Differential Equations

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Definition 1.1. There are three general approaches to solving differential equations.

- An *analytic* approach is to use a method that provides a formula for the exact solution.
- A *qualitative* approach is to look at different properties or sketch to get a feel for the solution.
- A *numeric* approach is approximate the solution with guess/check.

Some basics of differential equations:

- A differential equation relates some function to some derivative of itself.
- We solve for *functions*.
- Functions of one variable are "ordinary differential equations" (ODE's). Functions of multiple variables are called "partial differential equations" (PDE's).

2 Models

2.1 Exponential Model

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$$\frac{dP}{dt} = kP \tag{1}$$

/*solution*/ /*slope field*/

2.2 Logistic Model

/*graph motivation*/

$$\frac{1}{P}\frac{dP}{dt} = -\frac{k}{M}P + k$$
$$\frac{dP}{dt} = Pk(1 - \frac{P}{M}) \tag{2}$$

/*solution*/ /*slope field*/

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Theorem 3.1 (Fundamental Theorem of ODE's)**.** Suppose $f(t, y)$ is a function which x is continuous and has continuous $\frac{\partial f}{\partial y}$ in some neighborhood (t_0, y_0). Then, the IVP

$$\begin{cases} y(t_0) = y_0 \\ \frac{dy}{dt} = f(t, y) \end{cases}$$

has a unique solution.