Sean Laverty **2023-Fa-Laverty-MATH2153**
Assignment 0_review_packet due 08/31/2023 at 11:59pm CDT

---

1. **(1 point)** `set0_review_packet/function_notation.pg`
Record decimal answers with three digits after a decimal.

Enter the formula for the function $f(x) = x^2$.

$f(x) =$_____

Find the following function values, some may be numbers, some may be expressions containing variables.

$f(-1) =$_____

$f(\frac{5}{2}) =$_____

$f(x+1) =$_____

Solution: *(Instructor solution preview: show the student solution after due date.)*
Find the following function values, some may be numbers, some may be expressions containing variables.

$f(-1) = 1$

$f(\frac{5}{2}) = 6.25$

$f(x+1) = (x+1)^2$

Correct Answers:

- `x^2`
- `1`
- `6.25`
- `(x+1)^2`

---

2. **(1 point)** `set0_review_packet/composition.pg`
Find the following compositions using the functions $f(x) = x^2 + 3x$ and $g(x) = -x + 2$.

$(f \circ g)(x) =$__________

$(g \circ f)(x) =$__________

Solution: *(Instructor solution preview: show the student solution after due date.)*
Find the following compositions.

$(f \circ g)(x) = (-x+2)^2 + 3(-x+2)$

$(g \circ f)(x) = -\left(x^2 + 3x\right) + 2$

Correct Answers:

- `(-x+2)^2+3*(-x+2)`
- `-(x^2+3*x)+2`

---

3. **(1 point)** `set0_review_packet/solve.pg`
Solve $3x-4y=5$ for y.

$y =$ _____

Solution: *(Instructor solution preview: show the student solution after due date.)*
Solve $3x-4y=5$ for y.

$y = \frac{5-3x}{-4}$

Correct Answers:

- `(5-3*x)/(-4)`

---

4. **(1 point)** `set0_review_packet/linear.pg`
Record decimal answers with three digits after a decimal.

On average, the number of flowers on a particular species of plant is given as a function of time in days (starting now) by $f(t) = 5.2 + 2.6t$. Find each of the following.

$f(0) =$ _____

$f(10) =$ _____

The time at which the average number of flowers reaches 21.
$t =$ _____

Solution: *(Instructor solution preview: show the student solution after due date.)*

On average, the number of flowers on a particular species of plant is given as a function of time in days (starting now) by $f(t) = 5.2 + 2.6t$. Find each of the following.

$f(0) = 5.2$

$f(10) = 31.2$

The time at which the average number of flowers reaches 21.
$t = 6.07692$
$y =$

Correct Answers:

- `5.2`
- `31.2`
- `(21-5.2)/2.6`

Sean Laverty **2023-Fa-Laverty-MATH2153**
Assignment topic_1_lines due 09/10/2023 at 11:59pm CDT

---

1. **(2 points)** `problems/intro_lines/lines.pg`
Please round your answers to three decimal places. Your answer will be checked to two decimal places.

Determine the following linear functions.
a. Give the line whose slope is $m = 3$ and intercept is 5.

The appropriate linear function is $y =$____

b. Give the line whose slope is $m = 1$ and passes through the point $(8,9)$.

The appropriate linear function is $y =$____
Solution: *(Instructor solution preview: show the student solution after due date.)*

a. The line whose slope is $m = 3$ and intercept is 5 is $y = 3x+5$.

b. The line whose slope is $m = 1$ and passes through the point $(8,9)$ is $y = 1(x-8)+9$.

Correct Answers:

- `3*x+5`
- `1*(x-8)+9`

---

2. **(2 points)** `problems/intro_lines/intersection.pg`
Please round your answers to three decimal places. Your answer will be checked to two decimal places.

Consider the functions $f(x) = 7x+8$ and $g(x) = 4x+3$.
a. Solve the equation $7x+8 = 3$ for x.

Enter your solution $x =$____

b. Solve the equation $7x+8 = 4x+3$ for x.

Enter your solution $x =$____
Solution: *(Instructor solution preview: show the student solution after due date.)*

a. $f(x) =$ has a solution $x = -0.714$.

b. $f(x) = g(x)$ has a solution $x = -1.667$.
Correct Answers:

- `-0.714`
- `-1.667`

---

3. **(3 points)** `problems/intro_lines/glucose.pg`
Please round your answers to three decimal places. Your answer will be checked to two decimal places.
Consider the production of glucose as a function of mass in a growing plant. Mass is measured in grams and

glucose production is measured in milligrams (per day).

M mass, (grams)	G (glucose, milligrams)
4	7
4.5	8.2
9	19
11	23.8

a). Find the slope of the line that connects the point $(4,7)$ to one other point in the table.
The slope is $m =$ ____.

b). Give the formula of the line describing the relationship between the mass of the plant in grams and the corresponding glucose production. Be sure to use M as your variable in the formula.
The formula is $G(M) =$ ____.

c). What is the predicted glucose production for a 17 gram plant?
The predicted production is, ____ milligrams.

Solution: *(Instructor solution preview: show the student solution after due date.)*
a). The slope is $m = 2.4$.

b). The formula is $G(M) = (2.4 * M - 2.6)$.

c). The glucose production is predicted to be 38.2 milligrams for a 17 gram plant.

Correct Answers:

- `2.4`
- `2.4*M-2.6`
- `38.2`

---

4. **(3 points)** `problems/intro_lines/cricket.pg`
Please round your answers to three decimal places. Your answer will be checked to two decimal places.
The following relationship, presented by A. E. Dolbear in the entertaining scientific paper 'The Cricket as a Thermometer' allows one to estimate the temperature by counting the chirps made by a cricket each minute. All temperatures are recorded in Fahrenheit.

N (Number of chirps)	T (Temperature)
45	51.25
70	57.5
85	61.25
110	67.5

a). Find the slope of the line that connects the point $(45, 51.25)$ to one other point in the table.
The slope is $m =$ ____.

b). Give the formula of the line describing the relationship between the number of chirps and the predicted temperature. Be sure to use N as your variable in the formula.
The formula is $T(N) =$ ____.

c). If the crickets were totally silent, in other words $N = 0$, what is the predicted temperature?
The predicted temperature is, $T(0) =$____.

Solution: *(Instructor solution preview: show the student solution after due date.)*
a). The slope is $m = 0.25$.

b). The formula is $T(N) = (N/4+40)$.

c). The temperature is predicted to be 40 when the crickets are silent.

Correct Answers:

- `0.25`
- `N/4+40`
- `40`

---

1. **(1 point)** `set_topic2_power_poly/physics.pg`
Record decimal answers with three digits after a decimal.

As a function of velocity, v in meters per second, the function $K(v) = \frac{1}{2}mv^2$, describes the kinetic energy, K, for an object with a given mass, m in kilograms. Find each of the following.

Find the kinetic energy of an object with a mass of 5 kilograms and a velocity of 5.5 meter per second?
_____

Find the kinetic energy of an object with a mass of 5 kilograms and a velocity of 15.5 meter per second?
_____

Solution: *(Instructor solution preview: show the student solution after due date.)*
As a function of velocity, v in meters per second, the function $K(v) = \frac{1}{2}mv^2$, describes the kinetic energy, K, for an object with a given mass, m in kilograms. Find each of the following.

The kinetic energy of an object with a mass of 5 kilograms and a velocity of 5.5 meter per second is $K(5.5) = 75.625$.

The kinetic energy of an object with a mass of 5 kilograms and a velocity of 15.5 meter per second is $K(15.5) = 600.625$.

Correct Answers:
- `0.5*5*5.5^2`
- `0.5*5*(5.5+10)^2`

---

2. **(1 point)** `set_topic_1_poly_power/composition.pg`
Record decimal answers with three digits after a decimal.

The surface area of a roughly circular seasonal pond can be described by $A(r) = \pi r^2$, but the radius (in meters) changes according to $r(t) = 30 - 0.16t$.

a. Given $A(r) = \pi r^2$, what is the radius if the current area is 700 square meters?

$r =$__________

b. Given $r(t) = 30 - 0.16t$, how long is it until the pond has completely dried up (or until the radius reaches zero)?

$t =$__________

c. Write the composition $(A \circ r)(t) = A(r(t))$ that gives the area as a function of time.

$(A \circ r)(t) = A(r(t)) =$__________

d. Using the composition you just found, give the initial area of the pond.

(initial area) =__________

Solution: *(Instructor solution preview: show the student solution after due date.)*
The surface area of a roughly circular seasonal pond can be described by $A(r) = \pi r^2$, but the radius (in meters) changes according to $r(t) = 30 - 0.16t$.

a. Given $A(r) = \pi r^2$, if the current area is 700 square meters, the radius is $r = 14.9271$.

b. Given $r(t) = 30 - 0.16t$, the pond has completely dried up (or the radius reaches zero) at $t = 187.5$.

c. The composition $(A \circ r)(t) = A(r(t))$ that gives the area as a function of time is $(A \circ r)(t) = A(r(t)) = \pi(30 - 0.16t)^2$.

d. Using the composition we just found, the initial area of the pond is 2827.43.

Correct Answers:

- `sqrt(700/pi)`
- `30/0.16`
- `pi*(30-0.16*t)^2`
- `2827.43`

---

3. **(1 point)** `set_topic_2_power_poly/physics.pg`
The function $s(t) = -16t^2 + 9t + 18$ describes the height (in feet) of an object in free fall as a function of time (in seconds).

What is the initial height of the object?
height =_____

When does the object land? *Hint: consider when* $s(t) = 0$.
$t =$_____

When is the height when $t = 0.69$?
height =_____

Solution: *(Instructor solution preview: show the student solution after due date.)*
The function $s(t) = -16t^2 + 9t + 18$ describes the height (in feet) of an object in free fall as a function of time (in seconds).

What is the initial height of the object?

height = 18

When does the object land? *Hint: consider when* $s(t) = 0$.

$t = 1.37857$

When is the height when $t = 0.69$?
height = 16.5924

Correct Answers:

- 18
- [-9-sqrt(9^2-4*-16*18)]/(2*-16)
- 16.5924

Sean Laverty **2023-Fa-Laverty-MATH2153**
Assignment topic_3_exp_log due 09/18/2023 at 11:59pm CDT

---

1. **(2 points)** `problems/intro_exp/doubling.pg`
Consider a population of microorganisms that grows according to $s(t) = 18.8e^{1.5t}$.

a. Find the doubling time of the population, τ_D.

$\tau_D =$__________

b. Find the time that the population reaches a size of $s(t) = 34.8$.

$t =$__________

Solution: *(Instructor solution preview: show the student solution after due date.)*
We have:
a. The doubling time of the population $\tau_D = 0.462$.

b. The time that the population reaches a size of $s(t) = 34.8$ is $t = 0.411$.

Correct Answers:

- `[ln(2)]/1.5`
- `[ln(34.8/18.8)]/1.5`

---

2. **(3 points)** `problems/intro_exp/growth.pg`
Consider a population of microorganisms that grows exponentially.

a. The population doubles after 0.856 days, what is its growth rate α?

$\alpha =$_____

b. Given that growth rate, and an initial population size of 15.4, using this information enter the formula for the exponential function, $s(t)$, that describes population growth:

$s(t) =$_______________

Hint: it might help to think of your function as $(a\ number) * e^{(}anumber * t)^{(}anumber * t)^{(}anumber * t)^{(}anumber * t)$*, paying close attention to the par*

c. Given that growth rate, and an initial population size of 15.4*, how long until the population reaches a size of* 28.4*?*

Hint: it might help to use your result from part b.

$t =$_____
Solution: *(Instructor solution preview: show the student solution after due date.)*
Consider a population of microorganisms that grows exponentially.

a. The population doubles after 0.856 days, what is it's growth rate α?

$\alpha = 0.81$

b. Given that growth rate, and an initial population size of 15.4, enter the exponential function, $s(t)$, that describes population growth:

$s(t) = 15.4e^{0.81t}$

c. Given that growth rate, and an initial population size of 15.4, how long until the population reaches a size of 28.4?

$t = 0.756$

Correct Answers:

- `0.81`
- `15.4*e^(0.81*t)`
- `[ln(28.4/15.4)]/0.81`

---

3. **(2 points)** `problems/intro_exp/decay.pg`
Consider a dose of a radioactive medical tracer that vanishes according to $r(t) = 14.9e^{-0.52t}$.

a. Find the half-life of the sample, $\tau_{1/2}$.

$\tau_{1/2} =$_____

b. Find the time that the amount of tracer reaches a value of $r(t) = 3.725$.

$t =$_____

Solution: *(Instructor solution preview: show the student solution after due date.)*
We have:

a. The half-life of the sample, $\tau_{1/2}$, is $\tau_{1/2} = 1.333$.

b. The time that the amount of tracer reaches a value of $r(t) = 3.725$ is $t = 2.666$.

Correct Answers:

- `[ln(2)]/0.52`
- `-[ln(3.725/14.9)]/0.52`

---

1. **(4 points)** `problems/intro_dtds/cell.pg`
Enter all values to three digits *AFTER* the decimal place.

The concentration of a radioactive medical tracer changes according to the discrete-time dynamical system $y_{t+1} = 0.9 y_t$.

a. Starting from $y_0 = 8.5$ compute the next three values of the solution.

$y_1 =$_____

$y_2 =$_____

$y_3 =$_____

b. Find the equilibrium value of the concentration:

$y^* =$__________

Hint: solve the equation $y^* = f(y^*)$ for the equilibrium value y^*.

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

/opt/webwork/webwork2/htdocs/tmp/2023-Fa-Laverty-MATH2153/gif/b179c7f2-ad65-3562-a01d-14db5c23fce

Solution: *(Instructor solution preview: show the student solution after due date.)*
A population of yeast grows according to the discrete-time dynamical system $y_{t+1} = 0.9y_t$.

a. Starting from $y_0 = 8.5$ compute the next three values of the solution.

   $y_1 = 7.65$

   $y_2 = 6.885$

   $y_3 = 6.197$

b. Find the equilibrium value of the yeast population size:

   $y^* = 0$

c. Sketch a cobwebbing diagram to confirm your work.

Correct Answers:

- 7.65
- 6.885

- `6.197`
- `0`

---

2. **(4 points)** `problems/intro_dtds/equilib.pg`
Enter all values to three digits *AFTER* the decimal place.

Consider the discrete-time dynamical system $M_{t+1} = 0.4M_t + 1.5$ describing the concentration of medicine in a patient's bloodstream.

a. Starting from $M_0 = 1$ compute the next three values of the solution.

$M_1 =$_____

$M_2 =$_____

$M_3 =$_____

b. Find the value of the equilibrium drug concentration:

$M^* =$__________

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

/opt/webwork/webwork2/htdocs/tmp/2023-Fa-Laverty-MATH2153/gif/7052dcfe-683e-3b0b-aeb0-5c03555d450

Solution: *(Instructor solution preview: show the student solution after due date.)*
Consider the discrete-time dynamical system $M_{t+1} = 0.4M_t + 1.5$ describing the concentration of medicine in a patient's bloodstream.

a. Starting from $M_0 = 1$ compute the next three values of the solution.

   $M_1 = 1.9$

   $M_2 = 2.26$

   $M_3 = 2.404$

b. Find the value of the equilibrium drug concentration:

   $M^* = 2.5$

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

Correct Answers:

- `1.9`

- `2.26`
- `2.404`
- `1.5/(1-0.4)`

---

3. **(4 points)** `problems/intro_dtds/fake.pg`

Enter all values to three digits *AFTER* the decimal place.

Consider the discrete-time dynamical system $x_{t+1} = 3.5x_t - 8$, where x_t has no particular biological meaning.

a. Starting from $x_0 = 4.2$ compute the next three values of the solution.

$x_1 =$_____

$x_2 =$_____

$x_3 =$_____

b. Find the equilibrium value.

$x^* =$__________

Hint: solve the equation $x^* = f(x^*)$ for the equilibrium value x^*, where f is the updating function.

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

/opt/webwork/webwork2/htdocs/tmp/2023-Fa-Laverty-MATH2153/gif/e7b59d29-4982-3785-b0c8-a8c3b351228

Solution: *(Instructor solution preview: show the student solution after due date.)*
Consider the discrete-time dynamical system $x_{t+1} = 3.5x_t - 8$, where x_t has no particular biological meaning.

a. Starting from $x_0 = 4.2$ compute the next three values of the solution.

$x_1 = 6.7$

$x_2 = 15.45$

$x_3 = 46.075$

b. Find the equilibrium population size:

$x^* = 3.2$

c. Sketch a cobwebbing diagram to confirm your work.

Correct Answers:

- `6.7`
- `15.45`

- `46.075`
- `-8/(1-3.5)`

---

4. **(4 points)** `problems/intro_dtds/immig.pg`

Enter all values to three digits *AFTER* the decimal place.

A population of copepod grows according to the discrete-time dynamical system $c_{t+1} = 0.3c_t + 5$, where t is counted in weeks. In this case a fraction 0.3 of the previous week's population escapes predation and 5 new individuals hatch.

a. Starting from $c_0 = 10$ compute the next three values of the solution.

$c_1 =$_____

$c_2 =$_____

$c_3 =$_____

b. Find the equilibrium population size.

$c^* =$__________

Hint: solve the equation $c^* = f(c^*)$ for the equilibrium value c^*, where f is the updating function.

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

/opt/webwork/webwork2/htdocs/tmp/2023-Fa-Laverty-MATH2153/gif/eafd9332-77ef-3c7a-a7bc-97d07988efa

Solution: *(Instructor solution preview: show the student solution after due date.)*
A population of copepod grows according to the discrete-time dynamical system $c_{t+1} = 0.3c_t + 5$, where t is counted in weeks. In this case a fraction 0.3 of the previous week's population escapes predation and 5 new individuals hatch.

a. Starting from $c_0 = 10$ compute the next three values of the solution.

$c_1 = 8$

$c_2 = 7.4$

$c_3 = 7.22$

b. Find the equilibrium population size:

$c^* = 7.143$

c. Sketch a cobwebbing diagram to confirm your work.

Correct Answers:

- `8`
- `7.4`
- `7.22`
- `5/(1-0.3)`

---

5. **(4 points)** `problems/intro_dtds/fake2.pg`
Enter all values to three digits *AFTER* the decimal place.

Consider the discrete-time dynamical system $x_{t+1} = -x_t + 2$, where x_t has no particular biological meaning.

a. Starting from $x_0 = 0.6$ compute the next three values of the solution.

$x_1 =$_____

$x_2 =$_____

$x_3 =$_____

b. Find the equilibrium value.

$x^* =$__________

Hint: solve the equation $x^* = f(x^*)$ for the equilibrium value x^*, where f is the updating function.

c. Sketch a cobwebbing diagram to confirm your work. *There is nothing to turn in for this part.*

/opt/webwork/webwork2/htdocs/tmp/2023-Fa-Laverty-MATH2153/gif/e8deb140-d615-3aae-be83-98a4da9e1e5

Solution: *(Instructor solution preview: show the student solution after due date.)*
Consider the discrete-time dynamical system $x_{t+1} = -x_t + 2$, where x_t has no particular biological meaning.

a. Starting from $x_0 = 0.6$ compute the next three values of the solution.

$x_1 = 1.4$

$x_2 = 0.6$

$x_3 = 1.4$

b. Find the equilibrium population size:

$x^* = 1$

c. Sketch a cobwebbing diagram to confirm your work.

Correct Answers:

- `1.4`
- `0.6`

- `1.4`
- `2/(1--1)`

---

6. **(4 points)** `problems/intro_dtds/wflaw.pg`
Enter all values to three digits *AFTER* the decimal place.

The Weber-Fechner law describes our ability to detect small differences in various stimuli. For example, it can be applied to the sequence of audio frequencies we are able to distinguish between. Consider a person that can detect an initial frequency of $f_1 = 400$ Hertz. The next frequency that can be detected is $f_2 = 404$, meaning that $f_{n+1} = af_n$ where $a = 1.01$, or $f_{n+1} = 1.01f_n$.

a. What are the next two frequencies the person can detect?

$f_3 =$_____

$f_4 =$_____

b. Suppose a more preceptive person can hear a second frequency of $f_2 = 401$.

What is their value of α? ___

c. What are the next two frequencies that he can detect?

$f_3 =$_____

$f_4 =$_____

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 1*
Hint: if you are stuck, notice above that the ratio $\frac{f_2}{f_1} = a = \frac{404}{400} = 1.01$. *What is the corresponding value of a for the second person? If your answers are marked incorrect, and you believe them, be sure to use as many digits for the ratio as possible in your intermediate calculation.*
Solution: *(Instructor solution preview: show the student solution after due date.)*
The Weber-Fechner law describes our ability to detect small differences in various stimuli. For example, it can be applied to the sequence of audio frequencies we are able to distinguish between. Consider a person that can detect an initial frequency of $f_1 = 400$ Hertz. The next frequency that can be detected is $f_2 = 404$, meaning that $f_{n+1} = 1.01f_n$.

a. What are the next two frequencies the person can detect?

$f_3 = 408.04$

$f_4 = 412.1204$

b. Suppose a more preceptive person can hear a second frequency of $f_2 = 401$. What are the next two frequencies that he can detect?

$f_3 = 402.0025$

$f_4 = 403.0075$

Hint: if you are stuck, notice that our new ratio of successive values is $\frac{f_2}{f_1} = \frac{401}{400} = 1.0025.$

Correct Answers:

- 408.04
- 412.1204
- 1.0025
- 402.0025
- 403.0075