HW2

Sean Postan

1. $\forall a : (a \vee \neg a)$

- Gödel's Incompleteness theorem says we can't have complete & consistent. this is complete, so the system must be inconsistent.
- a complete system means that everything is true or false
- a must be true or false
- a must be a or $\neg a$
- thus, $a \vee \neg a$ is true for all a.

2. A complete system is one that expresses everything, and allows all true propositions to be provable from axioms.

- If our axiom is $a = a$, then we can easily prove this.
- If $a = a$, then $\neg a = \neg a$.
- Therefore, the system is complete because all true propositions are provable

3. $n \in \mathbb{N}$

- $\mathbb{N}$ is countably infinite
- since the set goes until (x_n), it continues countably infinitely
- therefore, the set is countably infinite.

4. we have the set of proof systems p such that $\langle p_1, p_2, p_3 \ldots p_n \rangle$, where $n \in \mathbb{N}$

- we can index these (like an array) such that: $\langle 1, 2, 3 \ldots n \rangle$
- since $\mathbb{N}$ is countably infinite, n is countably infinite
- therefore, the set of all proof systems is countably infinite

5. Gödel showed that a system is either inconsistent or incomplete.

- if there is an undecidable proposition, a system is incomplete and consistent.
- once the undecidable prop. is added as an axiom, a system is complete and inconsistent.
- to make the system consistent again, it must become incomplete.
- Therefore, consistent proof systems are recursively incomplete.