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## **Rating and Life Formulas for Tapered Roller Bearings**

**Werner K. Dominik**

The Timken Co.

Canton, OH

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# Rating and Life Formulas for Tapered Roller Bearings

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## ABSTRACT

In the fall of 1982, the author's company announced a revised dynamic load rating for its tapered roller bearings. Enhanced bearing steel quality brought about by improved steel making processes precipitated the revision in ratings. The paper presents the new rating formula for the basic dynamic load ratings of Timken tapered roller bearings. It provides the theoretical model of the rating equation with a summary of laboratory life test results that supplied the data to evaluate constants which correlate experiment with the model. It includes equations for equivalent loads, life calculations, and the rating reference conditions, plus discussions of life adjustment factors to guide the designer in applying the ratings in the analysis of bearing applications.

ANTI-FRICTION BEARING TECHNOLOGY has, in the decade of the nineteen-seventies, made some remarkable advances. Bearing companies and independent investigators researched and quantified many environmental effects on the fatigue life of anti-friction bearings. Examples include the effects of misalignment and of lubrication, (1, 2)\*. The results of these efforts led to life adjustment factors, which enable the designer to more reliably predict bearing performance, provided he can define the environment of his application.

Manufacturers are applying this knowledge to improve bearing designs, e.g., through modified internal contact geometry to better tolerate misalignment, or by better surface finishes to help performance with marginal lubrication, or with cleaner, more costly premium bearing steel to extend inclusion fatigue life (3).

The primary conventional material for tapered roller bearings traditionally has been a bearing quality vacuum degassed carburizing grade steel. This material is of special quality; still, it contains oxide type nonmetallic inclusions that can initiate fatigue type spalling in the bearing contact surfaces. Special processes are utilized to produce the cleaner premium steels. These steels contain significantly fewer and smaller oxide inclusions and, therefore, exhibit superior fatigue endurance qualities (4). The most widely used premium steel is produced by consumable electrode vacuum arc remelting of the conventional bearing material. The increased cost of the material limits its use to special high demand or high reliability applications (3).

More recent developments have led to alternate techniques for producing bearing quality steels. The new methods employ precipitation-deoxidation processes that are usually coupled with other in-ladle refining (4). They also are much less costly than vacuum remelting.

\*Numbers in parentheses designate references at the end of the paper.

The author's company has installed new ladle refining capability to produce premium quality air melt steels. Fatigue testing of production bearings made from material produced by the new processes has demonstrated consistently superior fatigue life compared to the material produced by the traditional process. The tests suggested an upgrading in the basic dynamic load rating of the bearings to be in order.

This observation prompted systematic testing and analysis of life test data which involved calculation of experimental ratings based on actual test life of each group of bearings adjusted for environmental operating conditions. The experimental ratings were used to evaluate two rating models in order to determine which model would best predict the observed performance. The first model relates capacity to contact stress and stress cycles per revolution (6, 7), modified for bearing size. The second relates capacity to subsurface shear stress, stress cycles and the volume of material stressed (8, 9). The latter involves the experimental constants of load-life exponent and Weibull slope.

Good agreement between experimental ratings and the stressed volume model was attained by utilizing a Weibull slope of 1.5 and a load-life exponent of  $10/3$ ; these values are also almost universally used by the roller bearing industry when adjusting life for reliability and load, respectively.

This work formed the basis for a revision to the basic dynamic load ratings for Timken tapered roller bearings which were published in the fall of 1982. All references in this paper to tapered roller bearings are with respect to product manufactured by The Timken Company. The same rating model is the basis of the Anti-Friction Bearing Manufacturers Association (AFBMA) and International Standards Organization (ISO) standard ratings for roller bearings (10, 11); however, the rating equation presented here differs from the standard because of differences in the experimental constants. Accordingly, a

rating proposal has been submitted to the AFBMA load rating subcommittee for consideration as the standard for tapered roller bearings.

This paper includes much of the information pertaining to the dynamic load rating of tapered roller bearings that was presented to the AFBMA committee. It presents a summary of the pertinent terminology, the rating equation, the equivalent load and life equations plus experimental data in support of the ratings. It defines and stresses the importance of environmental reference conditions and life adjustment factors. The Appendix to this paper contains, for the interested reader, the details of the theoretical rating model and the values of the constants that can only be derived experimentally.

For the present, the publication of the revised ratings is limited to standard tapered roller bearings through 180mm (7.125") bore; ratings of larger bearings and of thrust bearings are calculated using a 1962 rating equation (6), pending the generation and evaluation of sufficient test data to determine the experimental constants that are applicable to these sizes and bearing types.

#### DEFINITION OF BASIC TERMS

Bearing manufacturers assign load ratings to bearings that permit designers to calculate bearing life expectancy in applications. This so called "rating life"  $L_n$ , is commonly defined as the life in revolutions that (100-n) percent of a group of apparently identical bearings will complete or exceed. In terms of an individual bearing (100-n) is the reliability of that bearing; i.e., the probability that its life will exceed  $L_n$  revolutions. The standard rating life for the bearing industry is  $L_{10}$  or 90% reliability life.

The "basic load rating"  $C_m$  is defined as that calculated constant load (radial or thrust) which a group of apparently identical bearings can theoretically endure for a rating life of  $m$  million revolutions. A widely used rating is defined at  $m = 90$

million; i.e.,  $C_{90}$  is the reference for that load ratings. Another standard reference at which bearings are commonly rated is 1 million revolutions,  $C_1$ . Since bearings operate at loads other than either of the rating definitions, it matters only that the proper life equation is used with the respective rating. The  $C_{90}$  rating is in line with many application loads, whereas application loads approaching  $C_1$  are less common and excessive.

Tapered roller bearings are designed to carry combinations of radial and thrust loads. Special cases of loading are the case when the combined load is such that the bearing load zone approaches  $180^\circ$  (also referred to as "radial loading") and when only thrust is applied to the bearing. These special cases are the reference loading for the basic radial and basic thrust ratings, respectively.

Bearings routinely operate at other combinations of radial and thrust loads. Designers convert the combined load into an equivalent radial load  $P$  in order to estimate bearing life from the basic bearing rating. The "equivalent radial load" is defined as that calculated constant radial load which would give the same life as that which the bearing will attain under the actual conditions of load and speed.

#### THE RATING EQUATION FOR TAPERED ROLLER BEARINGS

**BASIC RADIAL RATING** - Figure 1 depicts a tapered roller bearing. Its tapered construction makes it well suited for carrying radial and thrust loads in any combination. It has both radial and thrust load ratings.

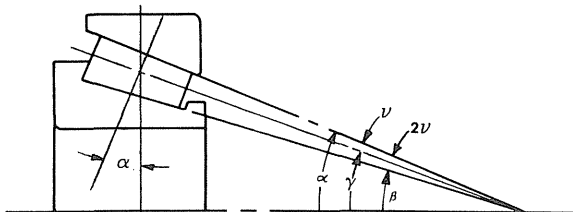


Figure 1 - Tapered Roller Bearing

Equation (1) gives the "basic radial load rating" for Timken tapered roller bearings.

$$C = f_c (i l_{\text{eff}} \cos \alpha)^{4/5} z^{7/10} D_{\text{wo}}^{16/15} \quad (1)$$

The factor  $f_c$  is the product of a material constant  $M$  and a geometry dependent factor  $H$ , i.e.:

$$f_c = M \times H \quad (2)$$

The Appendix to this paper contains a summary of the assumptions, the rating model plus a brief derivation of the rating equation, and a definition of  $f_c$  and its components,  $H$  and  $M$ . For an in-depth study of the subject, the reader is directed to references (8, 9).

Differences in the equations for  $H$  and  $C$  between the calculation presented here and references (8, 9) are in the experimental constants of the life dispersion parameter  $e$  (Weibull slope) and the load-life exponent  $p$ . When Lundberg and Palmgren (8, 9) developed their model, roller bearing experiments yielded an applicable Weibull slope  $e = 9/8$  and the load-life exponent  $p = 4$ ; these values are still implicit to the AFBMA rating equation for roller bearings. At the same time, the standards (10, 11), as well as the anti-friction bearing industry in general have since adopted  $e = 3/2$  and  $p = 10/3$  for reliability (above 90%) and life calculations, respectively. These values compare well with averages of the test data used for deriving the experimental constants presented here. Table 1 illustrates the above differences.

Table 1  
Experimental Constants

Constant	AFBMA Standard 11 (10)			Proposed Constant (Consistent)
	$C \& f_c$	Life	Reliability	
Load/Life Exponent $p$	4	10/3	-	10/3
Weibull Slope $e$	9/8	-	3/2	3/2

Again, the interested reader is referred to the Appendix which demonstrates how the experimental constants determine the exponents in the rating, equation (1).

The constant  $H$  in equation (2) is a function of bearing geometry or, specifically the ratio  $\tan \nu /$



$\tan \gamma$ . Table 2 lists the values of H for tapered roller bearings as a function of this parameter.

The ratio  $\tan \psi / \tan \gamma$  is the tapered roller bearing equivalent of  $D_{wo} \cos \alpha / d_m$  which is used in calculating the H equivalent for anti-friction bearings in (10, 11), for example. In fact, for all practical purposes the values of either calculations are the same.

Table 2  
Values of H

$\frac{\tan \psi}{\tan \gamma}$	H	$\frac{\tan \psi}{\tan \gamma}$	H
0.01	0.340	0.26	0.514
0.02	0.390	0.27	0.509
0.03	0.423	0.28	0.505
0.04	0.447	0.29	0.500
0.05	0.466	0.30	0.495
0.06	0.481	0.31	0.490
0.07	0.494	0.32	0.484
0.08	0.504	0.33	0.479
0.09	0.512	0.34	0.473
0.10	0.519	0.35	0.467
0.11	0.525	0.36	0.461
0.12	0.529	0.37	0.455
0.13	0.532	0.38	0.449
0.14	0.535	0.39	0.443
0.15	0.536	0.40	0.436
0.16	0.537	0.41	0.430
0.17	0.537	0.42	0.423
0.18	0.536	0.43	0.416
0.19	0.535	0.44	0.410
0.20	0.533	0.45	0.403
0.21	0.531	0.46	0.396
0.22	0.528	0.47	0.389
0.23	0.525	0.48	0.382
0.24	0.521	0.49	0.375
0.25	0.518	0.50	0.368

The material constant M is determined from life tests with tapered roller bearings. M correlates the experimental results with a theoretical equation. These are the values for M applicable to the rating equation 1, for a rating life of 90 million revolutions,  $C_{90}$ :

$M = 51.9$  to obtain  $C_{90}$  in newtons when  $\ell_{eff}$  and  $D_{wo}$  are given in millimeters

$M = 4890$  to obtain  $C_{90}$  in pounds when  $\ell_{eff}$  and  $D_{wo}$  are given in inches.

BASIC THRUST RATING - The "Basic Thrust Load Rating" for tapered roller bearings is given by equation (3):

$$C_a = f_{ca} (\ell_{eff} \cos \alpha)^{4/5} z^{7/10} D_{wo}^{16/15} \quad (3)$$

The  $f_{ca}$  factor is derived from the  $f_c$  factor -- equation (2) and Table 2 -- using

$$f_{ca} = 2.564 f_c \tan \alpha \quad (4)$$

The relationship between Basic Radial Load Rating and Basic Thrust Load Rating for tapered roller bearings is provided by the bearing K-factor, i.e.:

$$K = C/C_a = 0.39 \cot \alpha \quad (5)$$

TWO-ROW RATING - The "Basic Radial Rating" of two-row tapered roller bearings is determined from the system life of the two rows using a Weibull slope of  $e = 3/2$  and the load-life exponent of  $p = 10/3$ , i.e., consistent with the value in the rating equation.

$$C_{(2)} = 2 (2^{-1/e})^{1/p} C \quad (6)$$

$$C_{(2)} = 2^{0.8} C = 1.74 C \quad (7)$$

This rating applies to two-row bearings in which both rows are loaded equally and which constitute a single shaft support in conjunction with another support. In cases where the loads are unequal or where the rows can be treated as individual bearing supports, each row is considered a separate bearing, and the single-row rating applies. For similar reasons the basic radial load rating of a four-row assembly  $C_{(4)}$  is taken as two times the two-row rating; i.e.,  $C_{(4)} = 2 \times C_{(2)}$ .

RATINGS FOR ONE MILLION REVOLUTION RATING LIFE - As was noted earlier, many manufacturers of anti-friction bearings, as well as the AFBMA and ISO Standards (10, 11) publish ratings and rating equations for a rating life of 1 million revolutions,  $C_1$ .

For tapered roller bearings, to convert the ratings from 90 to 1 million revolutions multiply by  $90^{1/p}$  or  $90^{3/10}$ .

$$C_1 = 90^{3/10} C_{90} = 3.86 C_{90} \quad (8)$$

# RATING LIFE AND EQUIVALENT LOADING

**RATING LIFE** - The rating life  $L_{10}$  in millions of revolutions for tapered roller bearing applications is:

$$L_{10} = 90 \left( \frac{C_{90}}{P} \right)^{10/3} \quad (9)$$

Tapered roller bearing loads in applications can vary over a wide spectrum of light to heavy loading. The load-life exponent of  $p = 10/3$  applies if contact stress in the bearing at the mid point of the most heavily loaded roller does not exceed 2800 MPa (400 000 psi). For life estimates at higher loads, the manufacturer should be consulted.

**EQUIVALENT LOADS** - The equivalent load provides for life estimates of bearings under combined radial and thrust loads. It is calculated from

$$P = X F_r + Y F_a \quad (10)$$

Table 3 lists the values for X and Y as functions of the thrust to radial load ratio ( $F_a/F_r$ ). A radial load on a tapered roller bearing induces a thrust reaction whose magnitude depends on bearing load zone, cup angle, and the radial load. A minimum thrust is required to keep the cone-roller assembly and cup from separating. When the ( $F_a/F_r$ ) ratio is less than r (Table 3) the bearing is considered to be carrying the "radial load only", when in reality it is a special case of combined loading. When only thrust is applied to the bearing, (i.e., when  $F_r = 0$ ), then  $P = Y F_a$ . Alternately, life in million revolutions may be estimated using the thrust load and the basic thrust load rating.

$$L_{10} = 90 \left( \frac{C_{a90}}{F_a} \right)^{10/3} \quad (11)$$

Double-row bearings normally require equivalent loads for each row. Values of X and Y in Table 3 provide for calculating equivalent loads  $P_A$  and  $P_B$  on row A and row B respectively;  $F_a$  is on row A.

Table 3\*  
Values Of X And Y

Bearing Type	$\frac{F_a}{F_r} \leq r$		$\frac{F_a}{F_r} > r$		r
	X	Y	X	Y	
Tapered Roller Bearings	Single Row Bearings				0.6K
	1	0	0.4	K	
	Double Row Bearings				0.6K
Row A:	0.5	0.85K	0.4	K	
Row B:	0.5	0.85K	0	0	

\*Note,  $K = 0.39 \cot \alpha$

The single-row rating applies when calculating life of Row A and Row B. The combined life of the double row bearing  $L_{10AB}$  may be obtained by calculating the system life from:

$$L_{10AB} = (L_{10A}^{-3/2} + L_{10B}^{-3/2})^{-2/3} \quad (12)$$

**RATING REFERENCE CONDITIONS** - The  $f_c$  factor in the rating equation is derived from bearing life tests. The material constant M is a direct measure of the material quality. Implicit in H are the bearing load-life exponent and the life dispersion parameter or Weibull slope, both of which are determined experimentally. It is a generally recognized fact that operating parameters in addition to load influence bearing life. The rating reference conditions are the operating conditions at which the basic load ratings apply. The rating equations are valid for the following assumptions and reference conditions:

Weibull Slope:  $e = 1.5$   
 Material: Bearing quality hardened steel  
 Reliability: 90%  
 Internal Clearance: Nominal internal clearance, i.e., load zone  $150^\circ$  to  $180^\circ$   
 Lubrication: Oil Viscosity, 33cSt @  $55^\circ\text{C}$  (155 SUS @  $130^\circ\text{F}$ )  
 Operating Oil Temperature =  $55^\circ\text{C}$  ( $130^\circ\text{F}$ )  
 Bearing Speed = 500 rpm  
 Alignment: 0 to 0.001 radian  
 Spall-Size: 6 mm<sup>2</sup> (0.01 in<sup>2</sup>)  
 Loading: 0 to 3  $C_{90}$

If the operating conditions differ significantly from the reference conditions, if the material is nonstandard, or if the criterion for reliability changes, the life estimate should be adjusted accordingly.

## LIFE ADJUSTMENT FACTORS

**ADJUSTMENT FOR RELIABILITY** - The life estimate calculated from the rating and equivalent load -- equation (9) -- is  $L_{10}$  or 90% reliability life. Some applications require life definition at other reliabilities, usually higher than 90%.

To determine life for a different reliability, the rating life  $L_{10}$  must be adjusted by a factor  $a_1$  such that  $L_n = a_1 L_{10}$ .

$$a_1 = \left[ \frac{\ln(\frac{R}{100})}{\ln 0.9} \right]^{1/e} \quad (13)$$

For tapered roller bearings  $e = 1.5$ , and equation (13) reduces to

$$a_1 = 4.48 \left[ \ln \left( \frac{100}{R} \right) \right]^{2/3} \quad (14)$$

**ADJUSTMENT FOR MATERIAL** - The published ratings apply to the standard product, i.e., standard material and processing. Also available are bearings made from special steels. Examples include premium steel bearings for extended life or high reliability, such as consumable electrode vacuum remelted (CEVM) or vacuum induction melted, vacuum arc remelted (VIMVAR) steels, as well as special steel bearings for high temperature operating conditions.

Special materials usually require that the life be adjusted for a material factor such that

$$L_{10a} = a_2 90 \left( \frac{C_{90}}{P} \right)^{10/3} \quad (15)$$

The factor  $a_2$  is influenced by application related conditions and by the type of damage that is to be anticipated under prevailing conditions. The designer should consult the bearing manufacturer on questions concerning the needs for or the effects of special materials.

**ADJUSTMENT FOR APPLICATION CONDITIONS** - Rating life should be adjusted for application conditions if they are much different than the reference conditions defined earlier. Conditions that have a significant effect and which have been quantified include the internal bearing load distribution, lubrication, and alignment between inner and outer race. The

adjustment factor  $a_3$ , where  $L_{10a} = a_3 90 (C_{90}/P)^{10/3}$ , is made up of these three factors:

$$a_3 = a_{3k} a_{3\ell} a_{3m} \quad (16)$$

Where  $a_{3k}$ ,  $a_{3\ell}$ , and  $a_{3m}$  are the life adjustment factors for load zone, lubrication, and alignment, respectively. At the reference conditions these factors are 1.0.

**Load Distribution (Load Zone)** - The rating reference condition for radial ratings is a nominal internal clearance ( $150^\circ$  to  $180^\circ$  load zone); i.e., approximately one half the rollers are transmitting loads between inner and outer races. It assumes that inner and outer races remain perfectly circular. In most applications the load zone effect on life is approximated by the calculation of the equivalent radial load  $P$  (Table 3) which considers the effect of thrust on bearing loading. When that thrust is less than required for  $180^\circ$  load zone, (i.e., the bearing has axial clearance), the bearing is assumed to be operating within the nominal clearance of the reference condition and  $a_{3k} = 1$ , when life is calculated from  $L_{10a} = a_3 90 (C_{90}/P)^{10/3}$ .

In general, tapered roller bearings are mounted in pairs. The load distribution is affected by shaft loading, bearing contact angles, axial mounting clearance, temperature gradients, axial support stiffness, and support sections. Consideration of these effects to more accurately determine the load zone and its influence on life requires analytical techniques beyond the scope of this paper; therefore, the user should consult the manufacturer for evaluations and recommendations.

**Lubrication** - Lubrication and related operating effects include those of lubricant viscosity, operating temperature and bearing speed. When any of these deviate substantially from the reference conditions, life should be adjusted by the lubrication life factor  $a_{3\ell}$ . In general, higher viscosity lubricants, higher speeds and lower temperatures yield favorable life; i.e.,  $a_{3\ell}$  is greater than 1.0. Conversely,



lower viscosity lubricants, lower speeds or higher temperatures may reduce life; i.e.,  $a_3$  less than 1.0. Manufacturers' catalogs, e.g., (12), normally provide graphs and equations that allow the designer to adjust his application for lubrication related effects.

**Alignment** - While ideally tapered roller bearings should be perfectly aligned, some amount of misalignment is virtually always present in an application and is allowed for in the bearing rating. When misalignment exceeds the maximum reference value, its impact should be considered in the life analysis.

**LIFE ADJUSTMENT FOR USEFUL LIFE** - In laboratory life tests a spall size of 6 mm<sup>2</sup> (0.01 in.<sup>2</sup>) has been established as the damage criterion at which bearing life is terminated. Depending on tolerance levels for accuracy and sound, applications may tolerate larger spalls before bearings must be replaced. A useful life factor  $a_4$ , which is a function of spall area and which generally depends on application conditions, can be used to estimate the additional life; i.e.,  $L_{10a} = a_4 90 (C_{90}/P)^{10/3}$ . Again, the user should obtain specifics regarding the useful life from the bearing manufacturer.

**COMBINING LIFE ADJUSTMENT FACTORS** - A fatigue life formula embodying the foregoing life adjustment factors is:

$$L_{na} = a_1 a_2 a_3 a_4 90 (C_{90}/P)^{10/3} \quad (17)$$

The adjustment factors are not necessarily independent; for example, an adverse lubrication factor may not be overcome by improved material. Lubrication also influences the life adjustment factor for useful life. Since fatigue life is only one criterion for bearing selection, care must be exercised to select tapered roller bearings which are of sufficient size for other application requirements, such as rigidity, shaft strength, etc.

#### EXPERIMENTAL VERIFICATION OF RATING EQUATION

Extensive life testing of bearings made from bearing material manufactured to state-

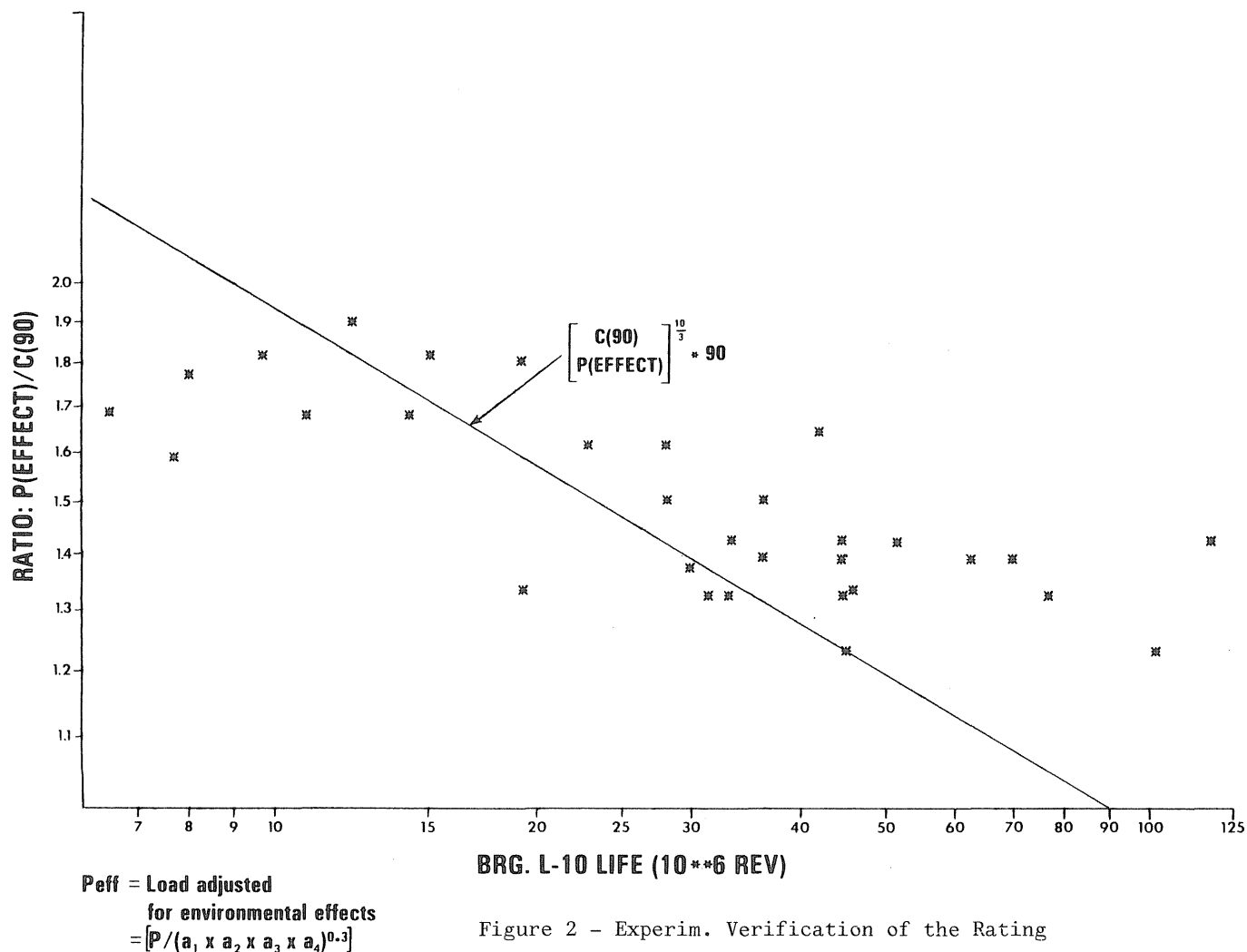
of-the-art steel melting practices and by standard manufacturing processes preceded the revision of the ratings.

Figure 2 is a plot of the results representing 30 tests and 17 bearing series ranging in size from 28 mm to 121 mm (1.1" to 4.8") pitch diameter. Each test point represents at least 32 bearings tested. The ordinate is the ratio of effective equivalent radial load over the basic radial load rating  $C_{90}$ . The effective equivalent load is the equivalent bearing load adjusted for the life adjustment factors:

$$P_{eff} = \frac{P}{(a_1 a_2 a_3 a_4)^{0.3}} \quad (18)$$

On the abscissa are plotted the  $L_{10}$  or 90% survival life. The solid line represents the predicted life using the basic load rating and life equations presented in this paper, as well as the life adjustment factors discussed.

Considering bearing  $L_{10}$  life scatter normally experienced from one bearing sample to another, the plotted points show how well actual life agrees with predicted life, or how well the basic load rating equation predicts the actual bearing capacity. The agreement is reasonable overall and somewhat conservative in the range of loading normally encountered in applications; e.g., survival life 20 million revolutions and above.



## CONCLUSION

The rating equation for tapered roller bearings presented here is based on a theoretical model that has wide acceptance in the rating of anti-friction bearings and, equally important, is backed by comprehensive fatigue testing of production bearings in the laboratory. Increased ratings, coupled with improved analysis methods that account for environmental influences on bearing life, provide the designer with the technology to meet today's demands for efficient, reliable, and economical designs. In machine design past "rule of thumb" approaches are being replaced by in-depth analytical solutions. The result is less conservatism with accompanying higher demands on machine components. The improvement in bearing quality and fatigue life must be welcomed solutions to some of these demands.

NOMENCLATURE			
$a_1$	Life adjustment factor for reliability	$C_{90(2)}$	Basic dynamic radial load rating of a two-row bearing for an $L_{10}$ of 90 million revolutions
$a_2$	Life adjustment factor for bearing material	$C(2)$	Basic dynamic radial load rating of a two-row bearing
$a_3$	Life adjustment factor for environmental conditions	$C(4)$	Basic dynamic radial load rating of a four-row bearing
$a_{3k}$	Life adjustment factor for load zone	$d_m$	Mean pitch diameter of the roller complement
$a_{3\ell}$	Life adjustment factor for lubrication	$d_o$	Mean inner race outside diameter
$a_{3m}$	Life adjustment factor for alignment	$D_{wo}$	Mean roller diameter
$a_4$	Life adjustment factor for useful life (spall size)	$e$	Measure of life scatter, i.e. Weibull slope
A, B, ...	Bearing position, used as subscripts	$f_c$	Bearing geometry-material constant as it influences the basic radial load rating
c	Experimental material characteristic (constant)	$f_{ca}$	Bearing geometry-material constant as it influences the basic thrust load rating
C	Basic dynamic radial load rating	F	Function describing probability of failure
$C_a$	Basic dynamic axial load rating	$F_a$	Thrust load on bearing
$C_{a90}$	Basic dynamic thrust load rating of a single-row bearing for an $L_{10}$ of 90 million revolutions	$F_{ae}$	External applied thrust load
$C_m$	Basic dynamic radial load rating for m million revolutions	$F_r$	Radial load on bearing
$C_1$	Basic dynamic radial load rating for 1 million revolutions	G	Mean circumference of raceway
$C_{90}$	Basic dynamic radial load rating of a single-row bearing for an $L_{10}$ of 90 million revolutions	h	Experimental material characteristic (constant)
		H	Geometry constant in rating equation (Table 1)

i	Number of bearing rows in assembly (1 or 2)	m	Subscript denoting the millions of revolutions ascribed to a rating, e.g., $C_m$
$J_r$	Sjovall's radial load integral		
K	K-factor. Ratio of basic dynamic radial load rating to basic dynamic thrust load rating in a single-row bearing	M	Material constant in the dynamic rating equation
$K_b$	Load function	n	Subscript denoting the percent failed of a population, e.g. $L_n$
$l_{eff}$	Effective roller contact length	N	Number of stress applications to a point on the raceway
$ln$	Logarithm to base e (where e = 2.718 approximately)	p	Load-life fatigue exponent
$L_n$	General term for life. The bearing life expectancy associated with (100-n) percent reliability	P	Equivalent dynamic radial bearing load
$L_{na}$	Adjusted life. The rating life adjusted for a reliability of (100-n) percent plus for any or all of the following: bearing material, operating conditions, and crack propagation	$P_{eff}$	Effective equivalent load, i.e., P adjusted for reliability, material, operating conditions, and crack propagation
$L_{10}$	Rating life. The bearing life expectancy associated with 90 percent reliability	q	Exponent defined in equation (13), Appendix
$L_{10a}$	Adjusted rating life. The bearing life expectancy associated with 90 percent reliability and adjusted for any or all of the following: material, operating conditions, and useful life	r	A function of the cup angle used to determine equivalent loads
$L_{10}(\text{SYSTEM})$	Rating system life of two or more bearings	R	Reliability (%)
		S	Probability of survival
		V	Volume of material under stress
		x	Time to failure
		X	Radial load factor
		Y	Axial load factors
		$z_o$	Depth to the maximum orthogonal shear stress
		Z	Number of rollers per bearing row
		$\alpha$	Bearing 1/2 included cup angle

$\beta$	Bearing 1/2 included cone angle
$\gamma$	Bearing 1/2 included roller centerline angle
$\theta$	Characteristic Life
$\kappa$	Ratio of inner to outer race material and geometry factor
$\rho$	Ratio $\tan \nu / \tan \gamma$
$\tau_0$	Maximum orthogonal shear stress
$\nu$	Bearing 1/2 included roller angle

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## APPENDIX

## THE MODEL FOR THE BASIC DYNAMIC LOAD RATING

The fatigue life of anti-friction bearings is a statistical phenomenon that has been observed to follow a Weibull distribution. The probability of failure  $F$  is given by:

$$F = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^e\right] \quad (1)$$

where  $e$  = measure of failure scatter as related to  $F$ ; i.e., Weibull slope.

$x$  = time to failure

$\theta$  = characteristic life

The probability of survival  $S$  then is:

$$S = 1 - F \quad (2)$$

$$S = \exp\left[-\left(\frac{x}{\theta}\right)^e\right] \quad (3)$$

$$\frac{1}{S} = \exp\left[\left(\frac{x}{\theta}\right)^e\right] \quad (4)$$

or

$$\ln\left(\frac{1}{S}\right) = \left(\frac{x}{\theta}\right)^e \quad (5)$$

Earlier investigators (8, 9) assumed that the time to failure  $x$  is a function of the maximum orthogonal shear stress  $\tau_0$ , the number of stress cycles  $N$ , the volume of material under stress  $V$ , and the depth to the maximum shear stress  $z_0$ , i.e.:

$$x = f(\tau_0, N, V, z_0) \quad (6)$$

where  $S$  = probability of survival

$\tau_0$  = maximum orthogonal shear stress amplitude

$N$  = number of stress applications to a point on the raceway

$V$  = volume of material under stress

$z_0$  = depth to the maximum orthogonal shear stress

Using the fact that  $\ln(1/S)$  is proportional to time,  $\ln(1/S) \sim x^e$ , the investigators assumed the following power function to develop the rating of anti-friction bearings:

$$\ln\left(\frac{1}{S}\right) \sim \frac{\tau_0^c N^e V}{z_0^h} \quad (7)$$

where  $h, c$  = material characteristics that must be determined experimentally.

For tapered roller bearings volume under stress ( $V$ ) is assumed to be proportional to the effective contact length ( $\ell_{\text{eff}}$ ), the mean circumference of the raceway ( $G$ ) under consideration, and the depth ( $z_0$ ) to the maximum orthogonal shear stress ( $\tau_0$ ).

$$V \sim \ell_{\text{eff}} G z_0 \quad (8)$$

where  $\ell_{\text{eff}}$  = effective contact length

$G$  = mean circumference of raceway

Substituting (2) in (1):

$$\ln\left(\frac{1}{S}\right) \sim \frac{\tau_0^c N^e \ell_{\text{eff}} G}{z_0^{h-1}} \quad (9)$$

For a given bearing  $\tau_0, N, G, \ell_{\text{eff}}$ , and  $z_0$  can be expressed in terms of bearing geometry, load, and revolutions. The relationship (9) is changed to an equation by inserting the constant of proportionality. Inserting a specific number of revolutions (e.g., 90 000 000) and a specific reliability (e.g.,  $S = 0.9$ ), the equation is solved for a special load which is designated the bearing rating. The constant of proportionality must be determined experimentally and is essentially the fatigue constant for the particular material of the bearing tested at given environmental conditions.

## BASIC DYNAMIC RADIAL LOAD RATING FOR TAPERED ROLLER BEARINGS

For tapered roller bearings of the design depicted in Figure 1, (main text) the result is the following:

$$C = f_c (i \ell_{\text{eff}} \cos \alpha)^{\frac{c-h-1}{c-h+1}} Z^{1-1/p} D_{wo}^{\frac{c+h-3}{c-h+1}} \quad (10)$$

Where  $C$  = Basic radial rating

$f_c = M H$  = material factor

( $M$ ) times geometry factor ( $H$ )

$i$  = number of rows (1 or 2)

$\alpha$  = contact angle (1/2

included cup angle)

$Z$  = number of rollers

$D_{wo}$  = mean roller diameter

$$\frac{c-h-1}{c-h+1} = 1-1/(pe) = \text{exponent for } (i \ell_{\text{eff}} \cos \alpha) \text{ term} \quad (11)$$

$$H = \frac{(\rho)^{1/(pe)} (1-\rho)^q}{[1+\kappa \left(\frac{1-\rho}{1+\rho}\right)^{e(1+pq)}]^{1/(pe)} (1+\rho)^{1/p}} \quad (12)$$

where  $\rho = \tan \nu / \tan \gamma$   
 $= D_{wo} \cos \alpha / d_m$   
 $\nu = 1/2$  included roller angle  
 $\gamma = 1/2$  included roller  
center line angle

$$q = \frac{c+h-3}{c-h+1} \quad (13)$$

$\kappa$  = ratio of inner to outer  
race material and geometry factor  
(line contact, no edge stress  
concentration)

$$\kappa = \left[ \frac{21.2}{20.4} \right]^{pe} \quad [\text{reference (8)}] \quad (14)$$

M = material factor (experimental)

Extensive life tests of  
tapered roller bearings have  
yielded the following experi-  
mental constants:

$$e = 3/2 \text{ (Weibull slope)} \quad (15)$$

$$p = 10/3 \text{ (load-life exponent)} \quad (16)$$

$$h = 7/3 \text{ (material characteristic)} \quad (17)$$

$$c = 34/3 \text{ (material characteristic)} \quad (18)$$

$$M = 51.9 \text{ to calculate rating in Newtons} \quad (19)$$

for 90 000 000 revolutions  
( $C_{90}$ ), when  $\ell_{eff}$  and  $D_{wo}$  are in mm

$$= 4890 \text{ to calculate rating in pounds for} \quad (20)$$

90 000 000 revolutions ( $C_{90}$ ) when  
 $\ell_{eff}$  and D are in inches.

Substituting (16), (17) &  
(18) into (10) yields the rating  
equation:

$$C = f_c (i \ell_{eff} \cos \alpha)^{4/5} Z^{7/10} D_{wo}^{16/15} \quad (21)$$

Substituting (17) and (18)  
into (13) and (15) and (16) into  
(16) and (12) yields, respectively

$$q = 16/15 \quad (22)$$

$$\kappa = (21.1/20.4)^5 \quad (23)$$

= 1.212

$$H = \frac{\rho^{1/5} (1-\rho)^{16/15}}{[1+1.212 \left(\frac{1-\rho}{1+\rho}\right)^{41/6}]^{1/5} (1+\rho)^{3/10}} \quad (24)$$

$$f_c = 51.9 H \text{ (to calculate rating in Newtons for} \quad (25)$$

90 000 000 revolutions,  $C_{90}$ )

$$= 4890 \text{ (to calculate rating in pounds for} \quad (26)$$

90 000 000 revolutions,  $C_{90}$ )

## BASIC DYNAMIC THRUST LOAD RATING FOR TAPERED ROLLER BEARINGS

The tapered construction of  
the bearing makes it suitable to  
carry thrust loads as well as  
radial loads and combinations of  
these.

Under pure thrust load the  
stresses and stress cycles per  
bearing revolution can be equated  
to those under radial loading  
(180° load zone) using the  
bearing cup angle to derive a  
thrust rating  $C_a$ :

$$C_a = A C \cot \alpha \quad (27)$$

Where A is dependent on the  
load integral  $J_r$  (Sjovall's) of  
the roller loading in the radial  
plane at 180° load zone and the  
corresponding load function  $K_b$ :

$$A = \left( \frac{J_{r180}}{K_{b180/2}} \right)^{3/10} \quad (28)$$

$$\text{where } J_{r180} = \frac{2}{\pi} \int_0^{\pi} \cos^{5/2} \psi d\psi$$

$$= 0.2288$$

$$K_{b180} = \frac{2}{\pi} \int_0^{\pi} \cos^5 \psi d\psi$$

$$= 0.3395$$

Then,  
 $A = 0.39$

Writing a material/geometry  
factor for thrust rating  $f_{ca}$ :

$$f_{ca} = 2.564 f_c \tan \alpha \quad (29)$$

$$C_a = f_{ca} (i \ell_{eff} \cos \alpha)^{4/5} Z^{7/10} D_{wo}^{16/15} \quad (30)$$

Tapered roller bearing  
catalogs contain a factor K,  
which defines the ratio of  
bearing radial rating to thrust  
rating:

$$K = C/C_a = 0.39 \cot \alpha \quad (31)$$

# DYNAMIC EQUIVALENT RADIAL LOAD FOR SINGLE ROW TAPERED ROLLER BEARINGS

Combinations of radial and thrust loads result in load-zones other than 180°. The effect on fatigue life in terms of equivalent radial load can be calculated for any combined load from:

$$P = F_r \left( \frac{K_b}{K_{b180}} \right)^{3/10} \left( \frac{J_{r180}}{J_r} \right) \quad (32)$$

$$\text{or } P = 0.3164 F_r \frac{K_b^{3/10}}{J_r} \quad (33)$$

$J_r$  and  $K_b$  are functions of the bearing cup angle  $\alpha$ , the bearing radial load  $F_r$ , and bearing axial load  $F_a$ . Tabulations of  $J_r$  and  $K_b$  versus  $F_r \tan \alpha / F_a$  can be found in references (8, 9).

In practical applications bearing operating clearance is usually small. With this consideration, the equivalent radial load equations that closely approximate equation (33) are as follows:

$$\text{When } F_a / F_r > 1.54 \tan \alpha \quad (34)$$

$$P = 0.4 F_r + 0.39 \cot \alpha F_a$$

$$\text{or } P = 0.4 F_r + K F_a$$

$$\text{When } F_a / F_r \leq 1.54 \tan \alpha \quad (35)$$

$$P = F_r$$

In the special case  $F_r = 0$ ,  $F_a \neq 0$ , the user may either use the equivalent load, equation (34), and compare against the basic radial load rating  $C$ , or he may use  $F_a$  directly and compare with the basic thrust load rating  $C_a$ .

# DYNAMIC EQUIVALENT RADIAL LOAD FOR DOUBLE ROW TAPERED ROLLER BEARINGS

Double row tapered roller bearings are typically mounted as either axially fixed or floating supports (Fig. 1A).

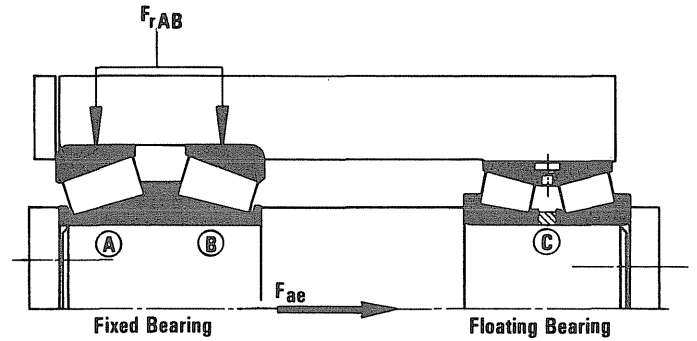


Figure 1A - Double Row Tapered Roller Bearing Mountings

One row of the fixed assembly (the "seated" row A in Fig. 1A) carries any unbalanced thrust in the system (thrust may be either externally applied or induced by the opposing bearing row - the "set-up" row B in Fig. 1A - in the fixed assembly). Assuming that the total radial load on that fixed assembly ( $F_r$ ) is constant ( $F_r = F_{rA} + F_{rB}$ ), it is reasonable to expect that the seated row carries a greater share of  $F_r$  than the set-up row. Depending on mounted clearance, support stiffness, bearing stiffness, and the magnitude of the external thrust  $F_{ae}$ , it is possible to "unload" the set-up row. The analysis considering all influencing parameters is complex; however, approximate equivalent loads can be calculated using the simplifying assumptions that the radial loads on each row are proportional to thrust loading; i.e., the thrust on each row is a constant ratio of its radial load and cup angle.

Designating the externally applied thrust  $F_{ae}$ , we can write:

$$F_r = F_{rA} + F_{rB} \quad (36)$$

$$F_{aB} = 1.54 F_{rB} \tan \alpha \quad (37)$$

$$F_{aB} = 1.54 F_{rA} \tan \alpha \quad (38)$$

$$F_{aA} = F_{ae} + 1.54 F_{rB} \tan \alpha \quad (39)$$

Equating (32) and (33) and solving simultaneously with (30):

$$F_{rA} = 0.5 F_r + 0.33 \cot \alpha F_{ae} \quad (40)$$

$$F_{rB} = 0.5 F_r - 0.33 \cot \alpha F_{ae} \quad (41)$$

As long as  $F_{ae}$  is such that  $F_{rB}$  is greater than zero, i.e.,  $F_{ae}/F_r < 1.54 \tan \alpha$ , the equivalent loads [corresponding to single row case, equation (35)] are:

$$P_A = F_{rA} \quad (42)$$

$$P_B = F_{rB} \quad (43)$$

If  $F_{ae}/F_r \geq 1.54 \tan \alpha$ , i.e.,  $F_{rB}$  calculates to be negative, row B carries zero load and equation [(34) applies]:

$$P_A = 0.4 F_r + 0.39 \cot \alpha F_{ae} \quad (44)$$

$$P_B = 0 \quad (45)$$

Obviously, different rating lives will be calculated for row A and row B. In many cases only the minimum life is of interest; however, users may calculate a system life for the bearing assembly using:

$$L_{10}(\text{SYSTEM})_{AB} = (L_{10A}^{-3/2} + L_{10B}^{-3/2})^{-2/3} \quad (46)$$

where  $L_{10}(\text{SYSTEM})_{AB}$ ,  $L_{10A}$ ,  $L_{10B}$  = rating life of system, row A and row B respectively.

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