_a^T F_a$$

reflect twist in one frame rel. to another using adjoint representation

↳ adjoint matrix brings a 4x4 trans. matrix T_{AB} into a 6x6 matrix so we can multiply by V

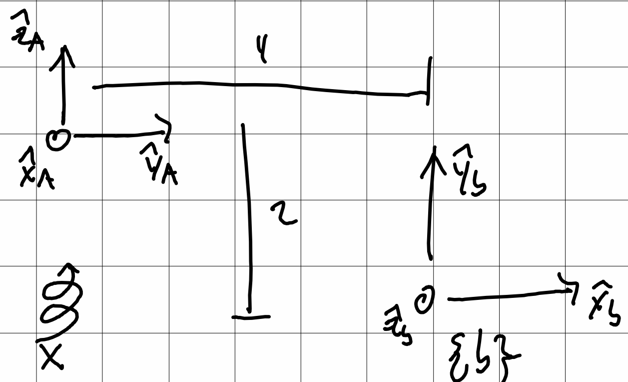
$$V_b^T F_b = (Ad_{T_{AB}} V_b)^T F_A$$

$$\cancel{V_b^T} F_B = \cancel{V_b^T} [Ad_{T_{AB}}]^T F_A$$

$$\boxed{F_B = [Ad_{T_{AB}}]^T F_A}$$

$$\to Ad_T = \begin{bmatrix} [R] & 0 \\ p[R] & [R] \end{bmatrix}$$

- Exercise: screw axis representation:



- pitch = 5mm/rad
- origin of $\{b\}$ at $(0, 4, -2)$

a) - Screw S_a:
  - $\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
  - $v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot 5$

$S_a = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 5 \text{ mm/rad} \end{bmatrix}$

- Screw S_b:
  - add an additional tangential velocity term $4 \cdot \omega_{yb}$
  - $\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ (about y axis for b)
  - $v = \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix} \begin{matrix} \\ \text{mm/rad} \\ \text{mm/rad?} \end{matrix}$ $\rightarrow$ $S_b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 5 \\ -4 \end{bmatrix}$

b) Transformation matrix T_{ab}:

- let's right-multiply, $T_{AB} = \text{Trans}(p)\,\text{Rot}(\hat{\omega}, \theta)$

= origin offset by $(0, 4, -2)$ rel. to. A coordinates, axes of b are $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ in a coords,

$$T_{AB} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \checkmark$$

c) Final configuration $T_{AB'}$:

$T_{AB'} = T\,T_{AB}$, T = transformation matrix corr. to. screw motion

$$T = \begin{bmatrix} [\omega] & v \\ 0 & 1 \end{bmatrix},$$

$[\omega]$ = so(3) version of ang. velocity in respect to $\{a\}$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$[\omega]$ = so(3) version of ang. velocity in respect to {a}

$$[\omega] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

Transformation matrix T corresponding to screw motion:

$$T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

X

... not what he was asking; goal is to exponentiate the screw $\cdot \theta$

↳ also: post-multiply, since transf. is happening in {b},

↳ $\boxed{T_{AB} \exp([S_b]\theta)}$ is mathematical expression

d) can be done by doing the rotation by π rad and looking at where it ends up