Now, for a given degree of accuracy, how many iterations do we actually need? Consider

$$2^{-x} = x \text{ on } \left[\frac{1}{3}, 1\right]$$

The bounds are given by

$$|p_n - p| \le k^n \max(p_0 - a, b - p_0)$$

using the initial guess, and by

$$|p_n - p| \le \frac{k^n}{1-k}|p_1 - p_0|$$

using the initial guess and first iteration. We will look at a few applications of the bounds to the problem above. First notice that $g'(x) = -\ln(2)2^{-x}$ and $|g'(x)| \le \ln(2)2^{-1/3} < k = 0.551$, where $k = 0.551$ is a bound on the magnitude of $g'(x)$.

The worst possible initial guess would be at one of the endpoints, so we will start there (this maximizes the term $\max(p_0 - a, b - p_0)$, which in this case we actually want to do in order to generate a conservative bound). Taking D as the desired accuracy (i.e., an accuracy within 10^{-D}), this gives,

$$\begin{aligned}
k^n \max(p_0 - a, b - p_0) &< 10^{-D} \\
(0.551)^n \left(\frac{2}{3}\right) &< 10^{-D} \\
(0.551)^n &< \left(\frac{3}{2}\right) 10^{-D} \\
n \log(0.551) &< \log\left(\frac{3}{2}\right) - D \\
n &> \frac{\log\left(\frac{3}{2}\right) - D}{\log(0.551)}
\end{aligned}$$

In the last line, the inequality has been reversed since we are dividing by a negative. With $D = 4$ this gives $n > 14.77277$ which requires $N = 15$ steps.

For the second bound, we actually need p_1 in addition to p_0. From $p_0 = \frac{2}{3}$, we have $p_1 = 2^{-1/3}$ (so $|p_1 - p_0| = |2^{-1/3} - \frac{1}{3}| \approx 0.4604$). Similarly, from $p_0 = 1$, we have $p_1 = \frac{1}{2}$ (so $|p_1 - p_0| = |\frac{1}{2} - 1| = 0.5$). We will use the second of these which is larger in value.

$$\begin{aligned}
\frac{k^n}{1-k}|p_1 - p_0| &< 10^{-D} \\
\frac{(0.551)^n}{1-0.551}(0.5) &< 10^{-D} \\
(0.551)^n &< \left(\frac{1-0.551}{0.5}\right) 10^{-D} \\
n \log(0.551) &< \log\left(\frac{1-0.551}{0.5}\right) - D \\
n &> \frac{\log\left(\frac{1-0.551}{0.5}\right) - D}{\log(0.551)}
\end{aligned}$$

For consistency, with $D = 4$ this gives $n > 15.63357$ which requires $N = 16$ steps. We have to do at least 16 steps to ensure we are within the bound, though we may satisfy this much more quickly. Notice that this is quite a bit more work than our bound for the Bisection method required.