CONJECTURE & PROOF

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1. Non Existence / "Show it is impossible" Proofs

Technique 1 (Invariant)**.** Find an invariant — some value that is impossible to change sufficiently.

Problems:
 (1) Show it is impossible to get from 4 to 6 using the operations $n \to n^2$, $n \to 2^n$ and $n \to n - 49$.
 (2)

Technique 2 (Contradiction)**.** Proof by contradiction. Assume that it does exist. This will give some information to work with in order to generate a contradiction.
Note: when possible, it does not hurt to assume some extra information such that the assumption is in some kind of minimal form.

2. Existence Proofs

Technique 3 (Directly)**.** Provide an example. Done.

3. Other Techniques

Technique 4. To arrive at a contradiction, generate a sequence a_n such that $a_n \to 0$ but that we can place a lower bound on. Note: the fact $a \in \mathbb{Z}, a > 0 \implies a \geq 1$ is often useful.

 (1) Show $\sqrt{2}$ is irrational. (Can take sequence $a_n = (1 - \sqrt{2})^n$).

Technique 5 (This or That)**.** Conclude that one thing in some set of things must be true. AND, in the whole set of things, the claim must hold.

 (1) Show there exists $a, b \notin \mathbb{Q}$ such that $a^b \in \mathbb{Q}$. Hint: consider $\left(\sqrt{2}^{\sqrt{2}} \right)^{\sqrt{2}}$