Takes force 29,272 lb to compress coil spring assembly from free height of 6 inches to fully compressed height of 2 inches.

a. What is assembly force constant

b. How much work to compress assembly first half inch? the second half inch?

Part a

1. Find d $\quad 6 - 2 = 4$ in

2. Known values

$F = kx \rightarrow 29{,}272 = k(4) \rightarrow \frac{29272}{4} = \frac{k(4)}{4}$

$\rightarrow 7318 = k$

∴ Force constant ≡ 7318 lb/in

Part b

1. Solve for work

$W = \int_a^b F(x)\,dx \rightarrow \int_0^{0.5} 7318\,x\,dx \rightarrow 7318 \int_0^{0.5} x\,dx$

$\rightarrow (7318)\,\frac{1}{2}x^2\Big|_0^{0.5} \rightarrow 7318 \cdot \frac{1}{2}(.5)^2 \rightarrow 914.75$ in-lb

for second half inch

$\int_{0.5}^{1} 7318\,x\,dx \rightarrow 7318 \int_{0.5}^{1} x\,dx \rightarrow$

$7318 \cdot \frac{1}{2}x^2 \Big|_{0.5}^{1} \rightarrow \left[7318 \cdot \frac{1}{2}(1)^2\right] - \left[914.75\right] =$

2744.25 in-lb

A mountan climber is going to haul 20-m length of hanging rope. How much work will it take if the rope weighs 0.4 N/m?

1. understand the incremation
   as x increases (i.e. rope is hauled up) the remaining rope weight formula = w(h − x) N
   $0.4(20-x)$ N
2. use work integration formula
   $$\int_a^b F(x)\,dx \longrightarrow \int_0^{20} 0.4(20-x)\,dx$$
   $$\left[0.4(20x) - 0.4\left(\frac{1}{2}x^2\right)\right]_0^{20} \equiv W = 80\text{ J}$$
3. Plug and play into $\frac{1}{2}x^2$ with simple problems

Electric Motor at top has cable weighing 3 lb/ft. When car is at first floor, 100 ft of cable are paid out, 0 ft are out when car is at top floor. How much work does motor do just lifting cable when car goes from first floor to top?

- Preserve x in the integral because you integrate

$$\int_0^{170} 5.5(170-x)\,dx \rightarrow 5.5\int_0^{170}\left(170x - \frac{1}{2}x^2\right)\Big|_0^{170}$$

$$5.5\left[\left(170\cdot170 - \frac{1}{2}170^2\right) - (0-0)\right]$$

A 3-kg bucket is lifted from ground into air, by pulling 80m rope at constant speed. The rope weighs 0.07 kg/m.

Bucket starts with 3 litres of water (3 kg) and leaks at constant rate. It finishes draining at top. How much work was spent lifting water alone?

1. Write function describing mass of water left in bucket with respect to height

$$M(x) = mx + b \longrightarrow mx + 3 \Rightarrow M(x) = mx + 3$$

2. Find slope

$$M(x) = 3 - \frac{3x}{80}$$

3. use mass function to find w/force

$$F(x) = 2.94 - \frac{29.4x}{80}$$

$g = 9.8\ m/s^2$
$3 \cdot 9.8 =$
2.94

4. Integrate.

$$\int_0^{80} 2.94 - \frac{29.4x}{80}\,dx = W \text{ Joules}$$

Find the Mass M and the center of mass $\overline{x}$ of a rod lying on the x-axis over the interval $[1,2]$ whose density is given by $\delta(x) = 2+3x^2$

- mass of rod is obtained by integrating density

$$M = \int_1^2 (2+3x^2)\,dx = \left[2x + x^3\right]_1^2$$

$$\rightarrow (4+8) - (2+1) = 9$$

$$M_0 = \int_1^2 x(2+3x^2)\,dx = 57/4$$

$$\frac{M_0}{M} = \frac{57/4}{9} = \overline{x}$$

$\delta \Rightarrow$ delta x density x