Problem 1. Show that the eigenvalues of the matrix $M = \begin{pmatrix} 0 & 1 \\ -\omega^2 & 0 \end{pmatrix}$ are $\pm i\omega$. (Recall that this matrix comes from the differential equation modeling the simple harmonic oscillator [SHO] in Section 1.3.) Also show that one can choose the corresponding eigenvectors to be

$$\begin{pmatrix} 1 \\ iw \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -iw \end{pmatrix}.$$

Problem 2. Please show that for any choice of vector $U_0 = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$ we can choose α and β so that

$$\alpha \begin{pmatrix} 1 \\ iw \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -iw \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}.$$

Problem 3. The last equation on page 12 of our course notes is a vector form general solution to the SHO equation. Use your results from Problem 2 to show that this general solution does actually provide a particular solution for any given initial condition.

Problem 4. The general solution to the SHO (at the bottom of page 12) involves complex numbers. Here we see how these nevertheless give rise to real-valued solutions.

1. Find the solution U_{c} to the initial value problem

$$\frac{d}{dt}U_c = MU_c \qquad U(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

   where M is the matrix from Problem 1 above. Use the Euler identity (**??**) to show that U_{c} is in fact real-valued.

2. Find the solution U_{s} to the initial value problem

$$\frac{d}{dt}U_{\mathrm{s}} = MU_{\mathrm{s}} \qquad U_{\mathrm{s}}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

   Show that U_{s} is also fact real-valued.

3. Explain why

$$U = \beta_{\mathrm{c}} U_{\mathrm{c}} + \beta_{\mathrm{s}} U_{\mathrm{s}}$$

   is another form of the general solution to the SHO differential equation written in matrix form,

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix},$$

   and that real initial conditions give rise to real-valued solutions.