

Consider the function $f(x) = e^x$ at a point $x_0 = 1$ and approximations to its derivative $f'(x_0) \approx 2.71281$. Approximations can be made by to somewhat straightforward approximations: the forward difference and the centered difference. The error in these approximations is illustrated below, with rounding to simulate calculations by single-precision and double-precision numbers.

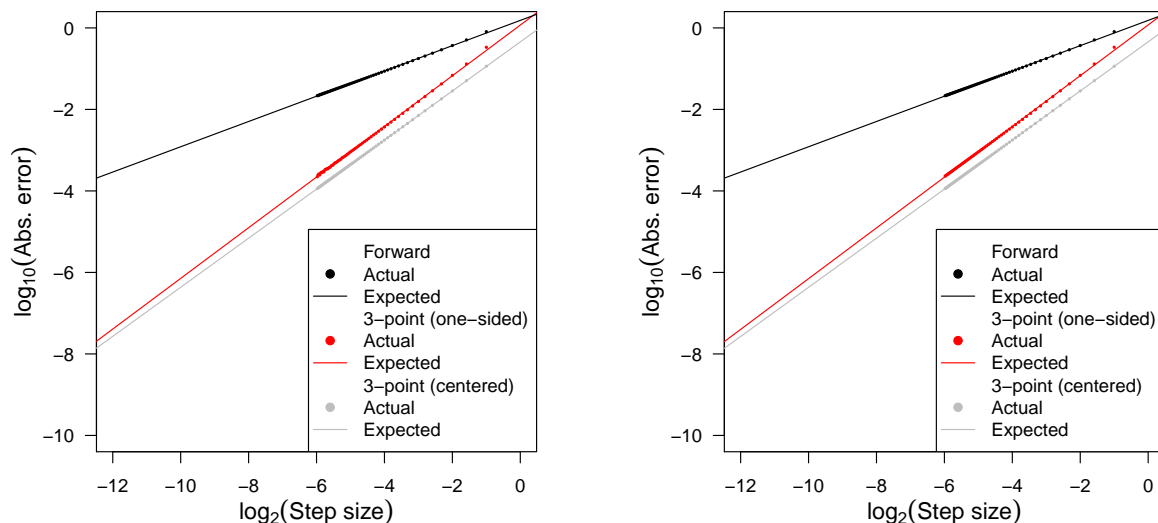


Figure 1: Points indicate calculated $\log_{10}(\text{abs. error})$ for forward (black) and centered (gray) difference approximations. Line illustrates predicted reduction in error as step size shrinks. **Left:** 8 digit calculation (simulated single-precision), **Right:** 16 digit calculation (simulated double-precision)

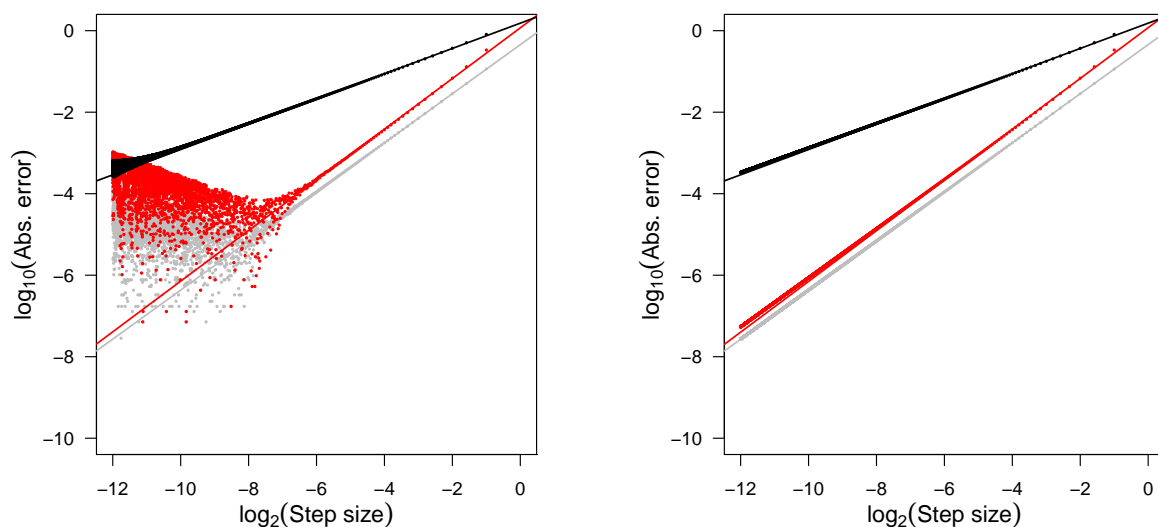


Figure 2: As in Figure 1, but with smaller step sizes included. Note that error *increases* due to roundoff for both methods, but especially for the centered method as the step size is reduced below $h = 0.01$.