# COMPLEX VARIABLES: EXAM 1

1. Compute all the values of $\sqrt[4]{-16} = (-16)^{\frac{1}{4}}$. Represent your answer geometrically.

2. Compute the following:
   (a) $\exp\left(1 + \frac{\pi}{2} i\right)$;
   (b) $\log(-ie)$;
   (c) $\cosh(i\pi)$;
   (d) $\sin(\frac{\pi}{2} + i)$;
   (e) $P.V(-1)^{1+i}$.

3. Describe in geometric terms the effect of the mapping $f(z) = (1+i)\, z^2$ on the right half-plane $\mathrm{Re}(z) \geq 0$. Specifically, sketch the *images* of the coordinate grid-lines of the "input" $z$-plane in the "output" $w$-plane.

4. Consider complex logarithm as a function defined on its Riemann surface. Find the pre-image, on the Riemann surface, of the horizontal strip $-2\pi \leq \mathrm{Im}(w) \leq 2\pi$. Please provide an accompanying illustration!

5. Find a branch $f(z)$ of the multivalued logarithmic function which is continuous along both negative and positive parts of the real axis and attains the value of $f(1) = 0$. Give an explicit (piece-wise) formula relating the branches to $\mathrm{Log}(z)$ and interpret in terms of the Riemann surface. In addition, sketch the range for your branch and emphasize the location of the branch cut.

6. Find the domain of the function $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$.

**Extra Credit:** Summarize in two-three sentences the reasons why Cauchy-Riemann equations hold.