Homework 5: More about the particle in the box (**Due Friday October 19**)

October 12, 2018

1 MORE ABOUT A PARTICLE IN THE BOX **(100 points)**

Consider a particle placed in the following potential:

$$V(x) = \begin{cases} V_0, & -L/2 < x < L/2 \\ \infty, & \text{otherwise} \end{cases} \tag{1.1}$$

1. **(30 points) Solve** the time-independent Schrodinger equation and **obtain** the eigenstates ψ_n and the corresponding energies E_n. **How** the eigenstates and energies of this problem compare to the problem we considered in the class?. **What** do you conclude?

2. **(15 points)** Let us define the parity operator $\hat{\mathscr{P}}$ as $\hat{\mathscr{P}} f(x) = f(-x)$, where $f(x)$ is any test function. Physically, the parity operator is a reflection operator which transforms an object into its mirror image. For the potential given above **show** that

$$\left[\hat{H}, \hat{\mathscr{P}}\right] = 0\,, \tag{1.2}$$

where $\hat{H}$ is the Hamiltonian of the system.

3. **(15 points) Show** that $\{\psi_n\}$, $n = 2, 4, 6, \ldots$ are eigenstates of $\hat{\mathscr{P}}$ with eigenvalue -1, and that $\{\psi_n\}$, $n = 1, 3, 5, \ldots$ are eigenstates of $\hat{\mathscr{P}}$ with eigenvalue 1.

4. **(10 points)** Given the initial wave function $\Psi(x, t = 0) = \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}} \sin\left[\frac{2\pi x}{L}\right] + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}} \sin\left[\frac{4\pi x}{L}\right]$, **calculate** $\Psi(x, t)$, $\langle x \rangle(t)$, and $\langle x^2 \rangle(t)$ at any time t. If a measurement of energy is performed, **what** are the outcomes of this measurement? **What** are the probabilities to obtain these energies?

5. **(10 points)** Given the initial wave function $\Psi(x, t = 0) = A\left(\frac{L}{2} - x\right)\left(\frac{L}{2} + x\right)$, **calculate** $\langle x\rangle(t)$. **Plot** $|\Psi(x, t)|^2$ and $\langle x\rangle(t)$ for $t = 0, T/4, 2T/4, 3T/4, T$, where T is the revival time.

6. **(20 points)** Using Mathematica or any other software, make a video for $|\Psi(x, t)|^2$, $\langle x\rangle(t)$, and $\langle x^2\rangle(t)$ and send it to my email.