1. P(x) = x is happy

a = Alice

P(a)

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P(a) P(x)

1. The model states that the Domain we are dealing with contains only a single element , the number 1 {1}.

a is our constant which refers to the 1.

P{1} is the extension of the predicate which defines for which elements P is true.

So with P(a), this becomes P{1}. From extension of predicates we can see that this evaluates to true.

From here P(a) = True

Where we can sub ~~P(a) = ~~True

We can first do ~True = False

Then for the remaining negation ~False

This becomes True

Therefore ~~Pa is True.

1. This states for some x not all P(x) if and only if not all x is P(x).

* There exists an x that is not P, there exists an x where P is false
* Not all x are P, not every x will be true for P

From this it says there exists an x where P is false, which is the same as not every x will be true for P. Hence it is a logical truth as it is true for every possible model

D = {1, 2}

a = {2}

P = {1}

For the first half there is an x that is not P, in this case this is True. As P(a) = False

There is an x that exists where P is not true.

For the second half not all x is P is True. As if P(a) not all possible x is True. X could be 1 or 2.

True True = True. There cannot exist a model that where this is not true even for an infinitely large D.

1. This says for all x in our domain (D), Q(x) will be true.

Therefore for P(a) or not P(a)

The premise is completely unrelated to our conclusion.

The 2 possible outcomes for P(a) is either True or False. By Saying

True or ~ True we are given True or False which always evaluates as True.

An attempt create a model to disprove this:

D = {1, 2}

a = {1}

P = {2}

In this case:

* P(a) = False
* ~P(a) = ~False = True
* True or False = True
* Even with a = {2} and P = 1 this would still be true.
* This would remain true any possible domain.

Ultimately the argument is valid as it is a tautology. This will always remain true regardless of the premises.