Day 2 notes

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Contents

1 Introduction to Calculus

1.1 Limits

A key concept in calculus is the limit. Below we will evaluate a few limits. For example, we might want to evaluate $\lim_{x\to 5}(1+x-2x^2)$. But, that is ugly to look at, so let's write it better, $\lim_{x\to 5}(1+x+2x^2)$. What happens if we keep writing? What happens if we keep writing? What happens if we keep writing? What happens if we keep writing? What happens if we keep writing? Instead, we might write,

$$\lim_{x\to 5} 1+x+2x^2 = 1+5+2(5)^2 = 56.$$

1.2 Derivatives

Consider the function $f(x) = x^3$. By the power rule for derivatives, $f'(x) = 3x^2$. In general we have for a function $g(x) = x^n$,

$$g'(x) = \frac{d}{dx}(x^n) = nx^{n-1}.$$

Recall that the power rule works as a direct result of the limit definition of the derivative which is given by,

$$f'(x) = \frac{df}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

We sometimes replace h by the symbol Δx.

We can also take higher-order derivatives of functions using the same rules. For $f(x) = x^{17}$ we have $f'(x) = \frac{df}{dx} = 17x^{16}$ and $f''(x) = \frac{d^2f}{dx^2} = 17\cdot 16x^{15}$.

1.3 Integrals

Finally the integral, of which we can discuss two types: indefinite and definite.

1.3.1 Indefinite integrals

As an example of an indefinite integral we have,

$$\int x^2\,dx = \frac{x^3}{3} + c$$

1.3.2 Definite integrals

(add some words)

$$\int_0^\pi \sin(x)\,dx = \ldots$$