Homework 2: Schrodinger's equation and the statistical interpretation of quantum mechanics (**Due Friday September 21**) (120 points)

September 14, 2018

1 BRUTE FORCE CALCULATIONS (1) **(20 POINTS)**

1. Given the wave function

$$\psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}, \tag{1.1}$$

calculate the probability P_1 that the particle exists between $-\infty$ and -1.

2. **calculate** the probability P_2 that the particle exists between -1 and 1.
3. **calculate** the probability P_3 that the particle exists between 1 and ∞.
4. **What** is the total probability $P_1 + P_2 + P_3$?

2 BRUTE FORCE CALCULATIONS (2) **(20 POINTS)**

Do problem 1.9 (parts a and b only) in Griffiths' book.

3 PROBABILITY CURRENT DENSITY **(35 POINTS)**

1. **(25 points)** As we mentioned in the class, the probability density is given by $\rho \equiv |\psi(x,t)|^2$. By taking the partial derivative of ρ with respect to t and using Schrodinger's equation **show that**

$$\frac{\partial \rho}{\partial t} = -\frac{\partial J}{\partial x}, \tag{3.1}$$

where

$$J = -\frac{i\hbar}{2m}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right). \tag{3.2}$$

The current J is called the probability current density. In fact, Equation (3.1) is a continuity equation for the probability: the rate of change of the probability density is equal to minus the gradient of the probability current density. Notice that it is crucial that the Schrodinger equation is a first order equation in time in order to get a continuity equation for the probability.

2. **(Bonus 10 points) Generalize** this equation to 3D and **show** that

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J}, \tag{3.3}$$

where $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$, and

$$\mathbf{J} = -\frac{i\hbar}{2m}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right). \tag{3.4}$$

This equation takes the exact same mathematical form of the continuity equation of a fluid.

4 WHEN WE CAN'T NEGLECT THE QUANTUM EFFECTS **(45 POINTS)**

1. **(Bonus 15 points) Calculate** the de Broglie wavelength of an electron accelerated through a 1000V potential step. Given this wavelength, **what** kind of experiment do you suggest to do if we want to see the wave nature of electrons?

2. **(Bonus 5 points) Estimate** the de Broglie wavelength of your body as you walk. **What** do you observe?

3. **(Bonus 25 points) Do Problem 1.18 in Griffiths' book.**