PROOF. Let $f : X \to Y$ denote a continuous function between two compact Hausdorff spaces.

First it must be verified that $J(f)$ is well-defined. Specifically, it must be shown that if $[E_1] = [E_2]$, then $J(f)([E_1]) = J(f)([E_2])$. That is, it must be shown that $E_1 \oplus \varepsilon^n \approx E_2 \oplus \varepsilon^n$ for some n implies $f^*(E_1) \approx_s f^*(E_2)$. First, note the following application of the distributivity of pullback over direct sum /*ref*/.

$$f^*(E_1) \oplus f^*(\varepsilon^n) \approx f^*(E_1 \oplus \varepsilon^n) \approx f^*(E_1 \oplus \varepsilon^n) \approx f^*(E_2) \oplus f^*(\varepsilon^n)$$

The result that the pullback of a trivial bundle is trivial combined with the above confirms $f^*(E_1) \approx_s f^*(E_2)$ and so $J(f)$ is well-defined.

With $J(f)$ well-defined, verifying that $J(f)$ is a semiring homomorphism follows easily from the properties of pullback. Specifically, the distributivity of pullback over direct sum directly gives the distributivity of $J(f)$ over the defined addition. Similarly, the distributivity of pullback over tensor product gives that $J(f)$ distributes over the defined multiplication. Lastly, the property that f^* maps the bundle ε^1 over X to the bundle ε^1 over Y implies that $J(f)$ maps the multiplicative identity to the multiplicative identity. $\square$