READING QUESTIONS 4 MARCH

Hi Iva! This week I gave page 57 a first pass, and I read the bottom of 60 through towards the bottom of page 61. I spent a good amount of time trying to fully understand the proof at the bottom of 60 – that S^{2k} is not an H-space if $k > 0$. This is the first result in which K-Theory leads to a concrete result! Very exciting! I mostly understand this proof, but I do have one question about it down below.

(1) Perhaps the most pressing question: Towards the top of page 61 in the paragraph beginning with "Now we specialize ...", Hatcher mentions a "cell e^{4n} attached by f". From google-ing, I found something called a "CW Complex" which I believe to be related. Here is my current understanding which may be completely wrong:
   - e^{4n} refers to a $4n$-disk.
   - We regard S^{4n-1} as the boundary of e^{4n} and so it is a subspace.
   - C_f is the quotient of the disjoint union $(S^{2n} \cup^* e^{4n})/\sim$ where the equivalence relation $\sim$ glues the boundary of e^{4n} to S^{2n} as according to the f.

   Even if this is correct, I feel that I do not have a good intuition of this construction. Further, in the proof of Lemma 2.18, Hatcher mentions the "characteristic map Φ of the $4n$-cell of C_f", and I am not sure what this refers to.

(2) Also, at the bottom of page 60 in the justification that S^{2k} is not an H-space if $k > 0$, Hatcher mentions that "i^* for i inclusion onto the first factor sends α to γ and β to 0". This makes intuitive sense to me, but I am having trouble formally justifying this. It seems this requires tracing the isomorphisms $K(S^{2k} \times S^{2k}) \approx K(S^{2k}) \otimes K(S^{2k}) \approx \mathbb{Z}[\alpha]/(\alpha^2) \otimes \mathbb{Z}[\beta]/(\beta^2) \approx \mathbb{Z}[\alpha, \beta]/(\alpha^2, \beta^2)$. From here, perhaps retractions are useful? I can semi-justify this piece, but I still don't have a proof.

I think that's it. I had some other questions, but I believe I figured them out in the process of trying to ask them.