PROOF. First it must be verified that the direct sum operation $\oplus$ is well-defined on the equivalence classes. So, consider vector bundles $E_1 \sim E_2$ and $F_1 \sim F_2$. Then let n_1, m_1, n_2, m_2 be the numbers such that $E_1 \oplus \varepsilon^{n_1} \approx E_2 \oplus \varepsilon^{n_2}$ and $F_1 \oplus \varepsilon^{m_1} \approx F_2 \oplus \varepsilon^{m_2}$. It then follows that $E_1 \oplus F_1 \approx E_2 \oplus F_2$

$$(E_1 \oplus F_1) \oplus (\varepsilon^{n_1+m_1}) \approx (E_1 \oplus \varepsilon^{n_1}) \oplus (F_1 \oplus \varepsilon^{m_1}) \approx (E_2 \oplus \varepsilon^{n_2}) \oplus (F_2 \oplus \varepsilon^{m_2}) \approx (E_1 \oplus F_1) \oplus (\varepsilon^{n_2+m_2})$$

Where the above computation used $\varepsilon^{n+m} \approx \varepsilon^n \oplus \varepsilon^m$.

With the group operation well-defined, the associativity and commutativity of the operation follows from direct sum associative and commutative on bundles.

The identity element in the group is given by the equivalence class $[\varepsilon^0]$, which is the set of all trivial bundles. Indeed, $[E] + [\varepsilon^0] = [E \oplus \varepsilon^0] = [E]$.

It only remains to show the existence of inverses, which appeals to /*ref*/. Then take any element $[E]$ and consider the promised bundle E' such that $E \oplus E' \approx \varepsilon^n$ for some trivial bundle of dimension n. Then, the element $[E']$ is the inverse element.

$$[E] + [E'] = [E \oplus E'] = [\varepsilon^n] = [\varepsilon^0]$$

$\square$