

# Homework 8

Math 198: Math for Machine Learning

Due Date:  
Name:  
Student ID:

## Instructions for Submission

Please include your name and student ID at the top of your homework submission. You may submit handwritten solutions or typed ones (L<sup>A</sup>T<sub>E</sub>X preferred). If you at any point write code to help you solve a problem, please include your code at the end of the homework assignment, and mark which code goes with which problem. Homework is due by start of lecture on the due date; it may be submitted in-person at lecture or by emailing a PDF to both facilitators.

## 1 Examples of Convex Functions

1. Give an example of a convex function.
2. Give an example of a non-convex function.
3. Give an example of a function which is strictly convex.
4. Give an example of a function which is convex, but not strictly convex.
5. Give an example of a function which is strictly convex, but not strongly convex.
6. Give an example of a function which is 2-strongly convex.
7. Give an example of a function which is convex but has no minima.

## 2 Feasible Sets

For the following problems, we will consider how the feasible set of solutions  $\mathcal{X}$  changes the feasibility of optimization. We will be attempting to optimize the strictly convex function  $f(x) = x^2$ .

1. Is there a unique global minimum of  $f$  if  $\mathcal{X} = \mathbb{R}$ ? If so, what is it? Is this set convex?
2. What if  $\mathcal{X} = \{1\}$ ? Is this set convex?
3. What if  $\mathcal{X} = \mathbb{R} \setminus \{0\}$ ? Is this set convex?
4. Let  $\mathcal{X} = (-\infty, -1] \cup [0, \infty)$ . Is this set convex? What are the local minima of  $f$  in this set? Are all local minima also global minima?
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### 3 Convexity Proofs

1. Let  $f$  and  $g$  be convex functions. Show that  $h(\mathbf{x}) = \max\{f(\mathbf{x}), g(\mathbf{x})\}$  is convex.
2. Let  $f$  be a convex function and  $g$  be a strictly convex function. Show that  $f + g$  is strictly convex.
3. Let  $f$  be a convex function and  $g$  be an  $m$ -strongly convex function. Show that  $f + g$  is  $m$ -strongly convex.