Outline - Quaternions

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Abstract

William Rowan Hamilton first described quaternions in 1843. Quaternions are used to describe transformation in 3-d space and have many applications in aeronautics, robotics, and computer graphics. This manuscript will provide a brief overview of quaternions and spatial geometries, specifically relating to algebra, geometry, and differential calculus. This will be followed by a comparision between quaternions, euler angles, and rotational matrices, and then a discussion of their applications.

1 Introduction

1.1 History of Quaternions

(a) Key Definition 1: Vector

(b) Key Definition 2: Quaternions [2]

1.2 Basic Geometric Transformations

(a) Key Definition 3: Translation

(b) Key Definition 4: Rotation

2 Discussion

2.1 Algebra and Quaternions

(a) Key Theorem 1: Addition & Subtraction of Quaternions

(b) Key Theorem 2: Muliplication of Quaternions

(c) Quaternions in Cartesian Form

(d) Quaternions as a Group

2.2 Geometry and Quaternions

(a) Key Theorem 3: Translation with Quaternions

(b) Key Theorem 4: Rotation with Quaternions

2.3 Differential Calculus and Quaternions

(a) Key Theorem 5: The Product Rule

(b) Key Theorem 6: The Chain Rule

(c) Key Theorem 7: $\frac{d}{dt}q(t)^{r(t)}$

3 Comparison

3.1 Other Non-Euclidean Transformation Methods

(a) Key Definition 5: Euler Angles

(b) Key Definition 6: Rotation Matrix

3.2 Comparisions Between Methods

(a) Euler Angles and Quaternions

(b) Rotation Matrices and Quaternions

4 Applications

4.1 Quaternions & Aeronautics

(a) Yaw, Pitch, & Roll

(b) Gimbal Lock

4.2 Quaternions & Computer Graphics

(a) Efficiency

(b) Interpolation

5 Conclusion

References

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[3] Michael Henle, *Modern geometries*, Prentice-Hall, Upper Saddle River, New Jersey, 2001.

[4] Lieut.-Colonel H. W. L. Hime, *Outlines of quaternions*, London, Longmans, Green & co., 1894.

[5] Charles Jasper Joly, *A manual of quaternions*, MacMillan and Co., Limited, New York, NY, 1905.