Key Player Policy in Stable and Dynamic Criminal Networks

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Research in Applied Economics

Department of Economics

May 2020

THE UNIVERSITY OF WARWICK

Contents

Acknowledgments

I would like to thank Dr. Jonathan Cave for supervising my project. It would have been impossible without his invaluable guidance and direction. I would also like to thank Dr. Andrew Harkins for his input and explanation of key concepts at the start of this project. Finally, I would like to acknowledge and thank Prof. Gianna Boero for her support and general feedback over this past year. This dissertation was typeset with LaTeX 2_ε[1].[2]

[1] LaTeX 2_ε is an extension of LaTeX. LaTeX is a collection of macros for TeX. TeX is a trademark of the American Mathematical Society. The style package *warwickthesis* was used.

[2] 4615 words.

Abstract

This paper outlines an endogenous network formation game, where agents have heterogeneous characteristics, and payoffs are linear-quadratic, and depend on the choice of action of the player and her the number of connections. I show that pairwise-stable networks exist, and they are stable states. Further, efficient networks are pairwise-stable, and stable states. The set of pairwise stable networks coincides with pairwise-Nash equilibrium networks. A central planner has the opportunity to disrupt the network before the formation game begins, in a way so as to influence the network formation process, and this paper gives a simple index of optimal targets. These results are relevant for a variety of applications (with appropriate calibrations and changes) such as criminal activity, trade networks etc.

Chapter 1

Introduction

Networks have recently gained prominence in popular discourse, due to the rise of social media and virtual social networks. However, there is an extensive body of knowledge in research of networks, and in their application to fields like epidemiology to study epidemics and pandemics, mathematics and graph theory, social network analysis and simulation, the network of the World Wide Web etc. In economics, networks have produced some interesting results about the distribution of wages (Arrow and Borzekowski, 2004), sharing of risk (Fafchamps and Lund, 2003), R&D networks (Goyal and Moraga-Gonzalez, 2001) and criminal activity (Calvo-Armengol and Zenou, 2004).[1] This paper develops a simple model of criminal networks, with heterogeneous agents having the opportunity to strategically form links to others and act on the resulting network. One of the central questions in any networked setting is whether equilibrium networks are stable, and whether efficient networks are stable, given the general tension between stability and efficiency (Jackson and

[1]For a comprehensive introduction, Dutta and Jackson's Networks and Groups (2002) is recommended. Alternatively, Matt Jackson's Social and Economic Networks (2008) is an excellent reference.

Wolinsky, 1996). This paper attempts to answer some of those questions.

Criminal networks have been the focus of sociological studies for a long time.[2] Economists have traditionally favoured the use of the bilinear quadratic model in this area (Calvo-Armengol and Zenou 2004, Ballester et al, 2006 etc.) Most models either deal with a static network with players embedded in it, and analysis is accomplished by purely game-theoretic equilibrium refinements (like Selten's trembling-hand, or Myerson's hierarchy of errors where more costly errors are less likely). Other models try to explain the process of network formation, by means of strategic decision-making by agents. Few models do both, i.e. endogenize network formation and then perform equilibrium analysis on the resulting network(s). This paper mostly focuses on the dynamics of the process that drives the formation of criminal networks, though it could be easily adapted to other settings like international trade, with some changes (see Chapter 6 for a more detailed discussion). This model uses a variant of the traditional linear-quadratic model, and as Ballester et al (2006) have noted, this can be extended to any C^2 payoff functions by first-order approximation.

Katz (1953) was one of the first to introduce a measure of prestige as a proxy for centrality in a network, in the context of sociological analysis. Bonacich (1987) established a family of centrality measures that could be defined in terms of walks from one node to another, with a decay factor for longer walks. These are geometric formulations to find the relative importance of nodes. Ideas like betweenness centrality, on the other hand, originated from a graph-theoretic background (Freeman, 1977). These notions have been extended with varying degrees of success, to groups of nodes. There are two

[2] C. Morselli, Crime and Networks (2013).

main classes of problems associated with indexing the key player in networks with strategic interactions: The Key Player Problem- Positive and Negative (henceforth KPP-P and KPP-N).[3] KPP-P tries to identify the key nodes who are maximally connected to the rest of the network to facilitate optimal diffusion of information throughout the network. KPP-N, which this paper deals with, is the problem of finding the key player whose removal would maximally disrupt the network, where disruption is defined as a specific optimization function.

Ballester et al (2006) pioneered the KPP-N solution where they develop a new metric of centrality- intercentrality- that reflects the planner's interests in optimally disrupting the network. This keeps track of the direct and indirect effects of the removal of the key player from the network- namely, the direct effect of removal of the key player's contribution to the aggregate effort of the network, and the change in the activity of the other players (the indirect effect). Variants on this theme include incomplete information about the state of the world, where agents receive independent, uncorrelated signals and update their beliefs using Bayes' rule. De Marti et al (2015) develop a model where the planner has a different prior and plays first. This is often the case with law-enforcement agencies having a more complete picture of the state of the world than individual agents (criminals). The difference in their analysis from the benchmark model is that under expectations, the key player index is not simply the intercentrality index.

[3]Bramoullé, Y., Galeotti, A., Rogers, B., Zenou, Y. (2016-04-14). Key Players. In The Oxford Handbook of the Economics of Networks. : Oxford University Press. Retrieved 5 May. 2020, from https://0-www-oxfordhandbooks-com.pugwash.lib.warwick.ac.uk/view/10.1093/oxfordhb/9780199948277.001.0001/oxfordhb-9780199948277-e-14.

In the dynamic network formation game, Liu et al (2012) assume that the agents play a two-stage game with the planner, where in stage one (the morning) they decide with whom to link (myopically), and the planner then removes the key player. In the afternoon, agents play the effort game after being embedded in the network. Konig et al (2014) then use this analysis to show that the steady state of the network takes the form of a nested split graph (separation into a maximal independent set and a clique). In Liu et al's analysis, the key player who reduces the aggregate effort by the most is not necessarily the player having highest intercentrality. It is important to note that these papers utilize Markovian solution concepts, which is not what this paper does. The contribution of this paper is to prove that given the model, stable networks exist, are attainable from the empty network, and efficient networks belong to the set of stable networks. Further, this paper utilizes the refinement of pairwise stability with pairwise-Nash equilibrium networks (Calvo-Armengol and Ilkilic, 2009), which is a stronger stability concept. Finally, this paper outlines a Key Player Policy given the payoffs and some directions for future research.

Chapter 2

The Game

There are $n + 1$ players (nodes) on an extant undirected network, that is disrupted by a central authority that does not have full knowledge of the structure of the network. Here, disruption simply means the planner intervening and removing a node. The process of selection of the node to be targeted will be discussed later. Thus, at the start of the network formation game, we have g^0 as our empty network. We assume that players go into hiding or at the very least, sever their connections temporarily for one period, post-intervention, due to the increased risk from the perceived increased activity of the planner. This could also be due to some exogenous event, like a pandemic, or impending disaster.

Here, $i = 1, 2, \ldots, n$ are our players in g^0. Let L denote the set of links, then by definition, $L(g^0) = \emptyset$. Players have full knowledge of the network structure. Now, players start re-wiring their connections strategically, but in a myopic fashion, having a utility function

$$u_i(x_i, n_i) = \frac{x_i(n_i)^{\frac{1}{2}}}{\beta} - \alpha_i {x_i}^2 - k{n_i}^2 \tag{2.1}$$

where x_i is choice of effort player i exerts. Note that we are dealing with an unweighted and undirected graph. The utility function is known to the planner. Here, α_i reflects individual heterogeneity, and $\alpha_i, \beta \in (0, 1]^2$. Lower α_i players exerting the same equilibrium effort, can afford to be more linked, making them hubs in equilibrium networks. The utility function is strictly concave in x, reflecting the fact that efforts have marginally diminishing returns to utility. Also, n_i reflects the degree of node i. Since players have a choice of linking to neighbours, there is a cost of doing so, reflected in the last term of 2.1. This function implies that higher effort exerted is subsidized by having more neighbours, but the agent has a trade-off due to the cost associated with linking. As described by Becker(1968), criminals act after careful consideration of expected costs and benefits. The gross benefit from our utility function is $b_i = h(x_i, n_i) = \frac{\sqrt{n_i} x_i}{\beta} - \alpha x_i^2$. The cost is characterized by $c_i = g(n_i) = kn_i^2$, where k is perceived planner activity and $k > 0$. u_i serves as our allocation rule. The best-response function is given by:

$$\mathbf{x}_i = \frac{(n_i)^{\frac{1}{2}}}{2\alpha_i \beta} \tag{2.2}$$

Post-intervention, each agent is selected at random, and given an opportunity to manage their relationships. Agents can update their links by linking to new neighbours, or they can unilaterally delete harmful links. The concept of Pairwise Stability developed by Jackson and Wolinsky (1996) is used here, to describe any pairwise-stable network g:

$$\forall ij \in g,\ u_i(g, x_i) \geq u_i(g - ij, x_i)\ and\ u_j(g, x_j) \geq u_j(g - ij, x_j),\ and \tag{2.3a}$$

$$\forall ij \notin g,\ u_i(g, x_i) < u_i(g + ij, x_i)\ then\ u_j(g, x_j) > u_j(g + ij, x_j). \tag{2.3b}$$

Thus the set of strategies of player i is given by $\sigma_i = X \times g^N$, and the set of all agents' strategies is simply $\Sigma = \sigma_1 \times \sigma_2 \times \cdots \times \sigma_n$, where $X = [0, +\infty)$. Here, pairwise stability is used as an equilibrium concept instead of the traditional Nash equilibrium because Nash equilibrium does not offer much when predicting equilibria in network formation, as it predicts that no single agent prefers to deviate in equilibrium. Pairwise stable implies the consent of two agents to form a link. As such, the empty network may be a Nash equilibrium, simply because single players do not wish to deviate from having no links, but when we consider pairs of players, it may be more profitable for both to have a link instead of none. Thus pairwise stability gives us more bite. Also, computing an agent's Nash equilibrium best response is an NP hard problem, but pairwise stability does not suffer from the same problem. As we will see later, the set of pairwise stable networks in our case coincides with the set of pairwise-Nash equilibrium networks. Pairwise-Nash equilibrium configurations are robust to multi-link severance, as opposed to single links allowed by pairwise stability. Intuitively, the set of pairwise-Nash networks are the intersection of Nash equilibrium networks, and pairwise stable networks. Yet another equilibrium concept, proper equilibria as introduced by Myerson (1978) have agents best responding to trembles in strategies of other agents, and where costly trembles are less likely in equilibrium. We shall not be using it here, but Calvo-Armengol and Ilkilic (2009) have a detailed discussion on its application to dynamic network formation games and its equivalence to pairwise-Nash and pairwise stable network configurations under certain conditions. There are yet stronger equilibrium concepts concerning coalitional deviations, that use strong stability as an equilibrium outcome (Jackson and Van den Nouweland, 2005). We shall not be using them in this paper.

It is assumed that the activity of the network on the path to equilibrium is not observed by the authority, but once equilibrium is reached, i.e. when a pairwise stable formation from g^0 via strategic link formation is attained, it is visible to the authority. Naturally, the next question is whether any pairwise stable equilibrium exists, and if so, is it reachable from the empty network? This will be discussed in later sections in detail.

Chapter 3

Analysis

Before we dive into a full-fledged analysis of the dynamics of the model and its stability, it would be worthwhile to briefly introduce the main tools used in the rest of the paper.

- Value function

  The net utility of any network configuration in our setting is simply given by $U(g,x) = \sum_{i=1}^{n} u_i(g,x)$, and as described earlier, $u_i : \{g|g \subset g^N\} \times \{x|x \in X\} \to \mathbb{R}$ and $X = [0, +\infty)$. Thus in our setting, g is efficient if $U(g) > U(g')$, $\forall g' \subset g^N$.

- Improving Paths

  This concept, borrowed from Jackson and Watts (2002), describes the sequence of networks evolving from the utility function described earlier, consistent with the rules of pairwise stability. Let $G = \{g_i\}_{i=1}^{t}$ represent an improving path, where G is finite. Then, every g_i and g_{i+1} in G are `adjacent`, meaning they differ by only one link. To formalise this, we define it as follows:

Definition 3.1. An improving path from g' to g is a finite sequence of networks $\{g_t\}_{t=1}^{T}$ where $g_t = g$ and $g_1 = g'$ such that for any $t \leq T - 1$,

either $g_{t+1} = g_t - ij$ for any ij that makes $u_i(g_t - ij) > u_i(g_t)$

or $g_{t+1} = g_t + ij$ for any ij that makes $u_i(g_t + ij) > u_i(g_t)$ and $u_j(g_t + ij) \geq u_j(g_t)$.

This represents the dynamic process of network formation.

- Defeat

  As outlined in Jackson and Watts (2002), a non-pairwise stable network g' is said to be defeated by another network g if g contains a link that, despite being profitable for both agents, was not formed in g', or g does not contain a link that was in g' despite being unprofitable for either of the agents then. Formally, $gdg' \iff g = g' + ij$ and 2.3a is violated for g', or $g = g' - ij$ and 2.3b is violated for g'. This paper has taken some liberty with the notation, but it simply means that g defeats g'.

- Cycles

  It is not necessarily the case that the dynamic process of network formation leads it to a single state that is pairwise stable. It may be the case that a number of networks are visited in succession since there exists no pairwise stable network. In this case, the set of networks visited is said to form a cycle. Formally, as defined in Jackson and Watts (2002),

Definition 3.2. A set of networks C forms a cycle if for some $g, g\prime \in C$, $\exists \{g_i\}_{i=1}^{T}$ that connects g' to g.

Thus there must be some improving path connecting g' to g. A cycle is closed if no network in C lies on any improving path leading out to some network not in C. The next lemma from Jackson and Watts (2002) is one of the primary building blocks of this paper.

Lemma 1. For any allocation rule and value function, there must exist either at least one pairwise stable network or a closed cycle.

Proof. As outlined in Jackson and Watts (2002). □

The question now is, do pairwise stable networks exist in our setting, or does the dynamic process lead us to a closed cycle of networks? We find proof of existence of at least one pairwise stable network (discussed in the next section).

The above theorem borrows the concept of Exact Pairwise Monotonicity from Jackson and Wolinsky (1996) with slight changes as outlined in Jackson and Watts (2002). Essentially, given our value and allocation functions, we find that individual incentives are exactly aligned with those of the whole network. This has one interesting implication, namely that efficient networks are pairwise stable.

- <u>Stable State</u>

  A pairwise stable network g^{PS} is said to be a stable state if it can be reached from the empty network. From Watts (2001), there must exist an improving path from g^0(the empty network) to g^{PS}(the stable state). The question for us, having established that a pairwise stable network exists, courtesy of 4.1, is whether this pairwise stable network can be reached from the empty set. For it may be the case that the empty network is itself pairwise stable, and consequently the dynamic process of network formation might be unable to nudge the network into creating links. For example, in our setting, if $c_i \geq b_i$, i.e.

$$k \geq \frac{x_i}{\beta} - \alpha_i x_i^2 \tag{3.2}$$

then the empty network cannot be nudged into creating new links. Far-sightedness of agents can potentially solve this problem, but we assume $k < \frac{x_i}{\beta} - \alpha_i x_i^2$ for $n_i = 1$, for any $i \in N(g)$ to get past the empty network.

- Single Peakedness of value function

  First used by Black (1948), this concept would help us overcome the previous problem, if certain conditions were true for the value function. For U to be single peaked, U must be monotonically increasing in the number of links, until a peak is hit, following which U monotonically decreases. As discussed in the next section, we find that our value function is single-peaked for appropriate values of k.

- α -submodularity As introduced by Calvo-Armengol and Ilkilic (2009), this condition states that a payoff function is α-submodular if for any $i \in N$, the joint payoff from any group of links present in a network is greater than the total marginal payoffs from the individual links in the group, in the network. Formally, for any u_i to be α-submodular, $\forall \ell \subseteq L_i, \forall i \in g, \forall g \subset g^N$.

$$\mu_i(g, \ell) \geq \alpha \sum_{ij \in \ell} \mu_i(g, ij) \tag{3.3}$$

where

$$\mu_i(g, \ell) = u_i(g) - u_i(g - \ell) \tag{3.4}$$

3.4 essentially describes the joint value of links in ℓ to i, and 3.3 states that such joint value must be weakly greater than the sum total of value from individual links in the group (scaled by α, making this a weaker concept than strict submodularity). We use this central result (established later in this paper) to establish that any pairwise stable network must also be pairwise-Nash stable.

Chapter 4

Results

Theorem 4.1. *Given U and u_i, $\exists f : \{g | g \subset g^N\} \to \mathbb{R}$ such that $[gdg'] \iff [f(g) > f(g')$ and $g = g' \pm ij$ for some $ij \in L(g^N)$]. Consequently, there exist no cycles.*

Proof. The proof is similar to the first part of Jackson and Watts (1998), and we proceed by contradiction. Let $g' \to g$ denote that g lies on an improving path starting from g'. We assume that a cycle exists so that $g \to g$, i.e. g lies on an improving path from itself. If this is true, then we have to merely prove that there cannot exist any function f. Assuming that f exists, f satisfies $f(g) > f(g)$ given the transitivity of the greater-than operator. However, this is a contradiction, and so there can be no cycle if $f(g)$ exists. Now we need to prove that such a function exists, given U and u_i.
A natural candidate for f is our utility function U. Since $U = \sum_{i=1}^{n} u_i$, if we assume that g and g' are adjacent, then if $g = g + ij$, it must be profitable for both i and j to have that link. If $g = g - ij$, it must be in the interest of both i and j to have that link removed. Given the formulation of u_i, every other agent in g is no worse off by these value adding operations. Further, if

$U(g) > U(g')$, that implies g contains some profitable link not in g', or that g does not have some unprofitable link (for some i) that was present in g'. Therefore gdg. Thus our value function U ensures that no cycles exist. Consequently, there must exist at least one pairwise stable network. □

The above theorem borrows the concept of Exact Pairwise Monotonicity from Jackson and Wolinsky (1996) with slight changes as outlined in Jackson and Watts (2002). Essentially, given our value and allocation functions, we find that individual incentives are exactly aligned with those of the whole network. This has one interesting implication, namely that efficient networks are pairwise stable.

From our discussion on the value function $U(g)$ we know that loosely speaking, $U = \sum_{i=1}^{N} u_i$. Thus, we can express U in terms of b_i and c_i as follows:

$$U(g, x) = \sum_{i=1}^{N} b_i - \sum_{i=1}^{N} c_i. \tag{4.1}$$

Adding links in our setting would only increase U monotonically to some point after which the squared degree cost scaled by the negative constant k would make U start decreasing monotonically, for appropriately chosen values of α, β and k. Thus we say that our value function is single-peaked. This helps us overcome the problem of the empty network being a stable state, by making it profitable for more links to be formed, until a point.

Theorem 4.2. *Given U and u_i, $PS(u_i) = PNE(u_i)$ as u_i is α-submodular on $PS(u_i)$, for some $\alpha \geq 0$.*

Proof. Theorem 1 from Calvo-Armengol and Ilkilic (2009) proved that

α-submodularity establishes $PS(u_i)$=$PNE(u_i)$. Thus we have to prove α-submodularity of u_i. Let x be equilibrium effort in g for agent i. Let ℓ links be considered for an arbitrary player i in g^{PS}. Then,
$\mu_i(g, \ell) = u_i(g) - u_i(g - \ell)$. Let new optimal effort of i be denoted by x'. We see from 2.2 that $\mathbf{x}$ is increasing in $n^{\frac{1}{2}}$, so $x' < x$.

$$u_i(g - \ell) = \frac{x'((n_i)^{\frac{1}{2}} - \ell)}{\beta} - \alpha_i(x')^2 - k(n_i - \ell)^2 \tag{4.2}$$

Therefore, $\mu_i(g^{PS}, \ell)$ will be given by

$$\mu_i(g^{PS}, \ell) = \frac{x(n_i)^{\frac{1}{2}} - x'\{(n_i)^{\frac{1}{2}} - \ell\}}{\beta} + \alpha_i\{(x')^2 - x^2\} + k\{(n_i - \ell)^2 - n_i^2\} \tag{4.3}$$

This gives us the joint utility derived from ℓ links to i. This is basically the difference of marginal benefit (first and second terms) and marginal cost (last term). In equilibrium, since $\mathbf{x}_i = \frac{(n_i)^{\frac{1}{2}}}{2\alpha_i\beta}$, equilibrium utility is given by

$$\mathbf{u}_i(g^{PS}, \mathbf{x}) = \frac{n_i}{4\beta^2\alpha_i} - kn_i^2 \tag{4.4}$$

It is clear that utility has a lower bound on 0, for any individual link, in equilibrium, for the network to be pairwise stable. If not, that link would be severed, setting the network off on an improving path, which violates our prior assumption of pairwise stability. Thus single-link marginal payoffs are non-negative. Similarly, if the cost of maintaining ℓ links is more than the benefit received, then the agent can always deviate and do better by severing at least one of the links in ℓ, which again violates our assumption of stability. Thus to be a PS network, marginal cost cannot be more than marginal benefit, implying $\mu_i(g^{PS}, \ell) \geq 0$. By this reasoning, $\mu_i(g^{PS}, ij) \geq 0$. Thus it follows from 3.3 that our utility function u_i is α-submodular on PS(u). Thus $PS(u_i) = PNE(u_i)$ by Theorem 1 from Calvo-Armengol and Ilkilic (2009). $\square$

Thus the sets of pairwise stable and pairwise Nash stable networks coincide. Given the dynamics of the model, is it possible for the central planner to optimally target a node in the network so as to minimize aggregate activity (effort) in the equilibrium network? Our model assumes that non-equilibrium activity is not observed by the authority, but once the network stabilizes to an equilibrium (pairwise-Nash stable) formation, can the planner optimally target the intervention towards a particular node in the network so as to influence re-wiring incentives and ultimately equilibrium network effort? This is discussed in the next section.

Chapter 5

Equilibrium and Disruption

We have previously found that due to α-submodularity, the set of pairwise stable and pairwise Nash equilibrium networks coincide. It is important to define the pairwise Nash- equilibrium concept used here, in the context of simultaneous choices of moves and action. Thus, when agents link to each other, their deviations in degree and new effort levels are considered best responses (*idem.* for link deletion). Deviations are denoted by x' and n'. Any strategy $\boldsymbol{\sigma^*} = (\boldsymbol{x}^*, \boldsymbol{g}^*)$ is a pairwise Nash equilibrium $\iff$

$$\forall i \in N, \forall \sigma_i \in \Sigma_i, \;\; u_i(\boldsymbol{\sigma^*}) \geq u_i(\boldsymbol{\sigma_i}, \boldsymbol{\sigma^*_{-i}}); \tag{5.1}$$

$$\forall ij \notin \boldsymbol{g}^*, \text{if}\, u_i(x'_i, \boldsymbol{g}^* + \boldsymbol{g_{ij}}) > u_i(\boldsymbol{\sigma^*}), \text{then}\, u_j(x'_j, \boldsymbol{g}^* + \boldsymbol{g_{ij}}) < u_j(\boldsymbol{\sigma^*}). \tag{5.2}$$

Thus, this equilibrium concept allows for any deviation where a player deletes a subset of her links and adjusts effort levels simultaneously. It is worth noting that any deviation where a player both deletes a subset of her extant links and adds a link, while adjusting her action, is not allowed.

Disruption is accomplished by having a key-player policy (Ballester et al, 2006). Such a policy would identify agents, whose removal from the graph would maximise the change in aggregate effort of the resulting graph from the original graph (equivalently, minimizing the aggregate activity in the resulting graph). Such a policy would not exist if the $PS(u) = \emptyset$. We have seen that the dynamics in this model allows for at least one pairwise stable (PS) or pairwise-Nash (PN) network in equilibrium, thus **a key player policy always exists** in our setting.
The optimisation problem for the planner is to solve

$$max_j\{\sum_{i\in N(g^{PS})} x^*(\boldsymbol{g^{PS}}) - \sum_{i\in N(g^{PS})\backslash j} x^*(\boldsymbol{g^{PS}_{-j}})\} \tag{5.3}$$

which is equivalent to

$$min_j\{\sum_{i\in N(g^{PS})\backslash j} x^*(\boldsymbol{g^{PS}_{-j}})\} \tag{5.4}$$

where $N(g)$ denotes the set of nodes in g. This is a finite minimisation, which has at least one solution. If j solves 5.4, then we call j **the key player**.

Definition 5.1. The Key Player is defined to be the player with the lowest α_i. Eliminating the Key Player reduces equilibrium network activity the most.

Now let us consider a simple example to illustrate the definition.
It is easy to see that 5.1 is pairwise stable, with $\beta = 0.4$ and $k = 0.8$ and aggregate activity given by $\sum_{i=0}^{3} x_i = 16.415$. If the planner targets node 1 before the beginning of the game, then the resulting PS network in

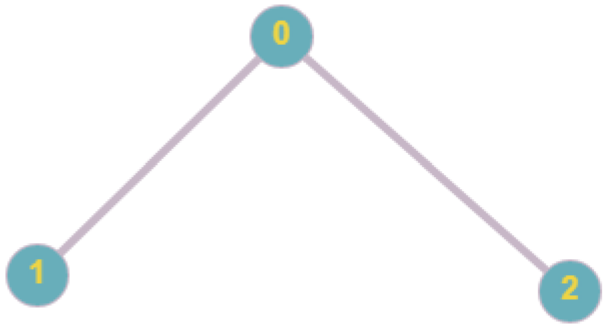

Figure 5.1: $\alpha_0 = 0.5, \alpha_1 = 1, \alpha_2 = 0.7$

equilibrium is given by 5.2. Aggregate activity now is given by

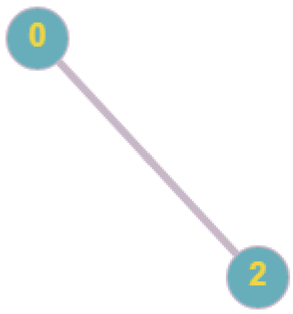

Figure 5.2: $\alpha_0 = 0.5, \alpha_2 = 0.7$

$x_0 + x_2 = 10.710$. The same will be the case if node 2 is disrupted. What if the planner targets node 0, the Key Player according to Definition 5.1? The PS network is given by 5.3. Here, the aggregate activity is given by

Figure 5.3: $\alpha_1 = 1, \alpha_2 = 0.7$

$x_1 + x_2 = 7.585$. Clearly, a greater reduction of aggregate activity is achieved by eliminating the player with lowest alpha, player 0 having the highest degree. As we have seen from 2.2, x_i is decreasing in α_i, and so the results are not too surprising. Thus, we have established an optimal targeting mechanism that takes into account how the network re-wires post-intervention.

Chapter 6

Discussion and Extensions

There exist a number of economic models that endogenise the dynamic process of network formation in a two-stage game. However, the model with the approach presented in this paper are new, to the best of the author's knowledge. This model can easily be extended to a non-criminal setting, eg. trade networks. In such a setting, starting from the empty network would have a different interpretation, where some exogenous event could temporarily sever trade links between trading partners, like a global pandemic. It could easily be adapted to reflect externalities and complementarities, although it remains to be seen if the efficient network would be pairwise-stable and a stable state. In such a setting, the targeting mechanism could be adapted to protect the most important player, although some different metric would be more suitable to protect the integrity of the network. Groups as targets have not been considered here, but Ballester et al (2010) show that the key-group selection is an NP-hard problem. Nevertheless, an exploit heuristic (greedy) is used and error bounds are established. Zhou et al (2015) in a slightly different framework have showed that an optimal leader group targeting problem is equivalent to the weighted MAX CUT (NP hard) problem in graph theory.

There are a number of potential extensions of this simple model. There is scope to generalize the utility function under appropriate constraints to any quadratic utility function. This specialised utility function is used in many models in economics, notably by Goyal and Moraga (2001) for RD networks and Akerlof (1997) to explain the role of social distance in social decision making. Ballester et al's (2006) model allowing for bilinear quadratic utility with local complementarities in actions (effort) and global strategic substitutes is the benchmark model for criminal activity. There exists robust empirical evidence that these models correspond to real-world networks, shown in Calvo-Armengol and Jackson (2004) and Calvo-Armengol and Zenou (2004) for delinquent criminal networks. Hiller (2013) generalizes this model and proves that in equilibrium, networks have a core-periphery structure (under certain conditions on minimum and maximum cost structures). Further, this can be extended to any C^2 payoff functions exhibiting nonlinear externalities by first-order approximation (Ballester et al, 2006).

The objective of the planning authority could also be different- it could be to simply optimise aggregate welfare. Further, if the assumption regarding the unobservability of activity on non-pairwise stable network to the planner is relaxed, the planner would care more about the net present expected value of actions induced by the dynamic configuration process. Another key assumption made in this paper- myopic utility maximising strategies of agents- is open for discussion. Perhaps results are different when agents are farsighted, and care about present discounted utility of some payoff stream, although the results would not change with a high enough discount factor, such that agents discount the future heavily. Also, β could be different for different clusters, but that is not explored by this paper. Capacity constraints would also be interesting to look at, since it is practically impossible to link to anyone in a network in reality. This could take the form of being able to link to agents in $N_i^{(2)}$ only, as shown in König, Tessone and Zenou (2009). It is suspected that this would merely reduce the total number of links in equilibrium, however König et al have found results related to assortativity in agents in equilibrium.

Further, the cost structure can be modified to reflect risk directed towards clusters than individual agents. Perhaps clusters with a high clustering coefficient would suffer from a higher k (more negative). Also, there is a cost to breaking a link unilaterally, as there is a risk of aggravating the agent who was better off, which could lead to her informing the authority about the other player. Thus, bilateral severance of links would seem like a more robust assumption to make, which would entail changes to the pairwise stability condition. Further, the resources of the planner could be limited, leading to costly interventions. Alternatively, links might be targeted instead of nodes. It would be interesting to see if and how the decision of the planner changes with limited resources in targeting nodes/edges.

Finally, it would be a logical next step to calibrate this model with data analysis from real-world datasets, in the spirit of Banerjee et al (2013). The

Add-Health datasets could prove useful here, as shown in Calvó-Armengol, Pattacchini and Zenou (2005 and 2009).

Chapter 7

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