

Outline - Quaternions

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February 27, 2017

Abstract

William Rowan Hamilton first described quaternions in 1843. Quaternions are used to describe transformation in 3-d space and have many applications in aeronautics, robotics, and computer graphics. This manuscript will provide a brief overview of quaternions and spatial geometries, specifically relating to algebra, geometry, and differential calculus. This will be followed by a comparison between quaternions, euler angles, and rotational matrices, and then a discussion of their applications.

1 Introduction

1.1 History of Quaternions

- (a) Key Definition 1: Vector
- (b) Key Definition 2: Quaternions [2]

1.2 Basic Geometric Transformations

- (a) Key Definition 3: Translation
- (b) Key Definition 4: Rotation

2 Discussion

2.1 Algebra and Quaternions

- (a) Key Theorem 1: Addition & Subtraction of Quaternions
- (b) Key Theorem 2: Multiplication of Quaternions
- (c) Quaternions in Cartesian Form
- (d) Quaternions as a Group

2.2 Geometry and Quaternions

- (a) Key Theorem 3: Translation with Quaternions
- (b) Key Theorem 4: Rotation with Quaternions

2.3 Differential Calculus and Quaternions

- (a) Key Theorem 5: The Product Rule
- (b) Key Theorem 6: The Chain Rule
- (c) Key Theorem 7: $\frac{d}{dt}q(t)^{r(t)}$

3 Comparison

3.1 Other Non-Euclidean Transformation Methods

- (a) Key Definition 5: Euler Angles
- (b) Key Definition 6: Rotation Matrix

3.2 Comparisons Between Methods

- (a) Euler Angles and Quaternions
- (b) Rotation Matrices and Quaternions

4 Applications

4.1 Quaternions & Aeronautics

- (a) Yaw, Pitch, & Roll
- (b) Gimbal Lock

4.2 Quaternions & Computer Graphics

- (a) Efficiency
- (b) Interpolation

5 Conclusion

References

- [1] Erik B. Dam et al, *Quaternions, interpolation, and animation*, 1998, <http://web.mit.edu/2.998/www/QuaternionReport1.pdf>.
- [2] William Hamilton, *Elements of quaternions*, London, Longmans, Green & co., 1866.
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- [4] Lieut.-Colonel H. W. L. Hime, *Outlines of quaternions*, London, Longmans, Green & co., 1894.
- [5] Charles Jasper Joly, *A manual of quaternions*, MacMillan and Co., Limited, New York, NY, 1905.