ICSI 210 - Discrete Structures

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Homework: 01.

1.1. Truth tables.

- Use a truth table to show that the following proposition is a contradiction:

$$(p \wedge q) \wedge (\neg p \vee \neg q). \qquad (1.1)$$

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(p \wedge q) \wedge (\neg p \vee \neg q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F.	F	T	T	T	F.

$\square$

1.2. Tautologies

- Is the following proposition a tautology? If yes, prove it without using a whole truth table. Otherwise, prove it by exhibiting truth values for p, q, and r such that the proposition evaluates to F.

$$(p \rightarrow (q \vee r)) \rightarrow (p \rightarrow q) \vee (p \rightarrow r). \quad \ldots \quad (1).$$

$$\rightarrow (\neg p \vee q) \vee (\neg p \vee r)$$

$$\rightarrow \neg p \vee q \vee \neg p \vee r.$$

$$\neg p \vee \neg p \vee q \vee r.$$

$$\neg p \vee (q \vee r)$$

$$p \rightarrow (q \vee r) \qquad \square$$

- Is the following English sentence a tautology? "I either attend class or I do not."

  Explain your answer by assigning letters to the elementary propositions in the sentence and writing the sentence as a logical proposition, then showing whether or not that proposition is a tautology.

Let p = I attend class.

Let q = I do not attend class.

Demonstrate the truth table:

p	q	$p \vee q$
T	F	T
F	T	T

By demonstrate the truth table, The sentence "I either attend class or I do not." is a tautology. $\square$

1.3 Quantifiers

Consider the following predicate $P(x,y)$:

$$P(x,y): x+y \geq 6, \text{ where } x, y \text{ are positive integers} \quad (1.3)$$

State whether each of the following propositions is true or false, and provide a proof of each:

- $\forall x \forall y \; P(x,y)$.

  Given that $x \in \mathbb{N}$ & $y \in \mathbb{N}$.

  The predicate given as $P(x,y): x+y \geq 6$, $x, y \in \mathbb{N}$.

  The proposition $\forall x \forall y \; P(x,y)$ is equivalent to "For all x, for all y, $x+y \geq 6$."

  Proof by Contradiction. For x and y, if $x+y < 6$, then $\forall x \forall y \; P(x,y)$ is false.

  Therefore, $1+2=3 < 6$, $2+3 < 6$ (counterexample). $\square$

- $\forall x \exists y \; P(x,y)$.

  $\forall x \exists y \; P(x,y)$. For all x and for some y, $x+y \geq 6$.

  $S = \{x \mid x \in \mathbb{N}\}$. since the universal of discouse of x is the set of natural number.

  Therefore, x could be any natural number. $x+y \geq 6$ is true for some y value.

1.4. Rules of inference.

Consider the following argument:

Hypotheses:

- If a person is a mathematician, then she is a philosopher.
- If a person is a philosopher, then she is prone to stomachaches.

Conslusion:

- Therefore, if a person is a mathematician, then she is prone to stomachaches.

Assign letters to the basic propositions, and write each hypothesis and the conclusion as an implication. Then explain which rule of inference (covered in Lecture 3) was used to derive the conclusion.

p: "A person is a mathematician".

q: "A person is a philosopher."

r: "she is prone to stomachaches".

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r. \end{array}$$

By Applying Hypothetical syllogism, we deduced that $((p \to q) \land (q \to r)) \Rightarrow (p \to r)$.

1.5 Implications and converse.

If $p \rightarrow q$, is it necessarily the case that $q \rightarrow p$? If so, provide a proof.

If not, exhibit truth value for p and q such that $p \rightarrow q$ but $\neg(q \rightarrow p)$.

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

By demonstration of truth table, $p \rightarrow q \not\equiv q \Rightarrow p$ necessarily.

1.6. Proof techniques

Provide a proof of the following statement.

Theorem 1.1. If a number $x \in \mathbb{Z}$ is even, then it is not odd. Conversely, if x is odd, then it is not even.

Let $x \in \mathbb{Z}$. and $\forall x$ s.t. $x = 2k$.

Let $y \in \mathbb{Z}$ and $\forall y$ s.t. $y = 2k+1$.

$2k = 2k+1$.

$0 \neq 1$.

Since "If a number $x \in \mathbb{Z}$ is even, then is is not odd." holds.

therefore, $p \longrightarrow q$. Conversely, the statement if x is odd, then is is not even is also true

$\forall x$, s.t. $x = 2k+1$

$\forall y$. s.t. $y = 2k$.

$2k+1 = 2k$.

$1 \neq 0$.

$\therefore$ Conversely, the statement $q \Rightarrow p$ is true.