1 Research Essay

1.1 Abstract

- Can you hear the shape of a drum? (Intuition): Given a specific drum, a skilled mathematician could predict the precise sound the drum will make when hit. However, if the mathematician could only listen to some drum in the neighboring room, is it possible to reverse engineer the shape of the drum? Or could there exist multiple drums produce the same sounds?
- (More Mathematical) This simple question sparked the field of *Inverse Spectral Geometry*, which my research resides in.
- Sound is Mathematically formalized through a list called the *Laplace Spectra*. This concept of sound then generalizes to not only drums and physical objects, but abstract mathematical constructs. In my research, we allowed the Laplace Spectra to extend to a class of objects called *orbifolds*, which are motivated by symmetries.
- Question: In this research, we ask the question: "Can you hear the shape of an orbifold?". Or, given the Laplace Spectra of an orbifold
- Tie in historic motivation?
- Applications (Imaging — detecting cracks in bridge support in Liz paper)

/*Give background as briefly as possible, to get into what we actually did*/

1.2 Orbifolds

This section addresses what an orbifold is:

An orbifold is a generalization of a manifold — some multi-dimensional surface. The local structure of manifold is restricted to euclidean space; however, we allow

1.3 Laplace Spectra

1.3.1 Result

The result of our research is the following new definition and theorem:

Definition 1 (Locally non-orientable)**.** We define the local structure of some orbifold to be *non-orientable* if the group action associated to the local structure contains a single orientation reversing element. We define an orbifold to be *locally non-orientable* if there exists any non-orientable local structure; otherwise, the orbifold is *locally orientable*.

Theorem 1. No locally orientable orbifold can have the same Laplace Spectra as any locally non-orientable orbifold. In other words, we can hear the local orientability of an orbifold.

1.4 Methods

1.4.1 Asymptotic Expansion of the Heat Kernel

- list of coefficients $a, b, c, \ldots$

1.5 Reflection on Research?

1.6 Process of finding result

- No huge "aha" moment.
- From our computations, we noticed a pattern within the class of 3-orbifolds.

2 Questionnaire

2.1 Career Goals

2.1.1 Brief

2.1.2 Specific

- Differential Geometry?
- Mathematical Physics?
- Mathematical CS?

2.1.3 Activity that helps enforce

I.S?

2.2 Research Activity

2.2.1 Cloud Research

2.2.2 Activities/Accomplishments

- xc/track

2.2.3

3 Questions

Current goal: Explain what contributions I made and give the necessary background to do this.

- How much historical motivation?
- How much application?

- Reflection on research?
- A lot will need to go into background information even if I try to cut it down. Is this okay?
- Will probably go into research, but not sure what field. — how apparent should I be with my uncertainty