> This PRACTICE exam is open book, open note. No electronic devices are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work. (But, actually, you don't need to hand in this exam.)

1. (10 points) Construct a truth table for the following proposition: $p \wedge \neg q \implies r$.

2. (10 points) Let A, B, C be sets. Show the following set identity. **A Venn diagram or examples are not valid proof techniques.**

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}. \tag{1}$$

3. (10 points) Write all elements of the power set of $\{1, 2, 3\}$.

4. (10 points) Prove the following claim by contradiction:

Claim 1. *Let $n \in \mathbb{Z}$. If $5n + 2$ is odd, then n is odd.*

5. (10 points) Let $A_j = \{i \mid i \in \mathbb{Z}, i \geq j\}$ Describe the elements of the following set without reference to set operations:

$$\bigcup_{j=1}^{\infty} A_j. \tag{2}$$

6. (10 points) Let r, ℓ be even numbers. Prove that $r + \ell$ is even.

7. (10 points) Consider the following relation R on $\mathbb{R}^2$: $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 \leq x_2$.

   Is this a partial order? Prove your answer.

8. (10 points) Prove that

$$(p \wedge q) \implies (p \vee q) \tag{3}$$

   is a tautology. That is, prove that the following logical equivalence holds:

$$(p \wedge q) \implies (p \vee q) \equiv T. \tag{4}$$

   Do not use a truth table.

9. (10 points) Consider a bijection $f : A \to B$. Suppose that $C \subseteq A$. Is it necessarily true that $g : C \to B$ defined by $g(x) = f(x)$ is injective? Is it necessarily true that g is bijective? Justify your answers.

10. (10 points) Consider the following recurrence. Determine a solution for it and prove it by induction.

$$T(n) = 2T(n-1) + 1 \tag{5}$$
$$T(0) = 0. \tag{6}$$

11. (10 points) Prove that the following relation on the integers other than 0 is an equivalence relation:

$$a \equiv b \iff ab > 0. \tag{7}$$

Describe all of its equivalence classes.