Math 305: An introduction to partial differential equations with applications to physics

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This document contains content that was created by Paul T. Allen. He gratefully acknowledges the assistance of Liz Stanhope.

Contents

Course goals

The purpose of this course is to serve as an introductory treatment of the wave equation and to provide an introduction to some basic ideas from Fourier analysis. Thus the first goal of the course is for students to understand a number of mathematical methods for studying the wave equation, including

- conservation of energy,
- traveling and standing waves,
- boundary conditions and natural frequencies, and
- the decomposition of waves in to "fundamental modes" for various spatial domains.

This last item in the list above leads naturally to the basic idea of Fourier analysis: decomposition of functions according to frequencies. The second goal of the course is for students to gain basic understanding of how this idea plays out in both the discrete and continuous settings, including:

- boundary conditions, the quantization of frequencies, standing waves on basic domains;
- Fourier-type series and the relation between physical and frequency spaces; and
- the Fourier transform.

In order to construct standing waves on basic domains (interval, rectangle, disk, sphere) the course takes a short foray into Sturm-Liouville theory. We furthermore include a discussion of the calculus of variations, and a variety of additional, optional topics.