1 Mathematical Model

We present a simple and accessible model to demonstrate the twisting phenomenon of the cat.

Consider two identical rods A and B, each with moment of inertia I, separated by angle θ, and the whole structure maintains 0 angular momentum. The primary twisting movement of the cat requires A and B to /*twist*/. Assume that both rods A and B have angular velocity ω in their respective direction $\hat{a}$ and $\hat{b}$ as depicted in /**/.

Then the angular momentums L_A and L_B of the respective rods is given by

$$\overrightarrow{L_A} = I\omega\hat{a} \quad \text{and} \quad \overrightarrow{L_B} = I\omega\hat{b} \quad \text{and} \quad \overrightarrow{L_C} = I_C\omega\hat{c} \tag{1}$$

However, we must have a counter rotation of the whole body by $\overrightarrow{L_C}$ such that angular momentum is conserved:

$$\overrightarrow{L_A} + \overrightarrow{L_B} + \overrightarrow{L_C} = \overrightarrow{0} \tag{2}$$

We let $L_C = I_C\omega_C\hat{c}$ where I_C is the moment of inertia at the center of mass about the $\hat{c}$ axis.

Take unit vectors $\widehat{A}$ and $\widehat{B}$ as shown in /**/. We define the orientation of the cat as the direction of $\widehat{A} + \widehat{B}$. The orientation can vary by an angle ϕ through a plane. We proceed to solve for $\frac{d\phi}{dt} = \omega - \omega_c$.

To solve for ω_C, we simply combine the information listed in (1) and (2):

$$I\omega\hat{a} + I\omega\hat{b} + I_C\omega_C\hat{c} = \overrightarrow{0}$$

And solving for ω_c,

$$\omega_C = -\frac{I}{I_C}\omega||\hat{a} + \hat{b}|| = -2\frac{I}{I_C}\omega\sin\left(\frac{\theta}{2}\right)$$

Then overall,

$$\frac{d\phi}{dt} = \omega\left(1 - 2\frac{I}{I_C}\sin\left(\frac{\theta}{2}\right)\right) \tag{3}$$