Homework 2

Sean Richardson

September 24, 2018

0.1

(e) This is the set of all palindrome binary numbers

(f) This is the empty set $\emptyset$

0.6

(a) $f(2) = 7$.

(b) $\text{Range}(f) = \{6, 7\}$
$\text{Domain}(f) = \{n \mid n \in \mathbb{Z}, 1 \leq n \leq 5\}$

(c) $g(2, 10) = 6$

(d) $\text{Domain}(g) = \{(p, q) \mid p, q \in \mathbb{Z}, 1 \leq p \leq 5, 6 \leq q \leq 10\}$
$\text{Range}(g) = \{n \mid n \in \mathbb{Z}, 6 \leq n \leq 10\}$

(e) $g(4, f(4)) = g(4, 7) = 8$

0.9

$G = \{V, E\}$ where V is the set of vertices and E is the set of edges.

$$V = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{(p, q) \mid \text{floor}\left(\frac{p}{3}\right) = \text{floor}\left(\frac{q}{3}\right)\}$$

0.10

The error in the proof is in the division by $(a - b)$ from both sides. Because $a = b$, this is division by 0.

0.13

Every graph with two or more nodes contains two nodes that have equal degrees. I assume we are not allowing self loops, for we would have the counter example $G = \{\{1, 2\}, \{(1, 1)\}\}$

Proof. We proceed by the method of induction.
We have the base case of $n = 2$ in which the graph $G = \{\{0, 1\}, E\}$ must have $E = \emptyset$ or $\{(0, 1)\}$. So, the base case holds.
We take the inductive hypothesis: that a graph of n nodes must have two nodes of the same order.
We will now make the inductive step: that under the inductive hypothesis, a graph of $n + 1$ nodes must have two nodes of the same order.
Consider a graph G with $n + 1$ vertices. Every node can connect to n other nodes. In the case that every node has at least order 1, then each node can have order d such that $1 \leq d \leq n$. We have $n + 1$ nodes and n possible degrees, so two nodes must share the same degree. In the case that there exists a node of order 0, then we have a subgraph of order n which must contain two nodes of the same degree by the inductive hypothesis.
This demonstrates the inductive step and concludes our proof by induction. $\square$