Homework 4: The time-independent Schrodinger equation (**Due Friday October 5**)

September 28, 2018

1 ONE-DIMENSIONAL QUANTUM DOTS **(100 POINTS)**, YOU HAVE SOLVED PARTS OF THIS PROBLEM IN HW3

Quantum dots are nano-technological devices that can be used to confine the electrons and control their motion. An electron is confined to move in a one dimensional quantum dot. The dot can be modeled with the potential

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{elsewhere} \end{cases} \tag{1.1}$$

1. **(5 points)** The wave function of an electron at time $t = 0$ is given by

$$\Psi(x, t = 0) = Ax^2(L - x)\,. \tag{1.2}$$

   Find the value of A.

2. **(15 points)** Given the electron's initial wave function $\Psi(x, t = 0)$, **Compute** the expectation value of the electron's total energy $\langle \hat{H} \rangle$.

3. **(15 points) Calculate** Δ_p and Δ_x and **show** that $\Delta_p \Delta_x$ satisfies the Heisenberg uncertainty principle.

4. **(15 points)** If the electron energy is measured, what are the probabilities, P_n, to find the electron at any of the energy levels $E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$. **(Hint: you do not need to do the integral by hand, you can either look it up in the mathematical tables or just use a software.)**

5. **(15 points) Perform** the sum $\sum_{n=1}^{\infty} P_n E_n$ **numerically**. **How this sum compares to** $\langle \hat{H} \rangle$?

6. **(5 points)** Given the electron's initial wave function $\Psi(x, t = 0)$, **find** $\Psi(x, t)$.

7. **(10 points) Show** that the revival time T of the wave function (that is $\Psi(x, T) = \Psi(x, 0)$) is given by $T = \frac{4mL^2}{\pi\hbar}$.

8. **(5 points)** At time t_1 we measure the electron energy to find that $E = E_2$. **What is the wave function** of the electron **immediately** after the measurement? **What is the wave function at any time** $t > t_1$?

9. **(15 points)** At time t_2 we measure the electron position to find $x = L/2$. **What is the wave function** of the electron **immediately** after this measurement? **What is the wave function at any time** $t > t_2$?