HOMEWORK SET #2 / CO2

Summer, 2015

June 22, 2015

1. Give an example of a bipartite graph G such that for a certain ordering of its vertices, the greedy algorithm uses 1000 colors for the proper coloring of G.
2. Prove that finite planes of order at least 3 have proper 2-colorings.
3. Suppose that $\mathcal{H} = (V, \mathcal{E})$ has no singleton edges and $|e \cap f| \neq 1$ for all $e, f \in \mathcal{E}$. Prove that the greedy algorithm colors V with at most two colors (in every ordering of V).
4. Prove that Steiner triple systems have no proper 2-colorings.
5. Prove that the n-element subsets of a $(2n + k)$-element ground set can be partitioned into $k + 2$ classes so that each class has pairwise intersecting sets.

Extra Problems, due July 6th

E3. Prove that the Mycielski graph M_i has no proper coloring with $i - 1$ colors.

E4. Modify the definition of Zykov graph Z_{n+1} so that instead of n copies of Z_n one copy of Z_i is used for $i = 1, 2, \ldots n$. Show that $\chi(Z_n) = n$ is still true.