Homework 11

Math 198: Math for Machine Learning

Due Date:
Name:
Student ID:

Instructions for Submission

Please include your name and student ID at the top of your homework submission. You may submit handwritten solutions or typed ones (LaTeX preferred). If you at any point write code to help you solve a problem, please include your code at the end of the homework assignment, and mark which code goes with which problem. Homework is due by start of lecture on the due date; it may be submitted in-person at lecture or by emailing a PDF to both facilitators.

1 More Probability Proofs

1. Show that for any random variables X and Y, $\mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.
2. Show that for any random variables X, Y, Z and constants α, β,
$$\text{Cov}(\alpha X + \beta Y, Z) = \alpha\text{Cov}(X, Z) + \beta\text{Cov}(Y, Z)$$
3. Show that for independent random variables X and Y, $\text{Cov}(X, Y) = 0$.
4. Show that for uncorrelated random variables $X_1, \ldots, X_n$, $\text{Var}(X_1 + \ldots + X_n) = \sum_{i=1}^{n} \text{Var}(X_i)$.
5. Show that for any random vector $\mathbf{X}$, its covariance matrix $\mathbf{\Sigma}$ is positive semi-definite.