So far:

Sec 1: Numbers

Sec 2: Algebra/Formulas/Functions

Sec 3: Geometry/Shapes and Spatial Relationships

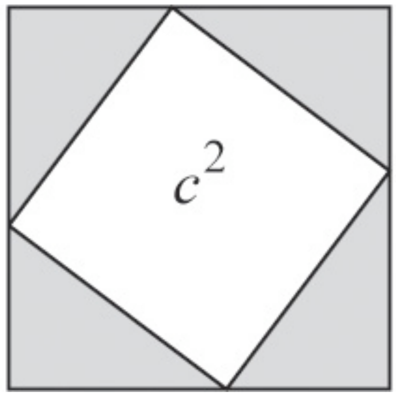
- #12 Square Dancing - Pythagorean Theorem
  (squares + triangles)

#13 Something from nothing - straightedge + compass

#14 Conic Conspiracy - round shapes

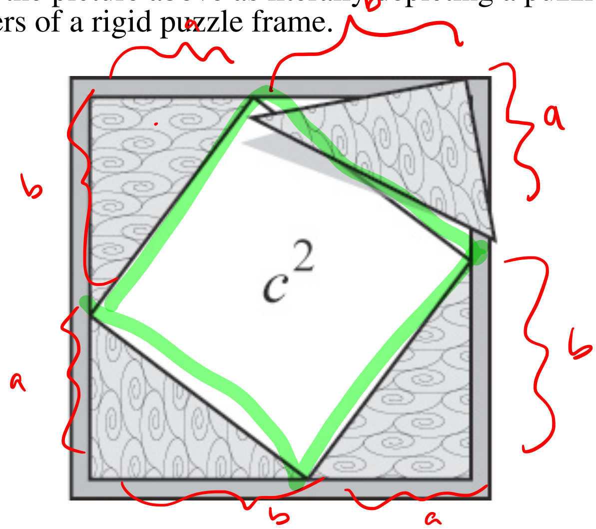
#15 Sine Qua Non - sine waves/curves

#16 Take it to the limit - the number π



Now recall what we're trying to prove: that the tilted white square in the picture above (which is just our earlier "large square"— it's still sitting right there on the hypotenuse) has the same area as the small and medium squares put together. But where are those other squares? Well, we have to shift some triangles around to find them.

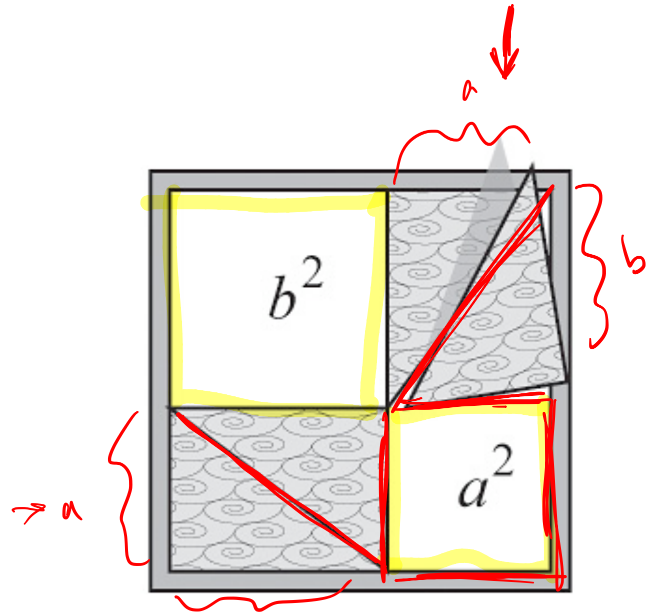
Think of the picture above as literally depicting a puzzle, with four triangular pieces wedged into the corners of a rigid puzzle frame.



In this interpretation, the tilted square is the empty space in the middle of the puzzle. The rest of the area inside the frame is occupied by the puzzle pieces.

Now let's try moving the pieces around in various ways. Of course, nothing we do can ever change the total amount of empty space inside the frame — it's always whatever area lies outside the pieces.

The brainstorm, then, is to rearrange the pieces like this:



All of a sudden the empty space has changed into the two shapes we're looking for — the small square and the medium square. And since the total area of empty space always stays the same, we've just proven the Pythagorean theorem!

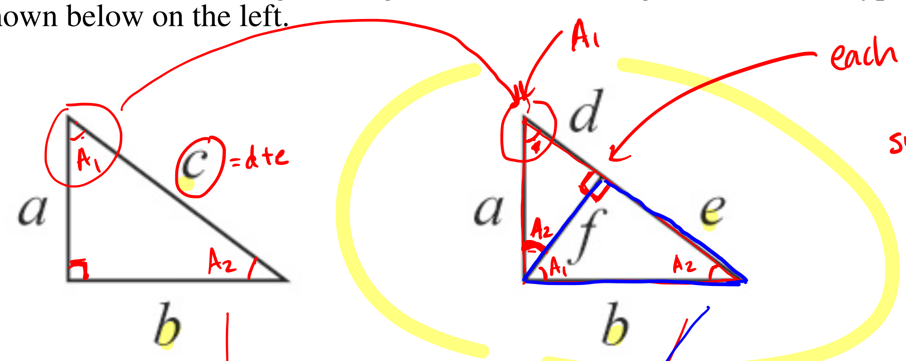
This proof does far more than convince; it *illuminates*. That's what makes it "elegant."

For comparison, here's another proof. It's equally famous, and it's perhaps the simplest proof that avoids using areas.

As before, consider a right triangle with sides of length *a* and *b* and hypotenuse of length *c*, as shown below on the left.

each 90° angles

sum of all angles is = 180



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Now, by divine inspiration or a stroke of genius, something tells us to draw a line segment perpendicular to the hypotenuse and down to the opposite corner, as shown above on the right.

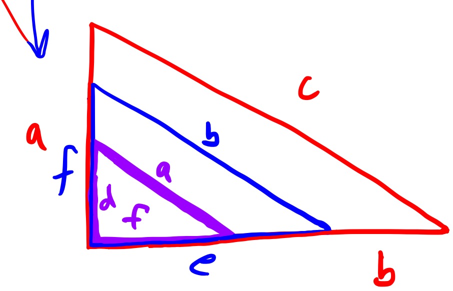
This clever little construction creates two smaller triangles inside the original one. It's easy to prove that all these triangles are "similar" — which means they have identical shapes but different sizes. That in turn implies that the lengths of their corresponding parts have the same proportions, which translates into the following set of equations:

also important!

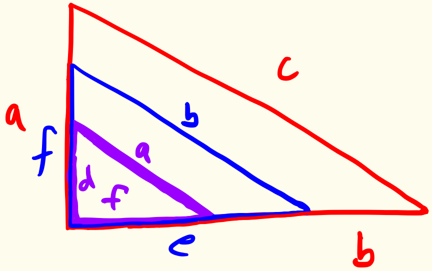
$$\frac{a}{f} = \frac{b}{e} = \frac{c}{b}$$

$$\frac{a}{d} = \frac{b}{f} = \frac{c}{a}$$

important!



with $c = d + e$ to show $a^2 + b^2 = c^2$.



a, f, and d are comparable

b, e, and f are comparable

c, b, and a are comparable

eg: $\frac{a}{f} = \frac{c}{b} = \frac{b}{e}$

We have

$c = d + e$

$\frac{a}{d} = \frac{c}{a}$ so $a^2 = cd$ and $d = \frac{a^2}{c}$

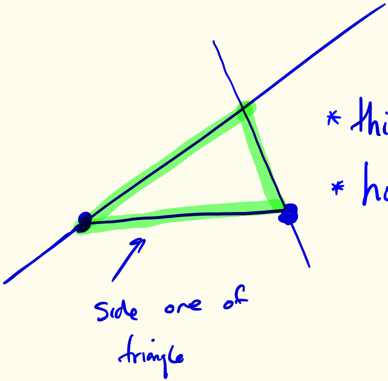
$\frac{b}{e} = \frac{c}{b}$ so $b^2 = ce$ and $e = \frac{b^2}{c}$

Since $c = d + e$,

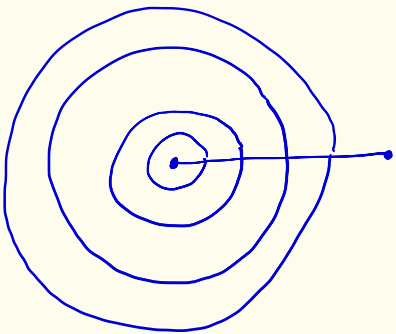
$c = \frac{a^2}{c} + \frac{b^2}{c}$

$c^2 = a^2 + b^2$

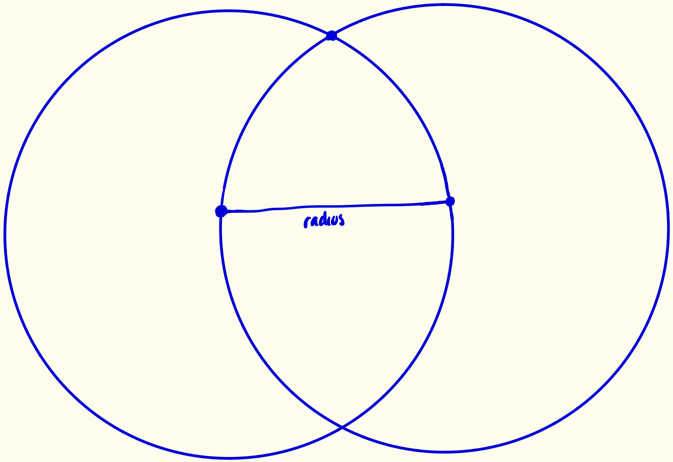
#13 Something from nothing



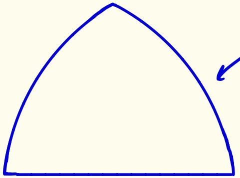
* this is a triangle, but sides are not all equal
* how might we construct an equilateral triangle - with only a straight edge and a compass?



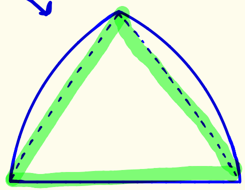
* by making circles bigger we are actually approaching a key step!



* overlapping circles gives us a way to measure radius
* all sides are the same length.



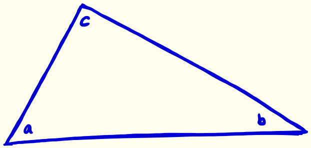
curved sides, but equilateral triangle lies beneath.
(look for this in windows of old buildings)

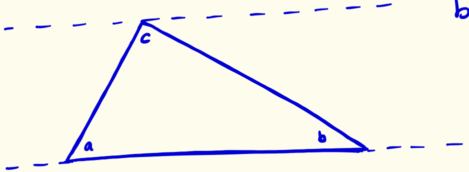


(Reuleaux triangle - a surprisingly elegant shape that could be plotted out with string and a nail.)

Consider the triangle and the claim:

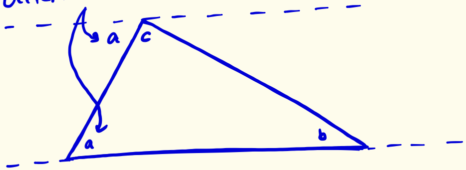
angles of a triangle sum to 180°



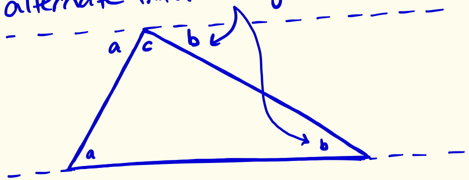


line parallel to base of triangle

alternate interior angles are equal

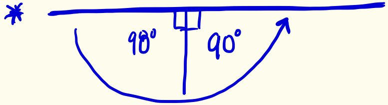


alternate interior angles are equal



By picture $a+b+c = 180°$. (* below)

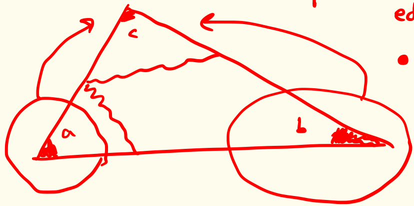
So the angles inside a triangle add to 180°. ✓

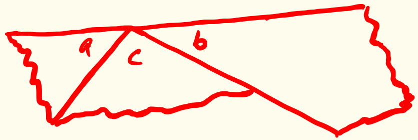


total angle of 180°

On paper

- draw a triangle (label corners)
- cut (or tear) corners
- align corners to form straight edge
- what happens for other shapes?

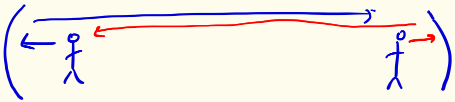




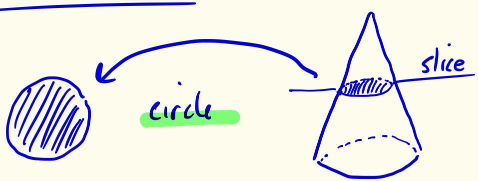
#14 Conic conspiracy

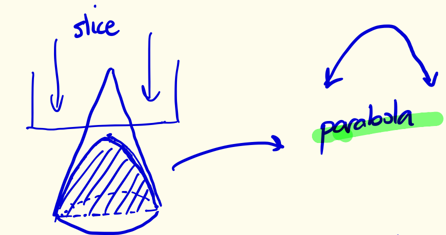
- certain shapes are part of a family called "conic sections"
- conic meaning "from a cone" (or cones)

Whisper dish: Omniplex Science Museum

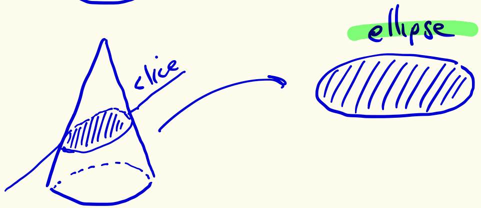


Types of conic sections





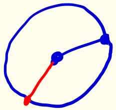
what other shapes are possible?



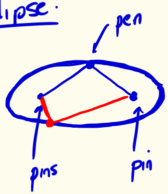
Other properties of conic sections:

circle:

every point on circle is the same distance (radius) from the center



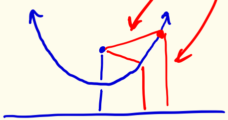
ellipse:



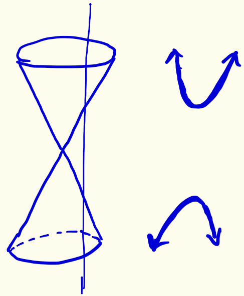
dots are called foci (each is called a focus)

parabola:

distance from focus to graph, graph to horizontal are the same

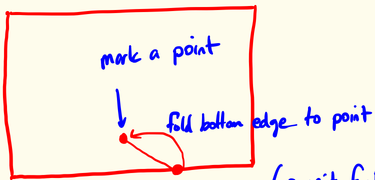


hyperbola:

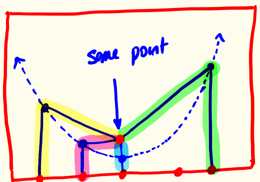


$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Different choices of A–F describe different conic sections.



(await further instruction)



The two yellow highlighted segments are both the same length, as are green, pink, blue.

The parabola traces points that are equally far from the bottom edge of the paper as they are the marked point!