

Neural Networks

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1 Introduction

The following will outline the mathematics behind neural networks

2 Forward Propagation

1. Training Matrix

$$\{x \in R^{m \times n} : m = \text{samples}, n = \text{features}\}$$

2. Weights Matrix

$$\{w \in R^{n \times p} : n = \text{features}, p = \text{nodes}\}$$

3. Bias Vector

$$\{b \in R^p : n = \text{features}\}$$

4. Forward Propagation

$$f := R^{n \times p} \mapsto R^{n \times p}$$

$$z = ((x @ w) + b) \tag{1}$$

$$a = f(z) \tag{2}$$

3 Back Propagation

Back propagation is done via gradient descent

$$\frac{dC}{dw} = (x^T @ (a - y) * g'(z)) / m \tag{3}$$

$$\frac{dC}{db} = (\sum (a - y)) / m \tag{4}$$

4 Optimization

$$w_i = w_i - (\alpha x \frac{dC}{dw_i}) \tag{5}$$

$$b = b - (\alpha x \frac{dC}{db}) \tag{6}$$