Student: Sean Villegas **Date:** 06/09/25	**Instructor:** Patrick Mitchell **Course:** MATH-04B-12830 (Summer 2025)	**Assignment:** 6.3 Homework - Arc Length

Find the length of the following curve. If you have a grapher, you may want to graph the curve to see what it looks like.

$$y = \frac{1}{3}\left(x^2+2\right)^{\frac{3}{2}} \quad \text{from } x=0 \text{ to } x=9$$

If f is continuously differentiable on the closed interval [a,b], the length of the curve (graph) y = f(x) from x = a to x = b is the following.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$$

To determine the length of the given curve use the formula from above with a = 0, b = 9, and $y = \frac{1}{3}\left(x^2+2\right)^{\frac{3}{2}}$. Begin by finding $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{1}{3}\cdot\frac{3}{2}\left(x^2+2\right)^{\frac{1}{2}}\frac{d}{dx}\left(x^2+2\right) \qquad \text{Apply the Chain Rule.}$$

$$= x\left(x^2+2\right)^{\frac{1}{2}} \qquad \text{Differentiate and simplify.}$$

Since $\frac{dy}{dx}$ is squared in the formula, square the result from above and simplify.

$$\left(\frac{dy}{dx}\right)^2 = \left[x\left(x^2+2\right)^{\frac{1}{2}}\right]^2$$

$$= x^2\left(x^2+2\right)$$

$$= x^4+2x^2 \qquad \text{Multiply.}$$

Now substitute the expression for $\left(\frac{dy}{dx}\right)^2$ into the formula for length along with the limits of the integral.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^9 \sqrt{1+x^4+2x^2}\,dx$$

Rearrange the terms of the expression in the radicand. Notice that $1+2x^2+x^4$ is a perfect square trinomial. In order to simplify further, rewrite the expression as a perfect square.

$$\int_0^9 \sqrt{1+x^4+2x^2}\,dx = \int_0^9 \sqrt{\left(1+x^2\right)^2}\,dx$$

Simplify and evaluate the integral. Find the antiderivative of $1+x^2$.

$$\int_0^9 \sqrt{\left(1+x^2\right)^2}\,dx = \int_0^9 \left(1+x^2\right)dx$$

$$= \left[x+\frac{x^3}{3}\right]_0^9$$

Evaluate.

$$\left[x + \frac{x^3}{3}\right]_0^9 = \left(9 + \frac{9^3}{3}\right) - \left(0 + \frac{0^3}{3}\right)$$

$$= 252$$

Thus, the length of the curve $y = \frac{1}{3}\left(x^2 + 2\right)^{\frac{3}{2}}$ from $x = 0$ to $x = 9$ is 252. The curve is shown in the accompanying graph.