World's weirdest math book

MATH 2753

Fall 2020

1 Introduction

The goal today is to standardize and clean up the work that was contributed by members of the class. Be gentle and constructive in your observations and any criticisms you discuss/comment.

2 Theorem-like environments

Corollary 0.1. *Krausz's Lemma. If G is a finite simple graph, then*

$$|L^2(G)| = \frac{1}{2}\sum_{v\in G}\deg_G(v)\left(\deg_G(v)-1\right) = \sum_{v\in G}\binom{\deg_G(v)}{2}.$$

Theorem 1 (The Litany Against Fear)**.** *I must not fear. Fear is the mind-killer. Fear is the little-death that brings total obliteration. I will face my fear. I will permit it to pass over me and through me. And when it has gone past I will turn the inner eye to see its path. Where the fear has gone there will be nothing. Only I will remain.*

Lemma 1. *Above all else, the mentat must be a generalist, not a specialist. It is wise to have decisions of great moments monitored by generalists. Experts and specialists lead you quickly into chaos. They are a source of useless nit-picking, the ferocious quibble over a comma. The mentat-generalist, on the other hand, should bring to decision-making a healthy common sense. He must not cut himself off from the broad sweep of what is happening in his universe.*

3 Definition-like environments

Definition 1 (Concavity)**.** Concavity *is defined if the graph of $f(x)$ is opening up or down. We can tell where the function $f(x)$ is concave up or down by finding the second derivative and setting it equal to* 0*. The second derivative of $f(x) = x^2$ is $f''(x) = 2$, this tells us the entire range of $f(x) = x^2$ is positive and concave up.*

Definition 2 (Human computers)**.** *Individuals trained as* human computers*, their minds developed to staggering heights of cognitive and analytical ability.*

Definition 3 (Integer Addition)**.** *You can* add *one integer to another integer by increasing the first number by the value of the second integer.*

Definition 4 (Partial Sum)**.** *The sum of the series $a_1 + a_2 + a_3 + \dots.$ is the*

$$\lim_{x\to\infty} S_n.$$

where $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is the n^{th} partial sum of the series.

4 Example-like environments

Example 4.1 (Algebra)**.** *Solve $10 = 2x + 2$ for x.*

$$\begin{aligned} 10 &= 2x+2 \\ 10-2 &= 2x \\ 8 &= 2x \\ 4 &= x \end{aligned}$$

Example 4.2 (Using the power rule). *Find the indefinite integral for the given function $f(x) = 3x^4+4x^2+72$. Using the power rule,*

$$\int 3x^4 + 4x^2 + 72\,dx = \left(\frac{3}{4+1}\right) x^{4+1} + \left(\frac{4}{2+1}\right) x^{2+1} + \left(\frac{72}{0+1}\right) x^{0+1} + c.$$

This simplifies to,

$$\frac{3}{5}x^5 + \frac{4}{3}x^3 + 72x + c.$$

Example 4.3 (Example of a Simple Derivative). *We can begin with a function $f(x) = x^2$. If we use the definition we learned earlier, we would know that by pulling the exponent down and subtracting the exponent by one, we can get the derivative. Therefore, the derivative of this would be $f'(x) = 2x$.*

Example 4.4 (Power rule for integrals). *Consider*

$$\int 3x^2\,dx.$$

Let's use the power rule to evaluate this integral.

$$\begin{aligned}\int 3x^2\,dx &= \left(\frac{1}{2+1}\right) 3x^{2+1} + c\\ &= \frac{1}{3}3x^3 + c\\ &= x^3 + c\end{aligned}$$

So, by the power rule, $\int 3x^2\,dx = x^3 + c$.

Example 4.5 (Power rule for integrals). *Use the power rule to find the integral of $f(x) = x^7$*

$$\int x^7\,dx = \frac{1}{7+1}x^{(7+1)} \tag{1}$$

$$= \frac{1}{8}x^8 + c \tag{2}$$

Example 4.6 (Picturing arithmetic operations). *Let's picture and solve two addition problems.*

a *What is $2+2$?*

b *What is $3+4$?*

Solution. *Below we solve our addition problems.*

a. *What's $2+2$? Well, we must start with drawing a number line. Pretend I drew a number line from 1 to 5. Start at 2. Now move up the number line by two digits. You're at four. Therefore, $2+2=4$*

b. *What's $3+4$? Just like last time, we will start with a number line. Pretend I drew a number line from 1 to 10. Start at 3. Now move up the number line by four digits. You're at 7. Therefore, $3+4=7$.*

Example 4.7. *Consider $f(x) = 3x^5 + 25x^2 + 5$ and find the derivative $f'(x)$. Using the power rule,*

$$f(x) = x^c, \quad f'(x) = cx^{c-1}.$$

Take each term and apply the power rule as follows,

$$\begin{aligned}\frac{d}{dx}\left(3x^5\right) &= (3\cdot 5)x^{(5-1)} = \mathbf{15x^4} && \textit{(First term)}\\ \frac{d}{dx}\left(25x^2\right) &= (25\cdot 2)x^{(2-1)} = \mathbf{50x} && \textit{(Second term)}\\ \frac{d}{dx}(5) &= \mathbf{0} && \textit{(Constant term)}\end{aligned}$$

Putting these together we find that

$$\frac{d}{dx}\left(3x^5 + 25x^2 + 5\right) = \mathbf{15x^4} + \mathbf{50x} + \mathbf{0}.$$