ECSE 543
Assignment 2

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Introduction

1 Finite Element Triangles

The equation for the α parameter for a general vertex i of a finite element triangle can be seen in Equation 1, where $i+1$ and $i+2$ implicitly wraps around when exceeding 3.

$$\begin{aligned}\alpha_i(x,y) = \frac{1}{2A}\big[&(x_{i+1}y_{i+2} - x_{i+2}y_{i+1}) \\ &+ (y_{i+1} - y_{i+2})x \\ &+ (x_{i+2} - x_{i+1})y\big]\end{aligned} \tag{1}$$

Using Equation 1, we can solve for the entries of the local S matrix, as shown in Equation 2. This was used in the program to compute every entry for both example triangles.

$$\begin{aligned}S_{ij} &= \int_{\Delta_e} \nabla\alpha_i \cdot \nabla\alpha_j dS \\ &= \frac{1}{4A}\big[(y_{i+1} - y_{i+2})(y_{j+1} - y_{j+2}) \\ &\quad + (x_{i+2} - x_{i+1})(x_{j+2} - x_{j+1})\big]\end{aligned} \tag{2}$$

The local S matrix for the first triangle can be seen in Equation 3.

$$S_1 = \begin{bmatrix} 0.50 & -0.50 & 0.00 \\ -0.50 & 1.00 & -0.50 \\ 0.00 & -0.50 & 0.50 \end{bmatrix} \tag{3}$$

The local S matrix for the second triangle can be seen in Equation 4.

$$S_2 = \begin{bmatrix} 1.00 & -0.50 & -0.50 \\ -0.50 & 0.50 & 0.00 \\ -0.50 & 0.00 & 0.50 \end{bmatrix} \tag{4}$$

2 Finite Element Coaxial Cable

2.a Mesh

2.b Electrostatic Potential

2.c Capacitance

The finite element energy equation can be seen in Equation 5.

$$W = \frac{1}{2}U_{con}^T S U_{con} \tag{5}$$

$$W = \frac{1}{2}CV^2 \tag{6}$$

3 Conjugate Gradient Coaxial Cable

3.a Positive Definite Test

3.b Matrix Solution

3.c Residual Norm

3.d Potential Comparison

3.e Capacitance Improvement

A Code Listings

B Output Logs