

* $C(t)$ is a parabola, $v(t)$ is linear

The change in $n(t)$ is given by its derivative,

$$\begin{aligned}\frac{dn}{dt} &= h'(t) = \frac{d}{dt}(c(t)v(t)) \\ &= c(t) \cdot \frac{dv}{dt} + v(t) \cdot \frac{dc}{dt} \quad (c(t)v'(t) + v(t)c'(t)) \\ &= (4-t^2)(3) + (1+3t)(-2t) \\ &\quad \uparrow \qquad \qquad \qquad \uparrow \\ &\quad \frac{d}{dt}(1+3t)=3 \qquad \frac{d}{dt}(4-t^2)=-2t\end{aligned}$$

$$\begin{aligned}\text{So } n'(t) &= 12 - 3t^2 - 2t - 6t^2 \\ &= -9t^2 - 2t + 12\end{aligned}$$

1) Since initially means $t=0$, $n'(0) = -9(0) - 2(0) + 12 > 0$
so initially, $n(t)$ is increasing.

2) A change in behavior can happen at a critical point,
where $n'(t) = 0$. We have to solve

$$\begin{aligned}-9t^2 - 2t + 12 &= 0 \quad \text{by the quadratic formula} \\ t &= \frac{2 \pm \sqrt{4 - 4(-9)(12)}}{-18} = -\frac{1}{9} \pm \frac{\sqrt{109}}{9} \approx -1.27, \uparrow 1.049\end{aligned}$$

the critical point is $t \approx 1.05$ sec

this one makes
sense since it
is positive