Take $\mathcal{O}_o$ to be some locally orientable orbifold and $\mathcal{O}_n$ to be some locally non-orientable orbifold.

It is well known that orbifold of different dimensions will have different Laplace Spectra. So, we only must consider the case in which $\dim(\mathcal{O}_o) = \dim(\mathcal{O}_n)$. Without loss of generality, we will take this dimension to be odd.

By Lemma /*4.5*/ $\mathcal{O}_o$ will have no even dimensional primary strata while $\mathcal{O}_n$ will have at least one even dimensional primary stratum.

We first claim that every integer power coefficient in the heat expansion of $\mathcal{O}_o$ is 0. We proceed by contradiction under the assumption that there exists some nonzero integer power coefficient. So, consider the asymptotic expansion given by

$$I_0 + \sum_{N \in S(\mathcal{O})} \frac{I_N}{|\operatorname{Iso}(N)|}$$

But, for an odd dimensional orbifold, the I_0 will not contribute to integer power terms. So, we consider only the second part of the sum. We use /**/ to expand the second half of the sum and arrive at the following expression.

$$\sum_{N \in S(\mathcal{O})} \frac{(4\pi t)^{-\dim(N)/2}}{|\operatorname{Iso}(N)|} \sum_{k=0}^{\infty} t^k \int_N \sum_{\gamma \in \operatorname{Iso}^{\max}(\widetilde{N})} b_k(\gamma, x) dvol_N$$

In studying the above, we find that in order for $\mathcal{O}_o$ to have some nonzero integer power coefficient, $\mathcal{O}_o$ must have some even dimensional primary strata. However, this contradicts our earlier conclusion and thus we reject our assumption and conclude that every integer power coefficient in the expansion of $\mathcal{O}_o$ is 0.

We now show that at least one integer power coefficient in the expansion of $\mathcal{O}_n$ is nonzero. We have that there exists at least one even dimensional primary strata in $\mathcal{O}_n$, so we consider all such strata of maximal dimension d in $\mathcal{O}_n$. Note that only these strata of maximal dimension will contribute to the $-d/2$ term in the heat expansion, which occurs in the $k = 0$ iteration in the sum. Furthermore, by Lemma /*4.1*/ the b_0 term for each contributing strata is strictly positive. Thus, the integer $-d/2$ term is the sum of strictly positive terms and is thus nonzero. This confirms the claim that $\mathcal{O}_n$ has at least one nonzero integer power coefficient.

So, $\mathcal{O}_o$ and $\mathcal{O}_n$ differ in at least one term in the heat expansion. Specifically, $\mathcal{O}_n$ will have some nonzero integer power coefficient in the expansion while the same term in $\mathcal{O}_o$ is guaranteed to be 0.

So, any orientable and non-orientable orbifold will have differing heat expansions, so we conclude that any orientable and non-orientable orbifold will have different Laplace Spectra. This concludes the proof that we can hear local orientability.

The proof for the case that $\mathcal{O}_o$ and $\mathcal{O}_n$ are of even dimension proceeds identically to the above with a few small differences. Instead of considering even dimensional primary stratum, take odd dimensional primary stratum. And, instead of looking at integer coefficients, take half-integer coefficient.