Rotations	**Rigid-Body Motions**
$R \in SO(3) : 3 \times 3$ matrices $R^{\mathrm{T}}R = I, \det R = 1$	$T \in SE(3) : 4 \times 4$ matrices $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$, where $R \in SO(3), p \in \mathbb{R}^3$
$R^{-1} = R^{\mathrm{T}}$	$T^{-1} = \begin{bmatrix} R^{\mathrm{T}} & -R^{\mathrm{T}}p \\ 0 & 1 \end{bmatrix}$
change of coordinate frame: $R_{ab}R_{bc} = R_{ac}, \;\; R_{ab}p_b = p_a$	change of coordinate frame: $T_{ab}T_{bc} = T_{ac}, \;\; T_{ab}p_b = p_a$
rotating a frame {b}: $R = \mathrm{Rot}(\hat{\omega}, \theta)$ $R_{sb'} = RR_{sb}$: rotate θ about $\hat{\omega}_s = \hat{\omega}$ $R_{sb''} = R_{sb}R$: rotate θ about $\hat{\omega}_b = \hat{\omega}$	displacing a frame {b}: $T = \begin{bmatrix} \mathrm{Rot}(\hat{\omega}, \theta) & p \\ 0 & 1 \end{bmatrix}$ $T_{sb'} = TT_{sb}$: rotate θ about $\hat{\omega}_s = \hat{\omega}$ (moves {b} origin), translate p in {s} $T_{sb''} = T_{sb}T$: translate p in {b}, rotate θ about $\hat{\omega}$ in new body frame
unit rotation axis is $\hat{\omega} \in \mathbb{R}^3$, where $\\|\hat{\omega}\\| = 1$	"unit" screw axis is $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$, where either (i) $\\|\omega\\| = 1$ or (ii) $\omega = 0$ and $\\|v\\| = 1$
	for a screw axis $\{q, \hat{s}, h\}$ with finite h, $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \hat{s} \\ -\hat{s} \times q + h\hat{s} \end{bmatrix}$
angular velocity is $\omega = \hat{\omega}\dot{\theta}$	twist is $\mathcal{V} = \mathcal{S}\dot{\theta}$

continued...

Rotations (cont.)	**Rigid-Body Motions (cont.)**
for any 3-vector, e.g., $\omega \in \mathbb{R}^3$, $[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \in so(3)$ identities, $\omega, x \in \mathbb{R}^3, R \in SO(3)$: $[\omega] = -[\omega]^{\mathrm{T}}, [\omega]x = -[x]\omega,$ $[\omega][x] = ([x][\omega])^{\mathrm{T}}, R[\omega]R^{\mathrm{T}} = [R\omega]$	for $\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$, $[\mathcal{V}] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \in se(3)$ (the pair (ω, v) can be a twist $\mathcal{V}$ or a "unit" screw axis $\mathcal{S}$, depending on the context)
$\dot{R}R^{-1} = [\omega_s], \;\; R^{-1}\dot{R} = [\omega_b]$	$\dot{T}T^{-1} = [\mathcal{V}_s], \;\; T^{-1}\dot{T} = [\mathcal{V}_b]$
	$[\mathrm{Ad}_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$ identities: $[\mathrm{Ad}_T]^{-1} = [\mathrm{Ad}_{T^{-1}}]$, $[\mathrm{Ad}_{T_1}][\mathrm{Ad}_{T_2}] = [\mathrm{Ad}_{T_1 T_2}]$
change of coordinate frame: $\hat{\omega}_a = R_{ab}\hat{\omega}_b, \;\; \omega_a = R_{ab}\omega_b$	change of coordinate frame: $\mathcal{S}_a = [\mathrm{Ad}_{T_{ab}}]\mathcal{S}_b, \;\; \mathcal{V}_a = [\mathrm{Ad}_{T_{ab}}]\mathcal{V}_b$
exp coords for $R \in SO(3)$: $\hat{\omega}\theta \in \mathbb{R}^3$	exp coords for $T \in SE(3)$: $\mathcal{S}\theta \in \mathbb{R}^6$
$\exp : [\hat{\omega}]\theta \in so(3) \to R \in SO(3)$ $R = \mathrm{Rot}(\hat{\omega}, \theta) = e^{[\hat{\omega}]\theta} =$ $I + \sin\theta[\hat{\omega}] + (1 - \cos\theta)[\hat{\omega}]^2$	$\exp : [\mathcal{S}]\theta \in se(3) \to T \in SE(3)$ $T = e^{[\mathcal{S}]\theta} = \begin{bmatrix} e^{[\omega]\theta} & * \\ 0 & 1 \end{bmatrix}$ where $* =$ $(I\theta + (1 - \cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)v$
$\log : R \in SO(3) \to [\hat{\omega}]\theta \in so(3)$ algorithm in Section 3.2.3.3	$\log : T \in SE(3) \to [\mathcal{S}]\theta \in se(3)$ algorithm in Section 3.3.3.2
moment change of coord frame: $m_a = R_{ab}m_b$	wrench change of coord frame: $\mathcal{F}_a = (m_a, f_a) = [\mathrm{Ad}_{T_{ba}}]^{\mathrm{T}}\mathcal{F}_b$

3.6 Software

The following functions are included in the software distribution accompanying the book. The code below is in MATLAB format, but it is available in other languages. For more details on the software, consult the code and its documentation.

```
invR = RotInv(R)
```