COMPLEX VARIABLES: HOMEWORK DUE MONDAY 1/22/2018

(1) Which of the following is true / false? Answer each one, but provide an explanation / proof for about a half. Please don't put too much time into the write-up ... just say or draw something that conveys understanding.

(a) $\mathrm{Re}(3z_1 + 2z_2) = 3\mathrm{Re}(z_1) + 2\mathrm{Re}(z_2)$;

(b) $\mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\mathrm{Re}(z_2)$;

(c) $\mathrm{Re}(z) = \frac{z+\overline{z}}{2}$;

(d) $\overline{\alpha z_1 + \beta z_2} = \alpha \overline{z_1} + \beta \overline{z_2}$;

(e) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$;

(f) $\overline{\frac{1}{z}} = \frac{1}{\overline{z}}$;

(g) $\overline{z^n} = \overline{z}^n$;

(h) $\mathrm{Re}(z^n) = \mathrm{Re}(\overline{z}^n)$;

(i) $\mathrm{Im}(z^n) = \mathrm{Im}(\overline{z}^n)$;

(j) $(\cos(\theta) - i\sin(\theta))^n = \cos(n\theta) - i\sin(n\theta)$;

(k) $|e^{i\theta}| = 1$;

(l) $|e^{in\theta}| = n$;

(m) $|z_1 + z_2| = |z_1| + |z_2|$;

(n) $|z_1 z_2| = |z_1||z_2|$;

(o) $|z^n| = |z|^n$;

(p) $\arg(z_1 + z_2) = \arg(z_1) + \arg(z_2)$;

(q) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$;

(r) $\mathrm{Arg}(z_1 z_2) = \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2)$;

(s) $\arg(\bar{z}) = -\arg(z)$;

(t) $\mathrm{Arg}(\bar{z}) = -\mathrm{Arg}(z)$.

(2) Explain / show the following version of the Triangle Inequality:

$$\big||z_1| - |z_2|\big| \leq |z_1 - z_2|.$$

(3) Draw the following:

(a) $\mathrm{Re}(z-1) > 2$;

(b) $\mathrm{Im}(z+1) < 3$;

(c) $1 \leq |z-1| \leq 3$;

(d) $|z+i| = 3$;

(e) $|2z-3| > 1$;

(f) $|(1+i)z+1| < 2$.

(4) Calculate the following. FAST. De Moivre's formula is your friend. And then some.
   (a) $(1+i)^5$;
   (b) $(1-i)^8$;
   (c) $(1+i)^5(1-i)^8$;
   (d) $\frac{(1+i)^5}{(1-i)^8}$;
   (e) $(3+\sqrt{3}i)^5$;
   (f) $(3-\sqrt{3}i)^5$.

(5) Draw me pictures for the following. Minus points for computing these.
   (a) $1+i$, $(1+i)^2$, $(1+i)^3$, $(1+i)^4$, $(1+i)^5$, ...
   (b) $\frac{1}{1+i}$, $(\frac{1}{1+i})^2$, $(\frac{1}{1+i})^3$, $(\frac{1}{1+i})^4$, $(\frac{1}{1+i})^5$, ...
   (c) $\cos(\theta)+i\sin(\theta)$, $\left(\cos(\theta)+i\sin(\theta)\right)^2$, $\left(\cos(\theta)+i\sin(\theta)\right)^3$, $\left(\cos(\theta)+i\sin(\theta)\right)^4$, ...

(6) Compute and <u>picture</u> **all** the values of the following. (Note: if your pictures suck your homework goes straight to the chicken coop.)
   (a) $1^{\frac{1}{6}}$;
   (b) $(-1)^{\frac{1}{4}}$;
   (c) $(1+i)^{\frac{1}{2}}$;
   (d) $i^{\frac{1}{3}}$;
   (e) $(1-i)^{\frac{1}{3}}$.

(7) Use the identity $\cos(2\theta)+i\sin(2\theta) = (\cos(\theta)+i\sin(\theta))^2$, which is basically just the re-statement of $e^{2i\theta} = (e^{i\theta})^2$, to get a formula for $\cos(2\theta)$ and $\sin(2\theta)$ in terms of $\cos(\theta)$ and $\sin(\theta)$. Then do this off top of your head.

(8) It is kind of useful to pause and think what would happen if the rules of complex number arithmetic were different. So here is an alternative multiplication; please keep addition the same. Also, keep conjugation the same.

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1x_2 + y_1y_2, x_1y_2 + x_2y_1).$$

   (a) Which, if any, of associative, commutative and distributive laws change? You don't need to prove much ... just say something intelligent.
   (b) What is the situation with $z \cdot \overline{z}$? In which ways does it behave differently from the corresponding quantity for "normal" complex numbers? (Draw me a picture.)
   (c) How would you perform division in this "new word"? Who do you get to divide by who?

   P.S. I know who was in abstract last semester. And you-who-were-in-abstract know what I am asking.