ECSE 543
Assignment 3

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Contents

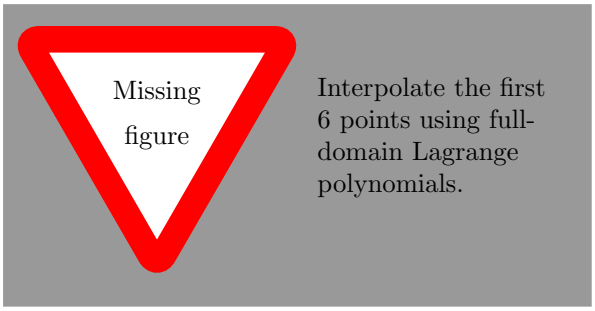
Introduction

The code for this assignment was created in Python 2.7 and can be seen in Appendix A. To perform the required tasks in this assignment, the `Matrix` class from Assignment 1 was used, with useful methods such as add, multiply, transpose, etc. This package can be seen in the `matrices.py` file shown in Listing 1. The only packages used that are not built-in are those for creating the plots for this report, i.e., `matplotlib` for plotting. The structure of the rest of the code will be discussed as appropriate for each question. Output logs of the program are provided in Appendix B.

1 BH Interpolation

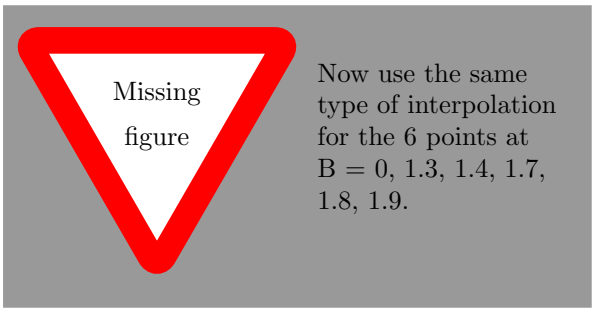
The source code for the Question 1 program can be seen in the `q1.py` file shown in Listing 2.

1.a Lagrange Polynomials



Interpolate the first 6 points using full-domain Lagrange polynomials.

1.b Full-Domain Lagrange Polynomials



Now use the same type of interpolation for the 6 points at B = 0, 1.3, 1.4, 1.7, 1.8, 1.9.

1.c Cubic Hermite Polynomials

2 Magnetic Circuit

The source code for the Question 2 program can be seen in the `q2.py` file shown in Listing 3.

2.a Flux Equation

The magnetic analog of KVL can be seen in Equation (1).

$$(\mathcal{R}_a + \mathcal{R}_c)\psi = \mathcal{F} \tag{1}$$

where $\mathcal{R}_a$ is the reluctance of the air gap, $\mathcal{R}_c$ is the reluctance of the coil, and $\mathcal{F}$ is the magnetomotive force. Plugging in the relevant variables from the problem, we obtain Equation (2).

$$\left(\frac{L_a}{A\mu_o} + \frac{L_c}{A\mu_c(\psi)}\right)\psi - NI = 0 \tag{2}$$

where $\mu_c(\psi)$ is a function of ψ given by Equation (3).

$$\mu_c(\psi) = \frac{B}{H} = \frac{\psi}{AH} \tag{3}$$

Plugging Equation (3) into Equation (2), we obtain Equation (4).

$$\left(\frac{L_a}{A\mu_o} + \frac{L_c H}{\psi}\right)\psi - NI = 0 \tag{4}$$

Simplifying the terms, we obtain Equation (5).

$$f(\psi) = \frac{L_a\psi}{A\mu_o} + L_c H - NI = 0 \tag{5}$$

Finally, if we plug in the values from the question, we obtain Equation (6), where the coefficients of the terms are calculated in the `q2.py` script shown in Listing 2.

$$f(\psi) = 3.979 \times 10^7\psi + 0.3H - 8000 = 0 \tag{6}$$

2.b Newton-Raphson

$$B = \frac{\psi}{A} \tag{7}$$

2.c Successive Substitution

3 Diode Circuit

The source code for the Question 3 program can be seen in the `q3.py` file shown in Listing 4.

3.a Voltage Equations

The current-voltage relationship for a diode is given by Equation (8).

$$I = I_s\left(\exp\left[\frac{qv}{kT}\right] - 1\right) \tag{8}$$

Let the nodal voltage at the anode of the A diode be denoted by v_A and that of the B diode by v_B. Let the current through the circuit be denoted by I. The diode equations for A and B can be seen in Equations (9) and (10).

$$I = I_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right] - 1\right) \tag{9}$$

$$I = I_{sB}\left(\exp\left[\frac{qv_B}{kT}\right] - 1\right) \tag{10}$$

By KVL, we also have Equation (11), relating V_A and I.

$$I = \frac{E - v_A}{R} \tag{11}$$

Equating Equations (9) and (11), we obtain the nonlinear equation for v_A, shown in Equation (12).

$$\begin{aligned} &f_A(v_A, v_B) \\ &= v_A + RI_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right] - 1\right) - E \\ &= 0 \end{aligned} \tag{12}$$

Equating Equations (9) and (10), we obtain the nonlinear equation for v_B, shown in Equation (13).

$$\begin{aligned} f_B(v_A, v_B) = I_{sA}&\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right] - 1\right) \\ - I_{sB}&\left(\exp\left[\frac{qv_B}{kT}\right] - 1\right) = 0 \end{aligned} \tag{13}$$

The total system of equations can then be expressed by Equation (14).

$$\mathbf{f}(\mathbf{v}_n) = \begin{bmatrix} f_A(v_A, v_B) \\ f_B(v_A, v_B) \end{bmatrix} = \mathbf{0} \tag{14}$$

3.b Newton-Raphson

To find an expression for the Jacobian matrix $\mathbf{F}$, we must first find expressions for all the partials of f_A and f_B. These are shown in Equations (15) to (18).

$$\frac{\partial f_A}{\partial v_A} = 1 + RI_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right]\frac{q}{kT}\right) \tag{15}$$

$$\frac{\partial f_A}{\partial v_B} = -RI_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right]\frac{q}{kT}\right) \tag{16}$$

$$\frac{\partial f_B}{\partial v_A} = I_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right]\frac{q}{kT}\right) \tag{17}$$

$$\begin{aligned} \frac{\partial f_B}{\partial v_B} = &- I_{sA}\left(\exp\left[\frac{q(v_A - v_B)}{kT}\right]\frac{q}{kT}\right) \\ &- I_{sB}\left(\exp\left[\frac{qv_B}{kT}\right]\frac{q}{kT}\right) \end{aligned} \tag{18}$$

With these equations, the Jacobian matrix $\mathbf{F}$ is given by Equation (19).

$$\mathbf{F} = \begin{bmatrix} \frac{\partial f_A}{\partial v_A} & \frac{\partial f_A}{\partial v_B} \\ \frac{\partial f_B}{\partial v_A} & \frac{\partial f_B}{\partial v_B} \end{bmatrix} \tag{19}$$

4 Function Integration

The source code for the Question 4 program can be seen in the `q4.py` file shown in Listing 5.

4.a Cosine Integration

Plot log10(E) versus log10(N) for N=1, 2,. . . , 20, where E is the absolute error in the computed integral.

4.b Log Integration

Repeat part (a) for the function loge(x), only this time plot for N=10, 20,. . . 200.

4.c Log Integration Improvement

A Code Listings

Listing 1: Custom matrix package (`matrices.py`).

```
from __future__ import division

import copy
import csv
from ast import literal_eval

import math


class Matrix:
    def __init__(self, data):
        self.data = data
        self.num_rows = len(data)
        self.num_cols = len(data[0])

    def __str__(self):
        string = ''
        for row in self.data:
            string += '\n'
            for val in row:
                string += '{:6.2f} '.format(val)
        return string

    def integer_string(self):
        string = ''
        for row in self.data:
            string += '\n'
            for val in row:
                string += '{:3.0f} '.format(val)
        return string

    def __add__(self, other):
        if len(self) != len(other) or len(self[0]) != len(other[0]):
            raise ValueError('Incompatible matrix sizes for addition. Matrix A is {}x{}, but matrix B is
             ↪  {}x{}.'
                             .format(len(self), len(self[0]), len(other), len(other[0])))

        return Matrix([[self[row][col] + other[row][col] for col in range(self.num_cols)]
                       for row in range(self.num_rows)])

    def __sub__(self, other):
        if len(self) != len(other) or len(self[0]) != len(other[0]):
            raise ValueError('Incompatible matrix sizes for subtraction. Matrix A is {}x{}, but matrix B
             ↪  is {}x{}.'
                             .format(len(self), len(self[0]), len(other), len(other[0])))

        return Matrix([[self[row][col] - other[row][col] for col in range(self.num_cols)]
                       for row in range(self.num_rows)])

    def __mul__(self, other):
        if type(other) == float or type(other) == int:
            return self.scalar_multiply(other)

        if self.num_cols != other.num_rows:
            raise ValueError('Incompatible matrix sizes for multiplication. Matrix A is {}x{}, but matrix
             ↪  B is {}x{}.'
                             .format(self.num_rows, self.num_cols, other.num_rows, other.num_cols))

        # Inspired from https://en.wikipedia.org/wiki/Matrix_multiplication
        product = Matrix.empty(self.num_rows, other.num_cols)
        for i in range(self.num_rows):
            for j in range(other.num_cols):
                row_sum = 0
                for k in range(self.num_cols):
                    row_sum += self[i][k] * other[k][j]
```

```
                product[i][j] = row_sum
        return product

    def scalar_multiply(self, scalar):
        return Matrix([[self[row][col] * scalar for col in range(self.num_cols)] for row in
        ↪  range(self.num_rows)])

    def __div__(self, other):
        """
        Element-wise division.
        """
        if self.num_rows != other.num_rows or self.num_cols != other.num_cols:
            raise ValueError('Incompatible matrix sizes.')
        return Matrix([[self[row][col] / other[row][col] for col in range(self.num_cols)]
                       for row in range(self.num_rows)])

    def __neg__(self):
        return Matrix([[-self[row][col] for col in range(self.num_cols)] for row in range(self.num_rows)])

    def __deepcopy__(self, memo):
        return Matrix(copy.deepcopy(self.data))

    def __getitem__(self, item):
        return self.data[item]

    def __len__(self):
        return len(self.data)

    def item(self):
        """
        :return: the single element contained by this matrix, if it is 1x1.
        """
        if not (self.num_rows == 1 and self.num_cols == 1):
            raise ValueError('Matrix is not 1x1')
        return self.data[0][0]

    def is_positive_definite(self):
        """
        :return: True if the matrix if positive-definite, False otherwise.
        """
        A = copy.deepcopy(self.data)
        for j in range(self.num_rows):
            if A[j][j] <= 0:
                return False
            A[j][j] = math.sqrt(A[j][j])
            for i in range(j + 1, self.num_rows):
                A[i][j] = A[i][j] / A[j][j]
                for k in range(j + 1, i + 1):
                    A[i][k] = A[i][k] - A[i][j] * A[k][j]
        return True

    def transpose(self):
        """
        :return: the transpose of the current matrix
        """
        return Matrix([[self.data[row][col] for row in range(self.num_rows)] for col in
        ↪  range(self.num_cols)])

    def mirror_horizontal(self):
        """
        :return: the horizontal mirror of the current matrix
        """
        return Matrix([[self.data[self.num_rows - row - 1][col] for col in range(self.num_cols)]
                       for row in range(self.num_rows)])

    def empty_copy(self):
        """
        :return: an empty matrix of the same size as the current matrix.
        """
        return Matrix.empty(self.num_rows, self.num_cols)
```

```
131
132     def infinity_norm(self):
133         if self.num_cols > 1:
134             raise ValueError('Not a column vector.')
135         return max([abs(x) for x in self.transpose()[0]])
136
137     def two_norm(self):
138         if self.num_cols > 1:
139             raise ValueError('Not a column vector.')
140         return math.sqrt(sum([x ** 2 for x in self.transpose()[0]]))
141
142     def save_to_csv(self, filename):
143         """
144         Saves the current matrix to a CSV file.
145
146         :param filename: the name of the CSV file
147         """
148         with open(filename, "wb") as f:
149             writer = csv.writer(f)
150             for row in self.data:
151                 writer.writerow(row)
152
153     def save_to_latex(self, filename):
154         """
155         Saves the current matrix to a latex-readable matrix.
156
157         :param filename: the name of the CSV file
158         """
159         with open(filename, "wb") as f:
160             for row in range(self.num_rows):
161                 for col in range(self.num_cols):
162                     f.write('{}'.format(self.data[row][col]))
163                     if col < self.num_cols - 1:
164                         f.write('& ')
165                 if row < self.num_rows - 1:
166                     f.write('\\\\\n')
167
168     @staticmethod
169     def multiply(*matrices):
170         """
171         Computes the product of the given matrices.
172
173         :param matrices: the matrix objects
174         :return: the product of the given matrices
175         """
176         n = matrices[0].rows
177         product = Matrix.identity(n)
178         for matrix in matrices:
179             product = product * matrix
180         return product
181
182     @staticmethod
183     def empty(num_rows, num_cols):
184         """
185         Returns an empty matrix (filled with zeroes) with the specified number of columns and rows.
186
187         :param num_rows: number of rows
188         :param num_cols: number of columns
189         :return: the empty matrix
190         """
191         return Matrix([[0 for _ in range(num_cols)] for _ in range(num_rows)])
192
193     @staticmethod
194     def identity(n):
195         """
196         Returns the identity matrix of the given size.
197
198         :param n: the size of the identity matrix (number of rows or columns)
199         :return: the identity matrix of size n
200         """
```

```
201             return Matrix.diagonal_single_value(1, n)
202
203         @staticmethod
204         def diagonal(values):
205             """
206             Returns a diagonal matrix with the given values along the main diagonal.
207
208             :param values: the values along the main diagonal
209             :return: a diagonal matrix with the given values along the main diagonal
210             """
211             n = len(values)
212             return Matrix([[values[row] if row == col else 0 for col in range(n)] for row in range(n)])
213
214         @staticmethod
215         def diagonal_single_value(value, n):
216             """
217             Returns a diagonal matrix of the given size with the given value along the diagonal.
218
219             :param value: the value of each element on the main diagonal
220             :param n: the size of the matrix
221             :return: a diagonal matrix of the given size with the given value along the diagonal.
222             """
223             return Matrix([[value if row == col else 0 for col in range(n)] for row in range(n)])
224
225         @staticmethod
226         def column_vector(values):
227             """
228             Transforms a row vector into a column vector.
229
230             :param values: the values, one for each row of the column vector
231             :return: the column vector
232             """
233             return Matrix([[value] for value in values])
234
235         @staticmethod
236         def csv_to_matrix(filename):
237             """
238             Reads a CSV file to a matrix.
239
240             :param filename: the name of the CSV file
241             :return: a matrix containing the values in the CSV file
242             """
243             with open(filename, 'r') as csv_file:
244                 reader = csv.reader(csv_file)
245                 data = []
246                 for row_number, row in enumerate(reader):
247                     data.append([literal_eval(val) for val in row])
248                 return Matrix(data)
```

Listing 2: Question 1 (`q1.py`).

```
1   def q1():
2       print('\n=== Question 1 ===')
3       q1a()
4
5
6   def q1a():
7       pass
8
9
10  if __name__ == '__main__':
11      q1()
```

Listing 3: Question 2 (`q2.py`).

```
1   import math
2
3   L_a = 5e-3
```

```
L_c = 0.3
A = 1e-4
N = 1000
I = 8
mu_0 = 4e-7 * math.pi


def q2():
    print('\n=== Question 2 ===')
    q2b()


def q2b():
    print('Flux equation: ')
    coeff_1 = L_a / (A * mu_0)
    coeff_2 = L_c
    coeff_3 = N * I
    eq = 'f(\psi) = \SI{{{:1.3e}}}{{}} \psi + {}H - {} = 0'.format(coeff_1, coeff_2, coeff_3)
    print(eq)
    with open('report/latex/flux_equation.txt', 'w') as f:
        f.write(eq)


if __name__ == '__main__':
    q2()
```

Listing 4: Question 3 (`q3.py`).

```
def q3():
    print('\n=== Question 3 ===')


if __name__ == '__main__':
    q3()
```

Listing 5: Question 4 (`q4.py`).

```
def q4():
    print('\n=== Question 4 ===')


if __name__ == '__main__':
    q4()
```

B Output Logs

Listing 6: Output of Question 1 program (`q1.txt`).

```

```

Listing 7: Output of Question 2 program (`q2.txt`).

```

```

Listing 8: Output of Question 3 program (`q3.txt`).

```

```

Listing 9: Output of Question 4 program (`q4.txt`).

```

```