

1. CATEGORY THEORY

- (1) Definition of Category Theory
- (2) homomorphism, monomorphism, epimorphism, isomorphism, automorphism?
- (3) Category of Sets
- (4) Category of Rings

2. LINEAR ALGEBRA

- (5) Category of Vector Spaces
- (6) Direct sum on V.S. definition
- (7) Complementary Definition
- (8) Tensor product on V.S. definition

3. TOPOLOGY

- (9) Category of Topological Spaces (1)
- (10) Hausdorff and Normal definitions [??]
- (11) Tietze Extension Theorem [??]
- (12) Homotopy things
 - (a) Homotopic functions
 - (b) Homotopy equivalence of spaces
 - (c) Retraction definition
 - (d) Contractible definition
- (13) Operations on topological spaces:
 - (a) Topological quotient
 - (b) Wedge Sum
 - (c) Smash Product
 - (d) Cone and Suspension

4. VECTOR BUNDLES

- (14) Category of vector bundles
- (15) Definition of V.B.
- (16) Definitions of $\Gamma(E)$, homomorphism, isomorphism.
- (17) Restriction and pullback of bundles
- (18) Applying functor T on vector spaces to functor on vector bundles [make less abstract?] [Need \oplus , \otimes , anything else?].
 - Do concretely for one and introduce others as an exercise?
 - (a) Definition of T on bundles
 - (b) Verify $T(E)$ has natural topology and $T(\varphi)$ is continuous.
- (19) Sub-bundle definition [??]
- (20) Quotient bundle definition (13) [??]
- (21) Hermetian bundle and metric on bundle definition [??]
- (22) Bundle form of Tietze Extension Theorem (11) [??]
- (23) Collapsing Operation (Lemma 1.4.7) (16, 13, 12)
- (24) Gluing/Clutching Construction (16, 13)

5. DEFINITION OF K-THEORY (AND REDUCED K-THEORY?)

- (25) Definition of $\text{Vect}(X)$ (16)
- (26) $\text{Vect}(X)$ equipped with \oplus and \otimes .

- (27) Stably isomorphic definition. (26)
- (28) Group/Ring Completion through Universal Property
- (29) Definition of $K(X)$ [and $\tilde{K}(X)$?] (25, 28, 27).
 - Verify relevant equivalence relations.
 - Computation of $K(X)$ and $\tilde{K}(X)$ for contractible bundles?
 - Proof of equivalence of multiple definitions. (Present one way and introduce another as an exercise).

6. K-THEORY AS A COHOMOMOLOGY THEORY

- (30) $f_t : Y \rightarrow X$ homotopy then $f_0^* E \cong f_1^* E$. (12)
- (31) If $f : X \rightarrow Y$ is a homotopy equivalence, $f^* : \text{Vect}(X) \rightarrow \text{Vect}(Y)$ is bijective. (Lemma 1.4.4 in Atiyah) (30)
- (32) Contractible Y and $f : X \rightarrow X/Y$ induces bijection $f^* : \text{Vect}(X/Y) \rightarrow \text{Vect}(X)$ (Lemma 1.4.8 in Atiyah, Lemma 2.10 in Hatcher) (12d, 25)
- (33) Exact sequences
 - (a) Definition of exact sequence of vector spaces
 - (b) Definition of exact sequence of vector bundles
 - (c) (?) If short exact sequence $0 \rightarrow E' \rightarrow E \rightarrow E'' \rightarrow 0$ then $E \cong E' \oplus E''$
- (34) Bott Periodicity Theorem

7. DIVISION ALGEBRA APPLICATION

8. OTHER

- V.B. homomorphism is isomorphism iff bijective
- Strict homomorphism definition
- $\Gamma(E)$ is a V.S.
- Prop 1.3.2 (Ker, Im, Coker and sub bundles)
- Natural isomorphism $f^* T(E) \cong T f^*(E)$
- Projection operator definition
- Fundamental Product Theorem in Hatcher

9. NOTES/QUESTIONS ON OUTLINE

- How much time should I spend on vector bundles?
- Try to talk about vector bundles with manifolds?
- Category of sets as example?
- Should I lean to abstract or to concrete? I'm thinking concrete.

10. QUESTIONS ON MATERIAL

- Stably isomorphic classes vs. group completion of $\text{Vect}(X)$.
- Wedge Sum, smash product, cone, suspension.