THESIS TALK OUTLINE

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(1) Give intuitive introduction to vector bundles with lots of examples.
  (a) Tangent bundle to sphere
  (b) Mobius Strip
  (c) Trivial bundle definition
(2) Give necessary background to understand definition of V.B.
  (a) Topology motivation: Motivate the definition of open sets and continuous functions with visual $\mathbb{R} \to \mathbb{R}$ example.
  (b) Give intuition for generalization with visual examples on sphere.
  (c) Briefly address formal definition of topology and continuous functions.
(3) Give formal definition of a vector bundle, but emphasize a vector bundle as a bunch of fibers.
(4) Go over goal of the talk: given a topological space, want to translate the vector bundles over the topological space into a ring.
(5) Necessary algebra:
  (a) /*ring completion now? maybe later when there is better motivation for it*/
  (b) Motivation: need addition and multiplication due to V.B's being a union of fibers, turn to V.S's.
  (c) Direct sum and tensor product on vector spaces with relevant properties.
(6) Extending direct sum and tensor product to V.B's by applying V.S. operations to each fiber.
  (a) Intuition for extending direct sum and tensor product
  (b) Address identity elements for each operation
  (c) Talk about $E \oplus E'$ trivial result. /*but maybe later when there is more motivation*/
(7) Go through operations of a ring and point out everything works except for additive inverses
(8) Another Algebra tangent:
  (a) What we have is a semiring . . . give definition of a semi ring.
  (b) Example of a semiring: $\mathbb{N} \cup \{0\}$. Adding in the additive inverses gives $\mathbb{Z}$.
  (c) This idea can be formalized so that every semiring has a unique ring extension so long as ... multiplication is commutative and ... there is the additive cancellation law
  (d) Commutativity is good, but note that we have no tools that would promise an additive cancellation law
(9) Quest for getting cancellation property

(a) However, here is a tool that could be of some use: ($E \oplus E'$ trivial theorem)

(b) Show some work trying to use this cancellation property:

$$E \oplus F \cong E' \oplus F$$
$$E \oplus (F \oplus F') \cong E' \oplus (F \oplus F')$$
$$E \oplus \varepsilon^n \cong E' \oplus \varepsilon^n$$

(c) However all we get is (above). BUT now introduce very convenient equivalence relation: $E \approx_s E'$ if we have what is given above. Using this, we do have cancellation property! Adding in equivalence relation brings some complications with it (well-defined, etc.) but it all works out.

(10) Summary slide for the definition of K-Theory