

READING QUESTIONS 02/11

Hi Iva, I read the Hatcher notes (attached) pages 40, 41, and the start of 42. I skipped 42 – 51, which is all the proof of theorem 2.2 (this theorem is the hard part of proving Bott Periodicity Theorem I believe), then I read starting from Section 2.2 pages 51, 52, 53, and starting 54. Here are my questions:

- (1) At the top of 41, the function $\mu : K(X) \otimes K(Y) \rightarrow K(X \times Y)$. Firstly, I don't think I'm familiar with the tensor product between groups; I imagine it is pretty similar to the stuff we did in Real 2, but should I know more? Secondly, this map is called an *external product*; is that the name of this particular map or is it a technical term I should know?
- (2) At the bottom of 52, the long chain of mappings $A \rightarrow X \rightarrow X \cup CA \rightarrow \dots$. What does $X \cup CA$ technically mean? Where exactly are these two spaces attached? My guess would be that if $CA = (A \times I)/(X \times \{0\})$, these two spaces share the “0” part of the interval. But perhaps it is a disjoint union; not sure.
- (3) On 53, the chain of mappings $\dots \rightarrow \tilde{K}(SX) \rightarrow \tilde{K}(SA) \rightarrow \tilde{K}(X/A) \rightarrow \dots$. How does do we get this? It is certainly a combination of Lemma 2.10 and the chain from my previous question. Perhaps the vertical maps from the chain of mappings on 53 can be expressed by some “obvious” quotients, which allows us to apply Lemma 2.10. Unfortunately, I have no clue what these quotient maps would be.
- (4) On 53, the short paragraph immediately after the chain of mappings starting with “for example”. What does it mean for the sequence to break up into short exact sequences and how is that true in this case? This gives us the result $\tilde{K}(A \vee B) \simeq \tilde{K}(A) \oplus \tilde{K}(B)$, which is used later so this seems important to know.
- (5) Hatcher often references $\mathbb{C}P^1$ and he tells me that this is the same thing as S^2 . He provided a proof in chapter 1 for this that mostly convinced me, but I think I am still not as comfortable with $\mathbb{C}P^1$ and visualizing it as I should be.

Here are some questions that I had, but I believe I have the answer to. I am mostly including these for my own reference, but perhaps you have some insights.

- (1) What is “ \approx ”? A: Apparently another way to write isomorphic.
- (2) Why does X compact Hausdorff and $A \subset X$ imply X/A compact Hausdorff? A: I believe I see the proof. Compact is easy and Hausdorff is easy

with the X compact Hausdorff $\implies X$ normal result.

- (3) On 53, why does A contractible imply bundle over A trivial? A: This proof is in Atiyah, and given the amount of time I spent trying to understand that proof I can't believe I forgot that result.