Discrete

Sean Richardson

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1 Combinatorics

Combinatorics studies methods of counting things.

Theorem 1 *Addition Principle: Consider some task T. If T can be accomplished by methods $M_1, M_2, \ldots, M_n$ which can each be accomplished in $a_1, a_2, \ldots, a_n$ ways, then T can be accomplished in $\sum a_k$ ways.*

Theorem 2 *Multiplication Principle: Consider some task T. If T can be broken down into necessary subtasks $t_1, t_2, \ldots, t_n$ which can be accomplished in $b_1, b_2, \ldots, b_n$ ways, then T can be done in $\prod b_k$ ways.*

Theorem 3 *An arrangment of n objects is called a "permutaion". There are $n!$ possible permutations.*

Theorem 4 *An arrangment of r objects out of a collection of n objects is called an "r-permutation". This can be done in $P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$ ways.*

Theorem 5 *An r-combination is how many combinations of r objects (ignoring order) you can choose from n objects. There are $C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ ways.*

We will now try to determine how many ways n balls can be put in to r boxes. To see this, we use the following trick. Think of balls distributed into boxes in the following structure: $\circ\circ / \circ\circ\circ / \circ / \circ\circ$ where "$\circ$" represents balls and the "/" symbols divide the balls into categories or boxes. So, the amount of ways to divide n balls into r categories is the amount of ways we can the distribute the dividers among the characters. This is equivalent to "*characters* choose *dividers*". There are $n + r - 1$ characters and $r - 1$

dividers. So, we have $\binom{n+r-1}{r-1}$ ways to divide n identical things into r categories. Additionally, we have the following useful equivalency:

$$\begin{aligned}\binom{n+r-1}{r-1} &= \frac{(n+r-1)!}{(r-1)!(n+r-1-(r-1))} \\ &= \frac{(n+r-1)!}{(n+r-1-(n))!(n)!} = \binom{n+r-1}{n}\end{aligned}$$

Theorem 6 *The number of ways to distribute n identical balls into r distinct boxes is $\binom{n+r-1}{r-1}$ or $\binom{n+r-1}{n}$ ways.*