Theorem 0.0.1. The regular tetrahedron is not equidecomposable in the geometric sense to any rectangular box of the same volume.

Proof. By problem 64, we have that α/π is irrational where α denotes the dihedral angle of the regular tetrahedron. Then, by problem 27, we can define an additive function f such that $f(\alpha) = 1$ and $f(\pi) = 0$. So, we define a function F as follows. For a polyhedron $P \subset \mathbb{R}^3$ with edge set E. For each edge $e \in E$, we denote the dihedral angle θ_e and the length $|e|$. Then, take

$$F(P) = \sum_{e \in E} |e| \, f(\theta_e)$$

We claim F is invariant between equidecomposable polyhedrons in the geometric sense.
Consider a polyhedron $P \subset \mathbb{R}^3$ with edge set E. Then, consider a planar cut of P into two polyhedrons P_1 and P_2. For each $e \in E$, there are three cases. Either the planar cut (1) preserved the edge, (2) cut the edge into two edges of smaller length, preserving dihedral angle, or (3) cut the edge into two edges with smaller dihedral angle but preserving the length.

In case 1, the value $|e| \, f(\theta_e)$ corresponding to e is preserved, being part of either $F(P_1)$ or $F(P_2)$.
In case 2, e is split into e_1 and e_2, but

$$|e| \, f(\theta_e) = (|e_1| + |e_2|) f(\theta_e) = |e_1| \, f(\theta_{e_1}) + |e_2| \, f(\theta_{e_2})$$

And so the quantity $|e| \, f(\theta_e)$ is preserved, being split between $F(P_1)$ and $F(P_2)$.
In case 3, if e is split into e_1 and e_2 we have

$$|e| \, f(\theta_e) = |e| \, f(\theta_{e_1} + \theta_{e_2}) = |e_1| \, f(\theta_{e_1}) + |e_2| \, f(\theta_{e_2})$$

by additivity of f. And so the quantity $|e| \, f(\theta_e)$ is preserved, being split between $F(P_1)$ and $F(P_2)$.
Further, Each edge in either P_1 or P_2 results from one of these cases, so we can conclude that $F(P) = F(P_1) + F(P_2)$. We can extend this to conclude that $F(P) = \sum F(P_i)$ if $P = \bigcup P_i$ with a decomposition from planar cutting. But, if we have a decomposition $\{X_i\}$ of P in general, then we can extend the planes of the polygons to make a refinement of planar cuts $\{Y_i\}$. Then, we by previous argument, $F(P) = \sum F(Y_i)$. And by gluing, we have $\sum F(Y_i) = \sum F(X_i)$. So, $F(P) = F(X_i)$ in general.
Then, if we have two equidecomposable polygons $X = \bigcup X_i$ and $Y = \bigcup Y_i$ with X_i, Y_i congruent. Then, $F(X_i) = F(Y_i)$ for each i and so:

$$F(X) = \sum F(X_i) = \sum F(Y_i) = F(Y)$$

confirming that F is an invariant over geometric equidecomposition.
Finally, we have that the tetrahedron T and any rectangle R have different values under this invariant. $F(T) = 6(1 \cdot f(\alpha))$ where α is the dihedral angle. Then, by definition of f, $F(T) = 6$. And, if R has side lengths l_1, l_2, l_3, then $F(R) = 4l_1 f(\pi/2) + 4l_2 f(\pi/2) + 4l_3 f(\pi/2) = 0$ by definition of f and f additive. $\square$