COMPLEX VARIABLES: HOMEWORK ON THE CAUCHY INTEGRAL FORMULA – DUE MONDAY MARCH 19TH

(1) Let $C_r(z_0)$ denote the counterclockwise circle of radius r centered at z_0. Compute the following integrals:
   (a) $\int_{C_1(1)} \frac{dz}{(z-1)(z+1)}$
   (b) $\int_{C_5(1)} \frac{dz}{z^3-1}$
   (c) $\int_{C_1(0)} \frac{dz}{z\cos(z)}$
   (d) $\int_{C_1(1+i)} \frac{dz}{z^4+4}$
   (e) $\int_{C_5(i)} \frac{\sin(z)\,dz}{z^2+1}$

(2) Let C be the counterclockwise circle of radius 3 centered at the origin. Use CIF to compute the following:
   (a) $\int_C \left(\frac{2z+1}{z-2}\right)^3 dz$
   (b) $\int_C \frac{1}{(z^2+1)^2}\,dz$
   (c) $\int_C \frac{1}{z^2(4z^2-1)}\,dz$
   (d) $\int_C \frac{1}{(z^2+5z+4)^2}\,dz.$

(3) Let f be holomorphic is a domain D of complex plane with no holes and let z_1 and z_2 be inside a closed contour C which in turn is completely within D. (Assume that C winds around z_1 and z_2 exactly once.) Show that

$$\frac{f(z_2)-f(z_1)}{z_2-z_1} = \frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{(z-z_1)(z-z_2)},$$

and imagine what happens if $z_2 \to z_1$.