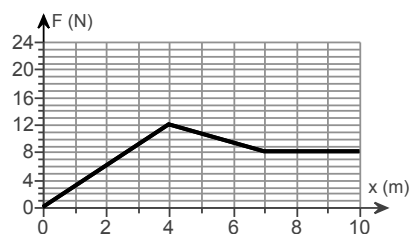


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**Date:** 06/14/25

**Instructor:** Patrick Mitchell  
**Course:** MATH-04B-12830 (Summer 2025)

**Assignment:** 6.5 Homework - Work

The graph of a force function (in newtons) is given. How much work is done by the force in moving an object 10 m?

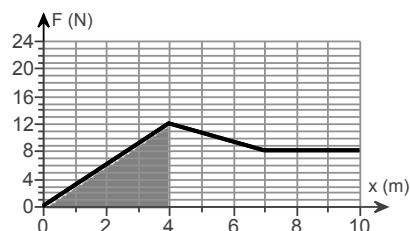


The work done is equal to the area bounded by the graph and the x-axis from 0 to 10. This follows from the definition of work. The work done

by a variable force  $F(x)$  in moving an object along the x-axis from  $x = a$  to  $x = b$  is  $W = \int_a^b F(x)dx$ .

To find the area under the graph, start by finding the graph from 0 to 4. The area bounded by the first line segment that goes from (0,0) to (4,12) and the x-axis forms a right triangle. Identify the base and the height of this triangle.

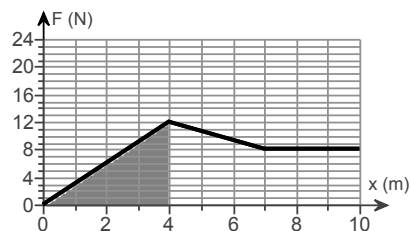
The base of the triangle is 4 m. The height of the triangle is 12 N.



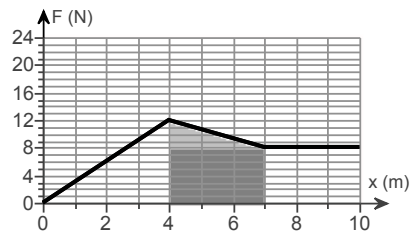
Find the area of this triangle.

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(12) \\ &= 24 \end{aligned}$$

So, the area of the shaded region in this graph is 24 J.

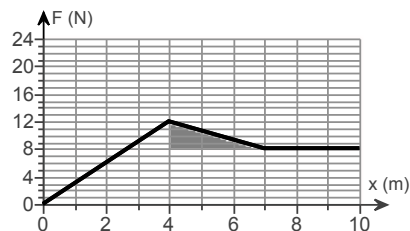


In SI units, the unit of force is a newton, the unit of distance is a meter, and the unit of work is a newton-meter ( $\text{N} \cdot \text{m}$ ). This combination appears so often it has a special name, the joule. The joule is represented by J. Now, look at the area bounded by the graph and the x-axis from  $x = 4$  to  $x = 7$ . Notice that this area can be broken into a rectangle and a right triangle.



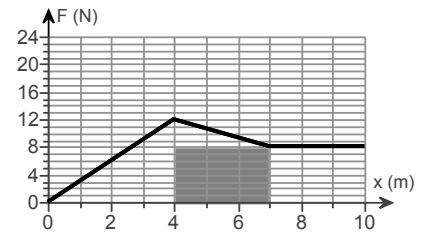
Start by looking at the right triangle in this region. Find the area of the triangle.

The area of the triangle is 6 J.



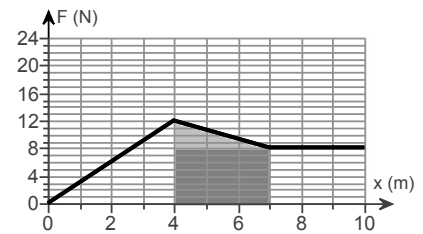
Now look at the rectangular portion. Find the area of this rectangle.

The area of the rectangle is 24 J.

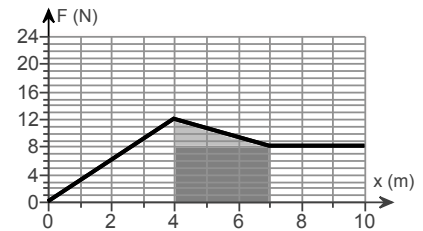


Now, to find the area of the entire shaded region, add the area of the rectangle and the area of the triangle.

$$6 + 24 = 30$$

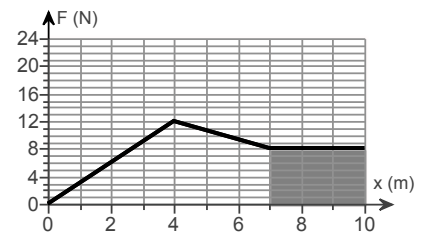


So, the area of the region bounded by the second line segment and the x-axis from  $x = 4$  to  $x = 7$  is 30 J.



Now, look at the area bounded by the graph and the x-axis from  $x = 7$  to  $x = 10$ . Notice that this region is in the shape of a rectangle. Find the area of this rectangle.

The area of this rectangle is 24 J.



So, the area of the region bounded by the line segment and the x-axis from  $x = 0$  to  $x = 4$  is 24 J. The area of the region bounded by the line segment and the x-axis from  $x = 4$  to  $x = 7$  is 30 J. The area of the region bounded by the line segment and the x-axis from  $x = 7$  to  $x = 10$  is 24 J. Add to find the total area bounded by the graph and the x-axis from  $x = 0$  to  $x = 10$ .

$$24 + 30 + 24 = 78$$

So, the amount of work done by the force in moving an object 10 m is 78 J.