Problems in QM

September 28, 2018

QUESTION 1

A particle is placed in a box of length L. The potential that the particle experiences can be expressed as

$$V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise.} \end{cases}$$

The initial wave function of the particle is given by

$$\Psi(x,0) = Ax(x-L).$$

1. **Find** A.
2. **What** is the expectation value of the particle's position at $t = 0$?
3. At $t = 0$ an observe measures the energy of the particle. **What** is the probability that the observer will find the particle in the ground state? **What** is the probability that the observer will find the particle in the first excited state?
4. At $t = 0$ the observer does the measurement and finds that the particle has energy $9E_1$, where E_1 is the ground state energy, **what** is the particle's wavefunction immediately after the measurement? **What** is the particle's wavefunction at any subsequent time t, i.e., $\Psi(x,t)$, after the measurement?
5. After the above measurement is done, now imagine that a student enters the lab and displaces the box very slowly a distance a (we assume that the displacement takes place

so smoothly that the particle inside the box does not experience any force). If the observer repeats the measurement, **what** is the probability that she will find the particle in the states that correspond to energies E_1, E_2, E_3, E_4?

You might need the integral

$$\int_0^L x(x-L)\sin\Big(\frac{n\pi x}{L}\Big)\,dx = -2L^3\frac{1-(-1)^n}{n^3\pi^3}.$$

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1 QUESTION 2

100 electrons are in a box of length L. Every particle has initial wave function given by (ignore the Coulomb repulsion between the electrons)

$$\Psi(x,0) = Ax(x-L) \tag{1.1}$$

1. **Find** A.
2. **What** is the probability that each electron can be found in the interval $[0, L/2]$ at $t = 0$?
3. **How** many electrons exist in the interval $[0, L/2]$ at $t = 0$?
4. **How** many electrons have energy $E = E_3$ at $t = 0$? **How** many electrons have energy $E = E_5$ at $t = 0$?
5. If an electron makes a transition from the energy states E_5 to E_3, **what** is the frequency of the emitted photon given that the length of the box is 10^{-5}m? **Give** your answer in electron volts.
6. **Calculate** $\langle E\rangle$
7. **Find** $\Psi(x,t)$.
8. **Calculate** $\langle E\rangle(t)$

Hint: you might need the integral $\int_0^L dx\,x(x-L)\sin\big(\frac{n\pi x}{L}\big) = -2L^3\frac{1-(-1)^n}{n^3\pi^3}$.