Mixing and Restratification in the Ocean Mixed Layer: Competing Mechanisms

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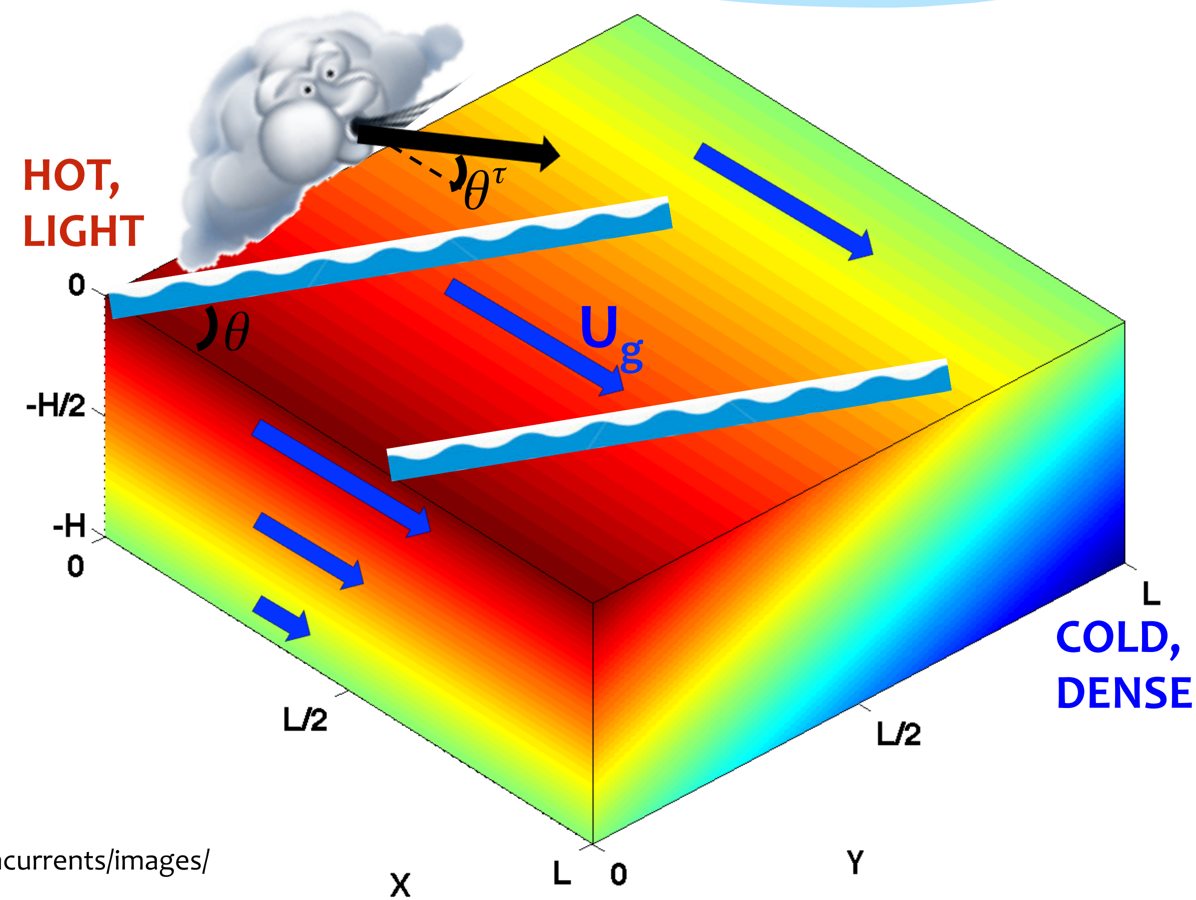
Outline

* Motivation and Objectives
* Problem Description
  * Schematic
  * Governing equations
* Approaches and Results
  * Linear stability
  * Nonlinear, Large Eddy Simulation (LES)
* Conclusions and Future Work

Motivation and Objectives

- Mixed layer dynamics may impede or enhance :
  - Heat, momentum, fresh water, and gas fluxes between ocean and atmosphere.
  - Vertical and horizontal stratification
- Which dynamical mixing and restratifying mechanisms, are important and under what combination of winds, waves, and fronts?
- How do the winds and waves stabilize or destabilize the typical the front?
- How does the front stabilize or destabilize the windy/wavy layer?

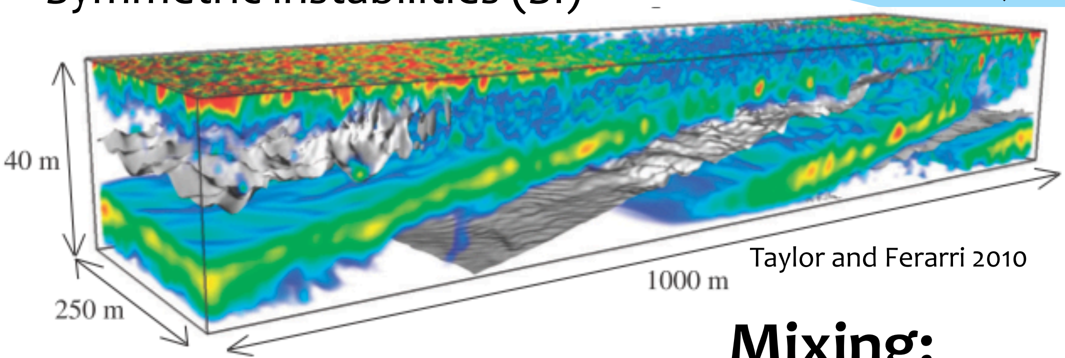


Cloud Art: http://www.seos-project.eu/modules/oceancurrents/images/currents_ch2_wind_cartoon.png

What's Out There?

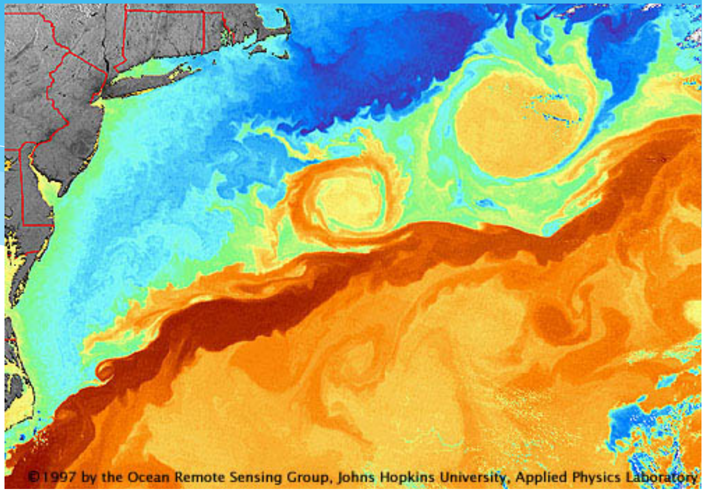
Restratifying:

Symmetric instabilities (SI)



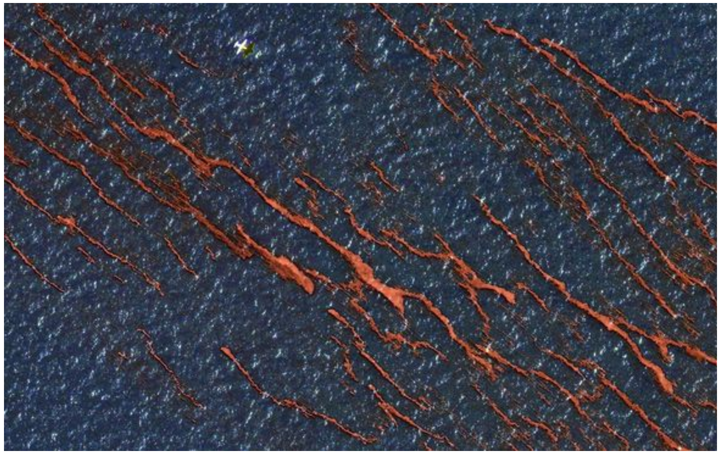
Taylor and Ferarri 2010

Mixed layer eddies (Geostrophic Instabilities; GI)



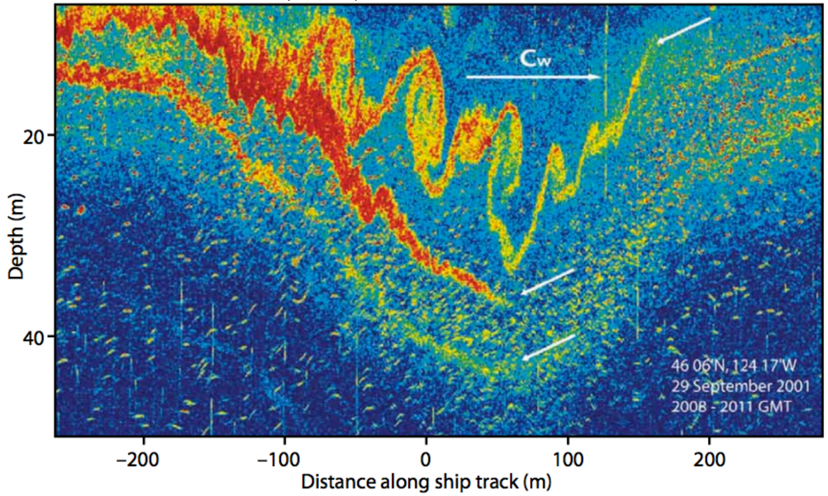
Mixing:

Langmuir Circulation/ mixing/turbulence (LC)



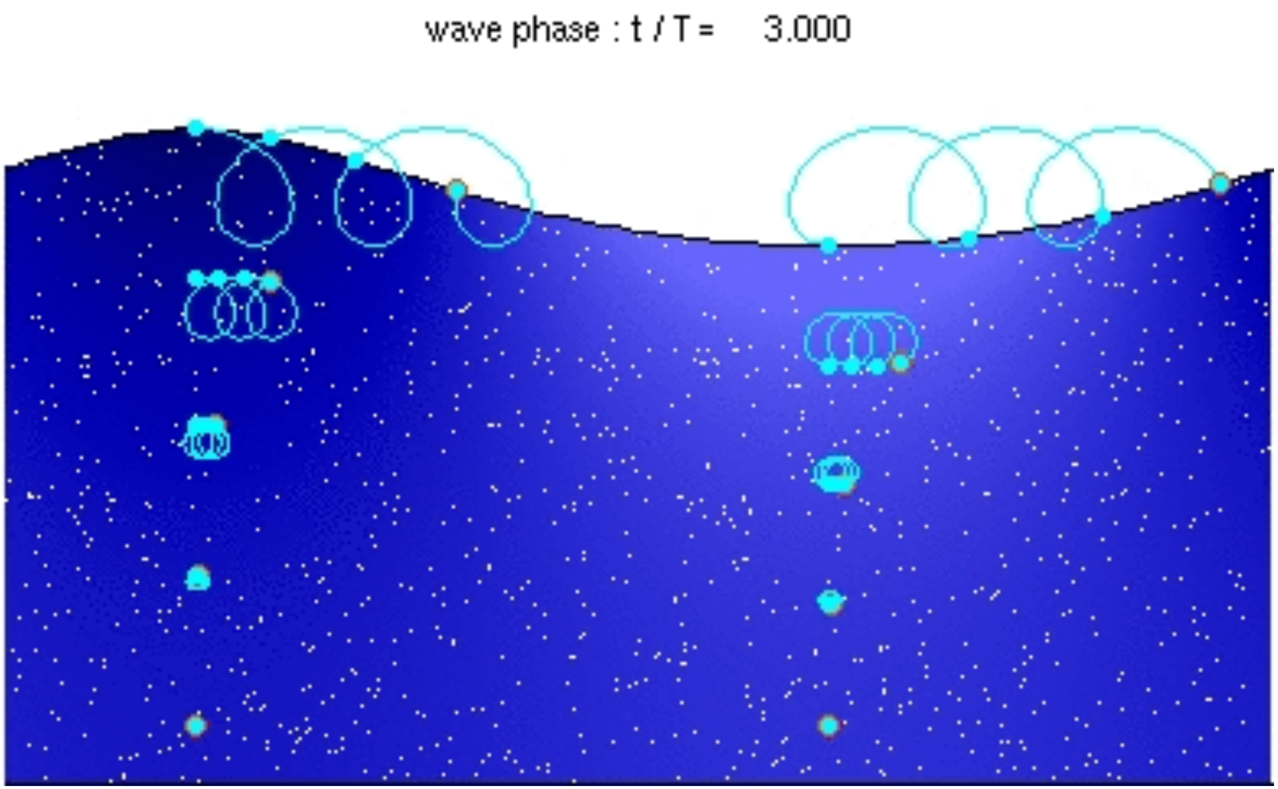
http://www.oilspillsolutions.org/evaluation.htm

Kelvin-Helmholtz instabilities (KH)



http://dx.doi.org/10.5670/oceanog.2012.37

Stokes Drift



http://en.wikipedia.org/wiki/File:Deep_water_wave.gif

- Stokes Drift is the wave averaged velocity following a particle.
- Lagrangian, not Eulerian.
- Decays steeply with depth.

Governing Equations and Scalings

- Boussinesq, rotating, surface gravity wave averaged equations

$$\underbrace{\partial_t \mathbf{u}}_{\text{Acceleration}} + \underbrace{(\mathbf{u}^L \cdot \nabla)\mathbf{u}}_{\text{Lagrangian Advection}} + \underbrace{f\hat{\mathbf{k}} \times \mathbf{u}^L}_{\text{Coriolis and Stokes Coriolis}} + \underbrace{\frac{\nabla p}{\rho_0}}_{\text{Pressure}} + \underbrace{u^{L,j}\nabla U^{S,j}}_{\text{Stokes Shear}} = \underbrace{b\hat{\mathbf{k}}}_{\text{Buoyancy}} + \nu\nabla^2 \underbrace{\mathbf{u}}_{\text{Dissipation}}$$

$$\partial_t \mathbf{u} + \left(\mathbf{u}^L \cdot \nabla\right) b = 0 \qquad \nabla \cdot \mathbf{u} = 0$$

BC's: $W = 0$ at $z = 0, -1$

$$\mathbf{U}_z = \frac{\tau}{\rho_0 \nu} \quad \text{at} \quad z = 0$$

Where, $\mathbf{u}^L = \mathbf{u} + \mathbf{U}^S$, and $\mathbf{U}^S$ is prescribed

Scalings and Approach

- Rescale equations

Non-dimensional number	Definition	Range of Values
Ro	$\frac{U^L}{fL}$	$(0,\infty)$
Ri	$\frac{N^2H^2}{U^{L,2}}$	$[0,\infty)$
μ	$\frac{U^S}{U^L}$	$[0,\infty)$
λ	$\frac{H}{H^S}$	$(0,\infty)$
γ	$\frac{U_z^g}{U_z^L} \equiv \frac{M^2H}{fU^L}$	$[0,1]$
Ek	$\frac{Ro}{Re} \equiv \frac{\nu}{fL^2}$	$[0,1)$

- Linear stability analysis
- Multiple scales of horizontal variation $x, X, y, Y, t, T.$
- Decompose into mean and perturbation:

$$\mathbf{u} = \bar{\mathrm{U}}(X, Y, z, T) + \mathbf{u}'(x, X, y, Y, z, t, T)$$

- Assume:

$$u' = \tilde{u}(z)e^{i(kx+ly+\sigma t)}$$

The Steady Background State

$$Ro >> 1, Ek > 0, \gamma = 0$$

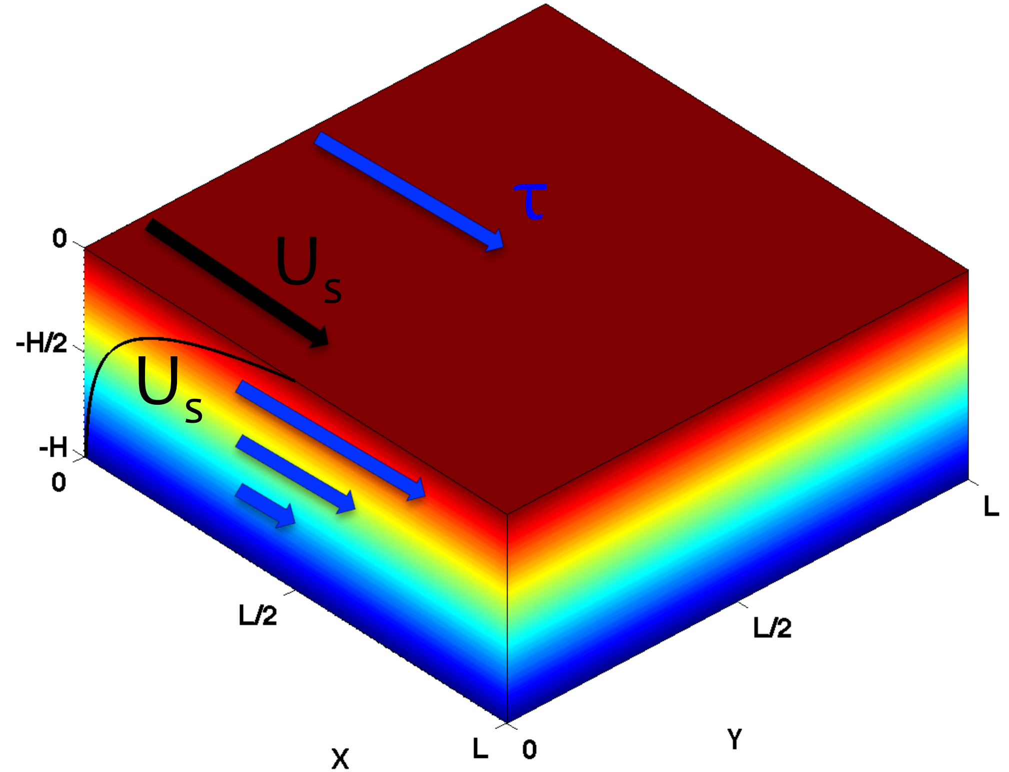
Weak Coriolis — Viscid — No front

Background Flow

$$\bar{\mathrm{U}} = z \quad \bar{W} = 0$$

$$\left.\begin{aligned}\bar{P}_z = \bar{B}\end{aligned}\right\}\text{Hydrostatic}$$

$$\bar{W} = 0$$



Reproduces "Classic" LC regime: Leibovich and Paolucci, 1980

The Steady Background State

$$Ro << 1, Ek = 0, \gamma = 1$$

Stong Coriolis — INviscid — Strong front

Background Flow

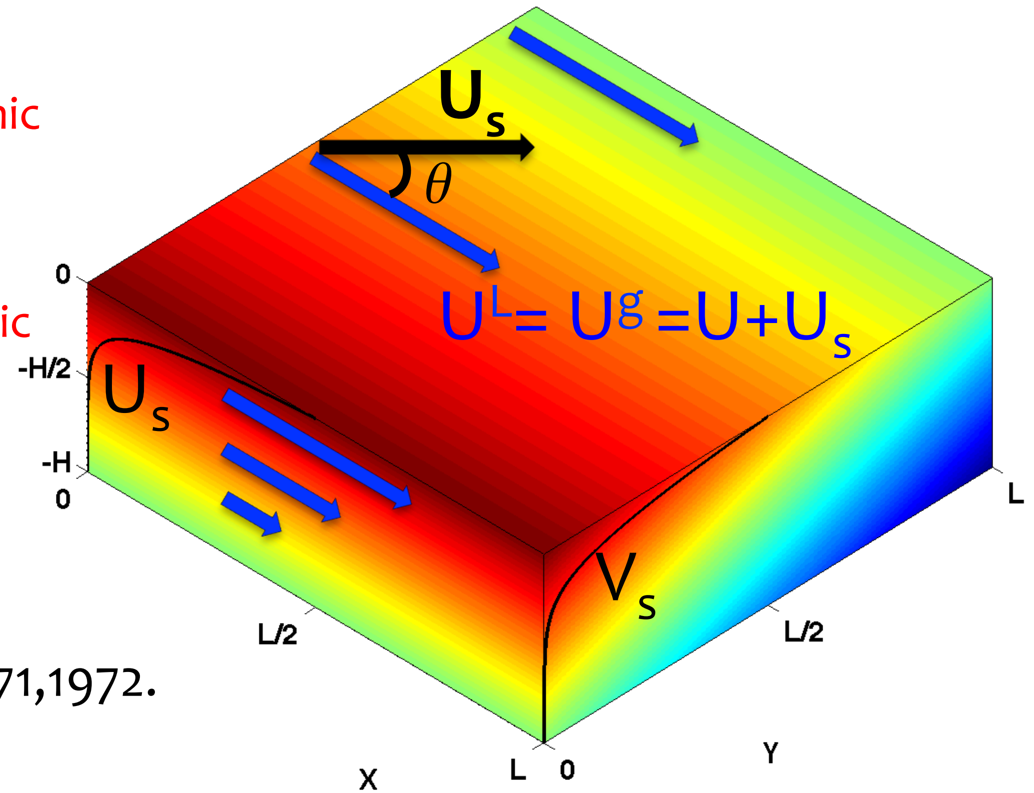
$$f(\hat{\mathbf{k}} \times \bar{\mathbf{U}}^L) = -\frac{\nabla \bar{P}}{\rho_0}$$ Geostrophic

$$\bar{P}_z = \bar{B}$$ Hydrostatic

$$f\bar{\mathbf{U}}_z^L = -\bar{B}_Y$$ Thermal Wind

$$\bar{W} = 0$$

$\mathbf{U}^S \to 0 \Rightarrow$ Stone, 1966,1970,1971,1972.



The Steady Background State

$$Ro << 1, Ek > 0, \gamma \sim O(1)$$

Stong Coriolis | Viscid | Strong front

Background Flow

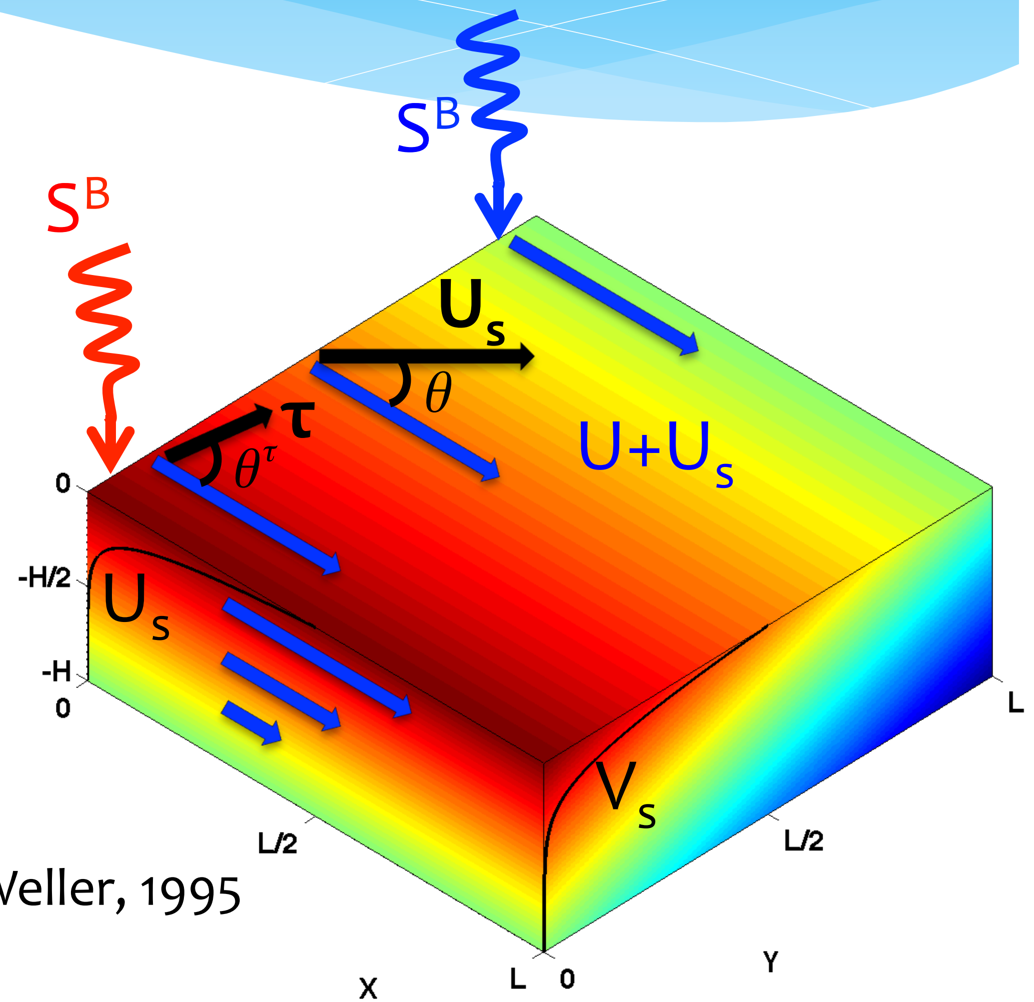
$$f(\hat{\mathbf{k}} \times \bar{\mathbf{U}}^L) + \frac{\nabla \bar{P}}{\rho_0} = \nu \bar{\mathbf{U}}_{zz}$$

Ekman-Stokes-Front Layer

$$\left.\begin{aligned}\bar{P}_z &= \bar{B}\end{aligned}\right\} \text{Hydrostatic}$$

$$\bar{W} = 0$$

$$\nabla \bar{B} \to 0 \Rightarrow$$ Gnanadesikan and Weller, 1995



Analytic Stability Criteria: Geostrophic Modes

* Charney, Stern, and Pedlosky showed, that geostrophic instability exists only if one of the following is true:

1. Q_y changes sign in the interior of the domain.
2. Q_y is the opposite sign to $U^L{}_z$ at the surface.
3. Q_y is the same sign to $U^L{}_z$ at the bottom.
4. $U^L{}_z$ has the same sign at the surface and bottom.

Where Q is the quasi-geostrophic potential vorticity:

$$\bar{Q} = \nabla_H^2 \bar{\psi} + \beta Y + \partial_z \left(\frac{f_0^2}{N^2} \bar{\psi}_z^L \right)$$

Analytic Stability Criteria: Symmetric Modes

* Hoskins (1974) showed that symmetric instability exists only if the Ertel potential vorticity (PV) is negative.

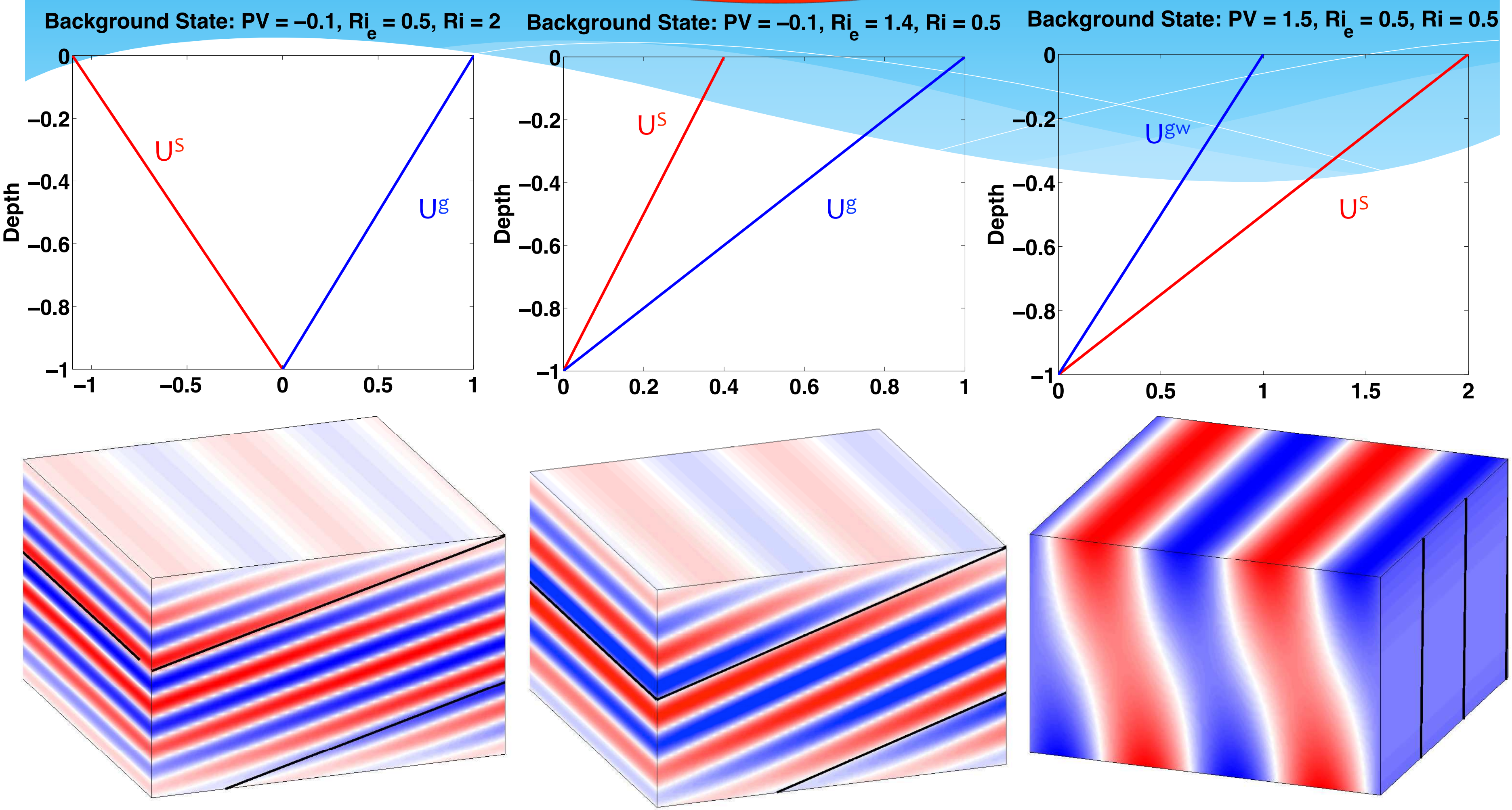
$$PV = \left(\nabla \times \bar{\mathbf{U}} + f\hat{\mathbf{k}}\right) \bullet \nabla \bar{B} < 0 \Rightarrow SI$$

* Proven for constant shear Stokes drift profiles as well.
* The Stokes drift modifies the PV by changing the Eulerian flow that balances the pressure gradient:

$$\bar{\mathbf{U}}_z = -\frac{\nabla_H \bar{B}}{f} - \mathbf{U}_z^S$$

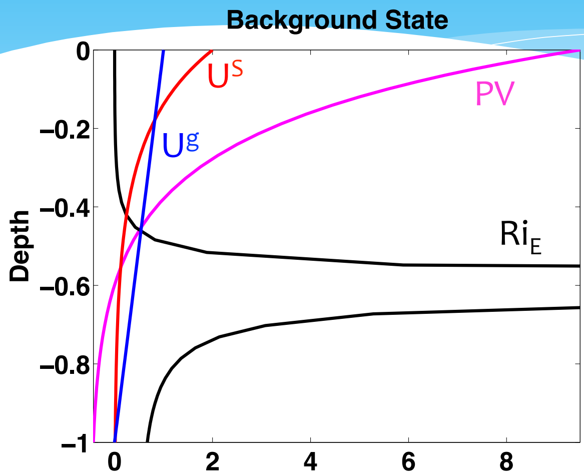
* Since the waves are assumed to be irrotational, the Stokes drift does not contribute directly to the PV

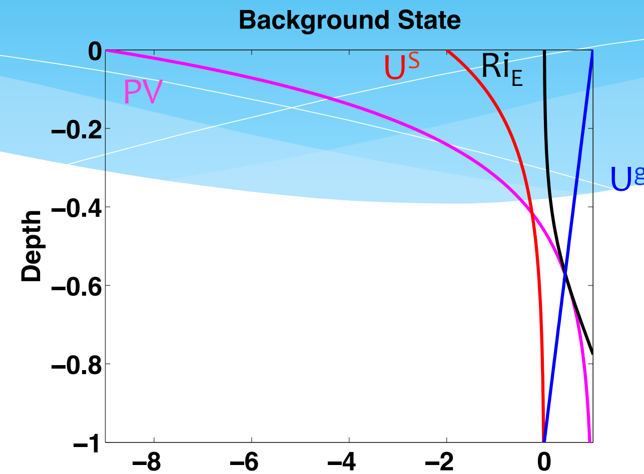
~~$Ri < 1 \Rightarrow SI$~~

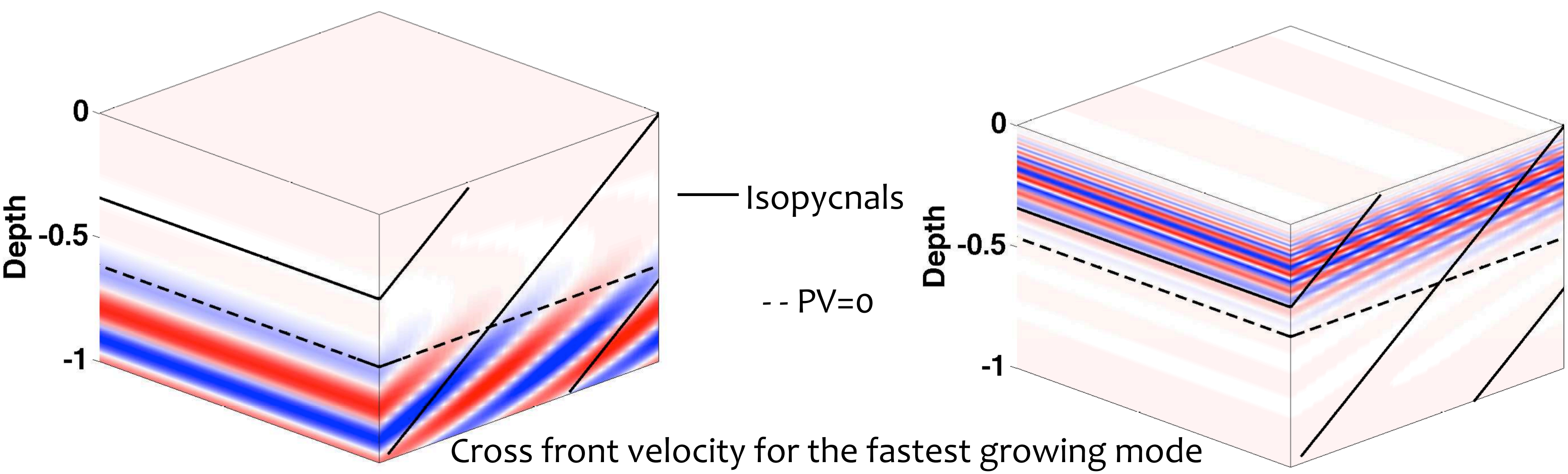


Cross front velocity for the fastest growing mode

$\cancel{Ri < 1 \Rightarrow SI}$







Cross front velocity for the fastest growing mode

Energetics

- Energetics are used to distinguish modes

$$\frac{D\overline{e'}}{D_t} = -\underbrace{\overline{\mathbf{u}'w'}\cdot\overline{\mathbf{U}}_z}_{\text{ESP}} - \underbrace{\overline{\mathbf{u}'w'}\cdot\mathbf{U}_z^S}_{\text{SSP}} - \underbrace{\overline{w'b'}}_{\text{BP}} \underbrace{- \overline{\nabla_h\cdot\mathbf{u}'p'} - \overline{\partial_z\left(w'p'\right)}}_{\text{PW}} + diss$$

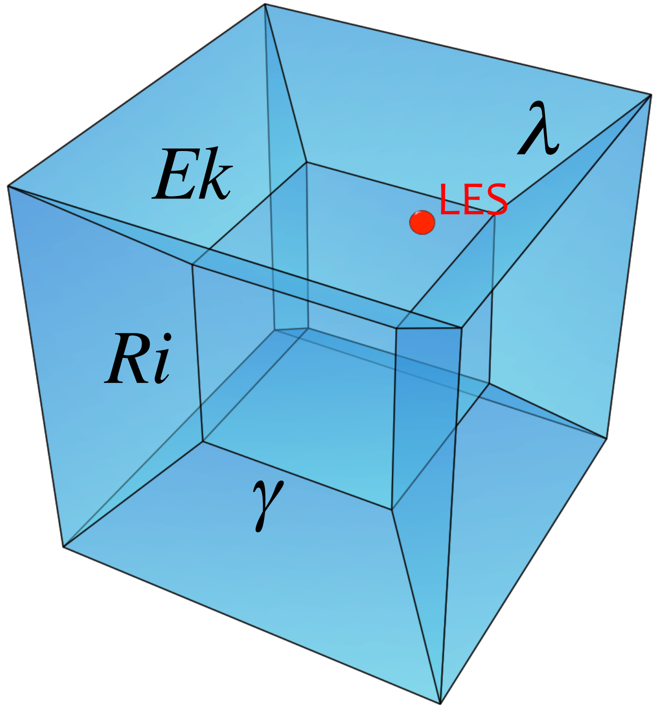
- BP dominant: instability extracts potential energy to RE-stratify the mixed layer (typical of geostrophic instabilities).
- SSP, ESP dominant: instability extracts kinetic energy (typical of SI, LC, KH)
- Hybrid modes with various mixed of energy production terms exist.

Why Linear Stability?

* In general,

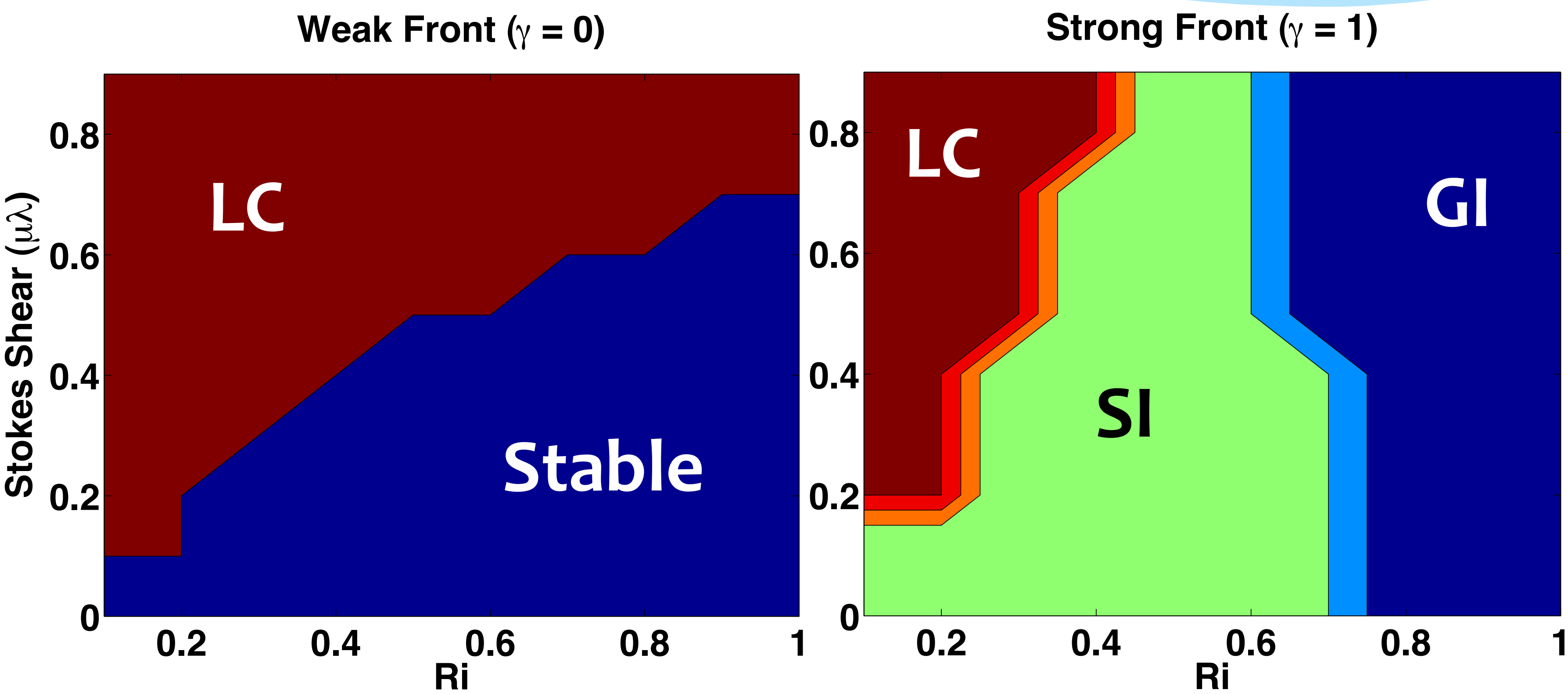
$$\sigma = F\left(Ri, \mu, \gamma, \lambda, Ek, Ro, \alpha\right)$$

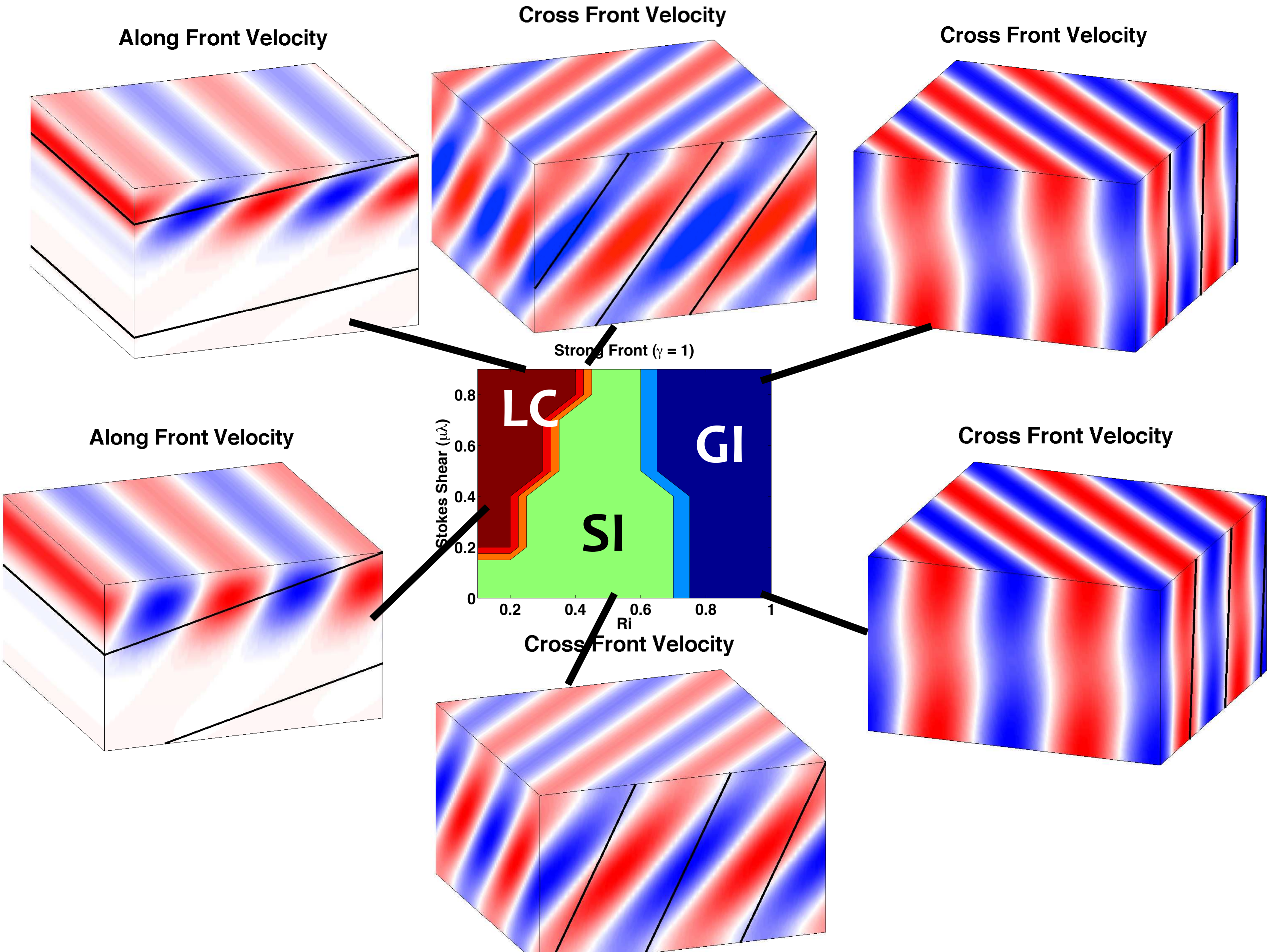
* Furthermore, the vertical structure, and dominant energy production terms are functions of the same non-dimensional numbers.



Hypercube Art: http://upload.wikimedia.org/wikipedia/commons/thumb/2/22/Hypercube.svg/943px-Hypercube.svg.png

Stability Regimes



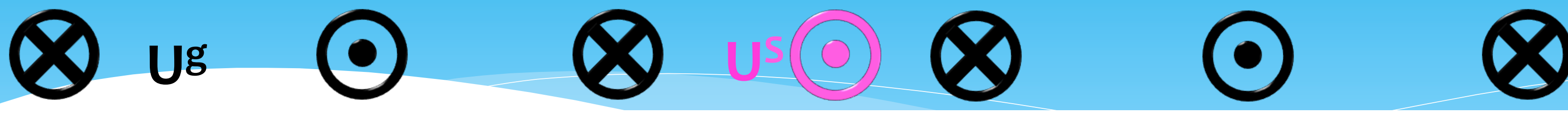


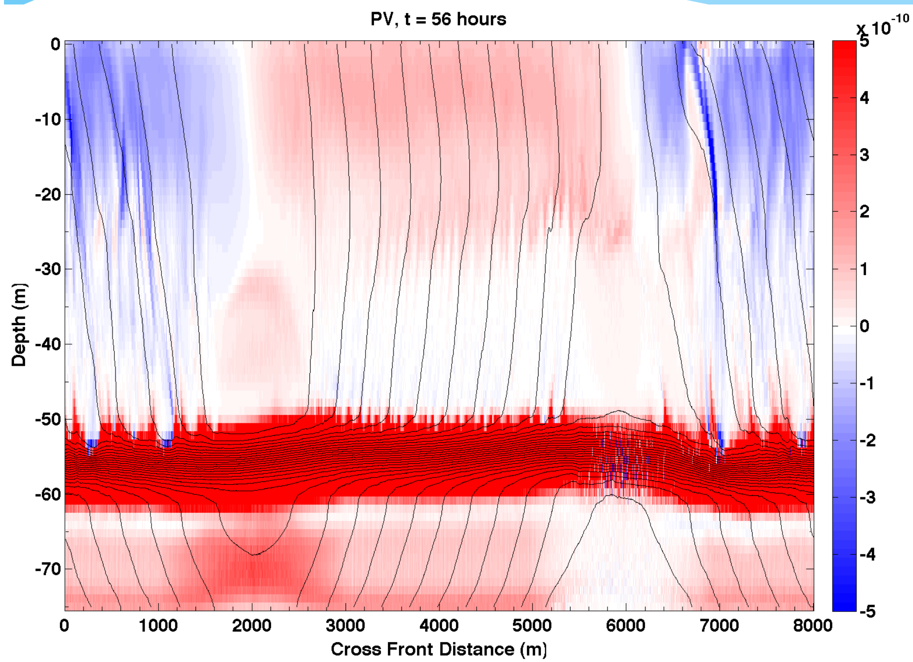
Along Front Velocity
Cross Front Velocity
Cross Front Velocity
Strong Front (γ = 1)
Stokes Shear (μλ)
Ri
0
0.2
0.4
0.6
0.8
1
LC
SI
GI
Along Front Velocity
Cross Front Velocity
Cross Front Velocity

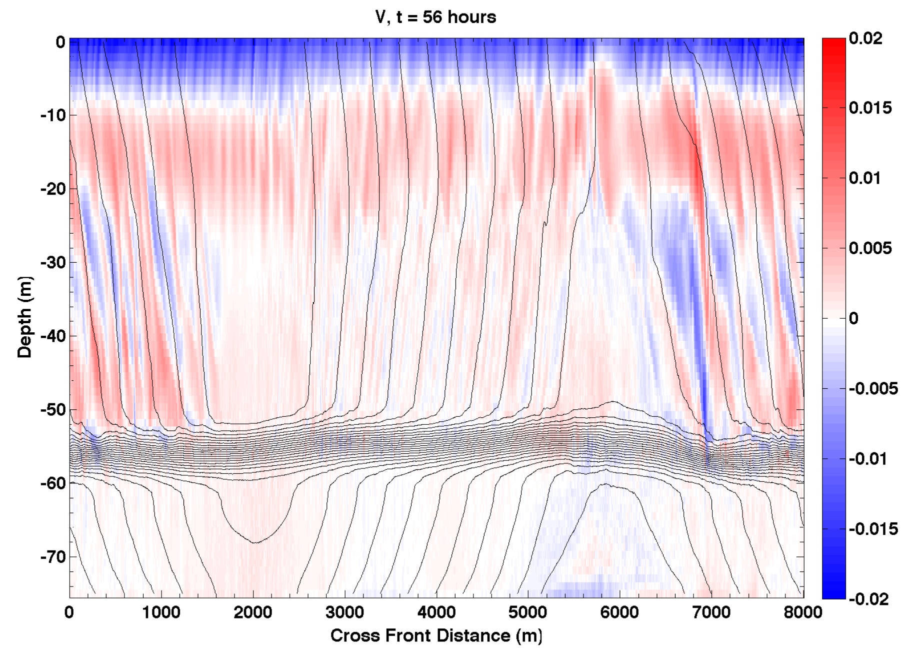
Nonlinear Simulations

* National Center for Atmospheric Research (NCAR) LES model.
* Configuration
  * 500m (along front) x 8km (cross front) x 75m deep
  * ~4m horizontal x ~1m vertical resolution.
  * Periodic BC's in the horizontal (requires simulating 2 fronts)
  * No flux on top and bottom
  * No wind stress on top
* Initialization
  * $\mu \sim 2$, i.e. $U^S \sim 2U^L \sim 2U^g$ at the surface
  * Ri = 0.5

PV<0 ⇒ SI



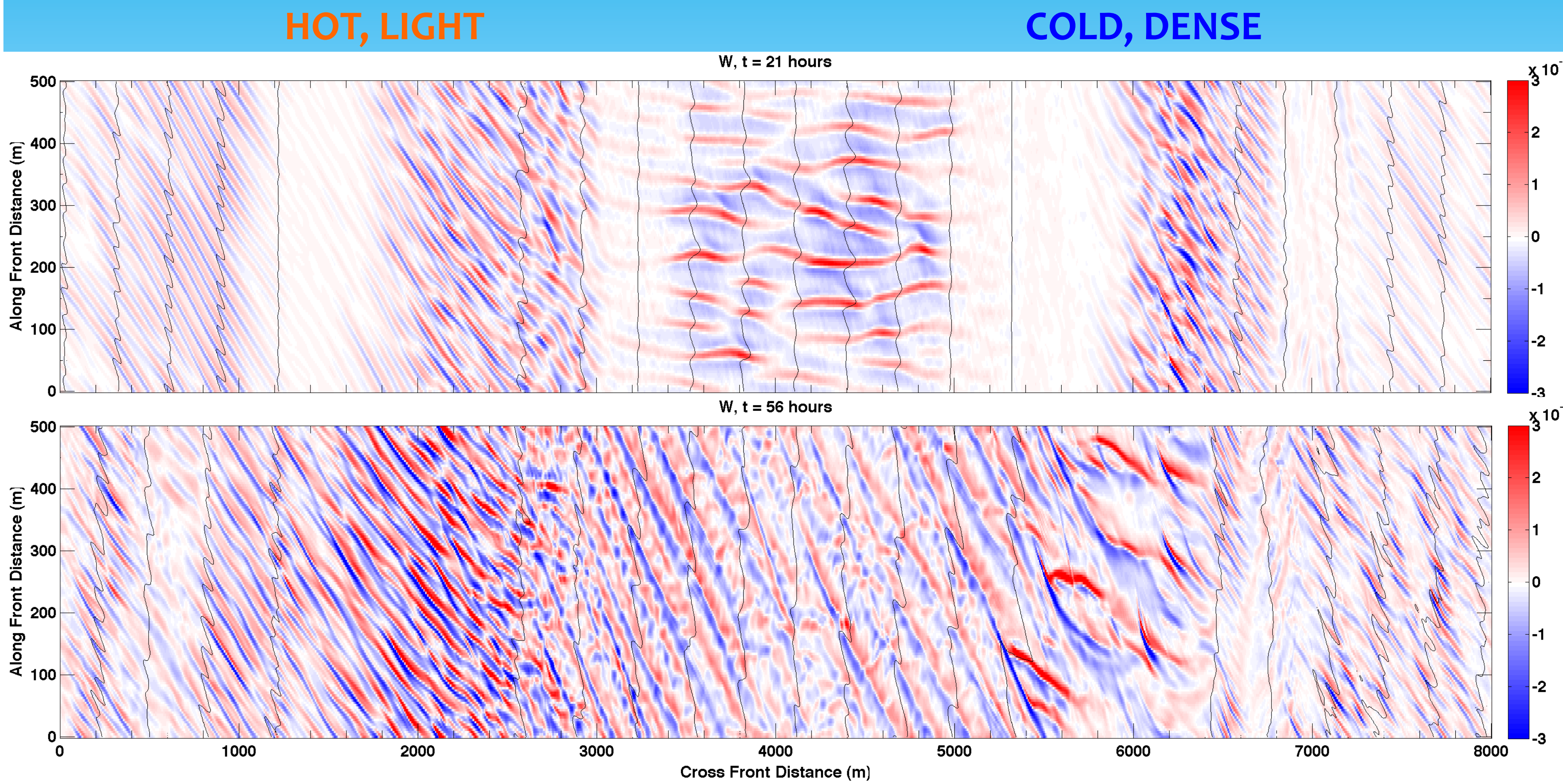




- A "no [wind] stress" Ekman layer develops due the the surface geostrophic stress.
- SI develop only in regions of negative PV, and are stronger for more negative PV.
- SI restore the PV to zero by exchanging negative PV for positive PV in the pycnocline.

Fronts Slow LC

Horizontal slice of vertical velocity at ~ 5m deep.



- LC develop fastest in regions without horizontal stratification
- Unstable stratification in central front yields convective KH rolls
- LC align with the Lagrangian shear direction

Conclusions and Future Work

- GI are only weakly affected by Stokes drift, but their instability depends on the Lagrangian, not just geostrophic shear.
- PV criteria for SI remains the same, however, the PV is altered by the Stokes drift such that $PV<0 \neq Ri<1$.
  - Observational estimates of PV must be based on Eulerian shear if SI are of interest.
- LC dominance requires: Strong Stokes shear, weak stratification, AND weak geostrophic shear (weak front).
- Near surface effects of the Stokes drift on the PV are noticeable, but generally dominated by LC or Ekman effects.

Future Work:

- More extensive analysis of LES results
  - Explore the onset, and then decay of KH instability
  - Diagnose from LES energetics why LC are suppressed
- Apply linear stability results to observations:
  - Where do we expect to find what types of instabilities, and do we really see them there?