# Demand-induced regime shift in fishery: A mathematical perspective

Building a bio-economic model to model a fish population

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09/04/2024

# Contents

# 1 Abstract

This paper takes a unique perspective on how a regime shift in fishery may be induced. A regime-shift in fisheries involves a significant and sudden change in the behaviour of the fish population which results in a change having to be made in the fishery management decisions. While previous papers have cited over-fishing and climate change as the main reasons for a regime shift in fishery this paper examines how demand and taxation may cause a regime shift in fishery. To do this a four-dimensional bio-economic fishery model is considered and analysed to explore the systems dynamic behaviour. Our analysis showed how low taxation and high demand can cause the fish population to become disease free. However, if the demand passes a threshold the price of healthy fish becomes unbounded and the fish market collapses. This paper therefore demonstrates how demand which is dependent on taxation, fish price and harvesting effort can play a major role in the regime-shift of fisheries and therefore must be accounted for when developing management systems for fisheries. Using the bio-economic model we also discussed the dynamics of revenue generation in the fishing industry as taxation is varied. Increasing taxation always increased the revenue generated in the system. However this rise in revenue came at a cost to the individual fisher as their income decreased with taxation.

# 2 Introduction

Fish play a crucial role in our society. Fishery is an important sustainable livelihood .In 2020, an estimated 58.5 million people were engaged as full-time, part-time, occasional or unspecified workers in fisheries and aquaculture.With roughly 3.4 billion employed people worldwide, fisheries and aquaculture makes up 1.7205% of this.[1] Fish account for a significant proportion of the world's food supply. They provide 17% of the world's meat consumption. [2] Global average consumption of fish and other seafood reached a record high of 20.5 kilogram in 2019, continuing a continuous upward trend from 9.9 kg in the 1960s to 11.4 kg in the 1970s, 12.5 kg in the 1980s, 14.4 kg in the 2000s 19.6 kg in the 2010s.[3]Fish also play a crucial role in keeping the ocean healthy. Beyond their nutritional value, fish also play a crucial role in maintaining ocean health, which is vital for mitigating climate change and preserving the diverse marine ecosystem, home to over 243,613 species.[4]

The reasons outlined above is why it is a global responsibility to maintain the health of the world fisheries and aquaculture. To maintain the health of world fisheries and aquaculture control measures are often put into place to ensure over-fishing doesn't occur. These measures often include imposing quotas on fish species, implementing taxes on catches, and regulating fishing periods for certain species. Real-world examples of such measures include recent legislation in Italy regulating the harvesting of specific shellfish species in the Adriatic Sea and the application of taxes on the anchovy fishery in Peru[5][6]. However, developing effective control measures necessitates accurate modelling to assess their impact comprehensively.

A robust model should consider the multifaceted dynamics of fisheries management, accounting for the interests of various stakeholders. Economic and biological factors are the primary determinants affecting fish population dynamics.Economic factors, such as fish prices, demand, supply, and taxation, influence harvesting efforts and ultimately impact fish populations. Conversely, biological factors, including birth rates, mortality rates, disease prevalence, and environmental carrying capacity, directly influence the population dynamics of fish species.

Several existing models predominantly focus on open-access fisheries, where harvesting is unregulated. While these models provide valuable insights into harvesting strategies and market dynamics, they often overlook the interplay between economic and biological factors.[7] explores a fishery model which incorporates a switching of harvesting strategy. This model explores how fisher's will change their harvesting strategy based on profit margins with no restrictions in place. In [8]they introduce the idea of the fish population depending on a variable market price. In [9] a simple predator prey model is built to model two separate fish species populations. The prey population grows logistically, and the predator population grows dependent on the prey. All these models fail to effectively integrate both the economic and biological aspects.

The bioeconomic model emerged as a comprehensive framework that harmonizes ecological and economic considerations in fisheries management. These models allow for analysis of the biological and economic changes caused by human activities. The bioeconomic models are often developed at farm, country, and global scales, and are used in various fields, including agriculture, fisheries, forestry, and environmental sectors .In [10] the idea of a bioeconomic model is used to model a fish population. This model consists of three major parts and is the framework for the paper that I studied. A biological part where the fish population grows logistically, and a proportion of the fish population becomes infected with rate lambda. An exploitation part that links the fish caught to fishing effort and price and an economic part that links the effort fishing to price and demand.

While this study makes significant strides in modelling fish populations using a bio-economic framework, it also has many limitations [10]. One limitation is in the assumption of harvesting effort following a CPUE (catch per unit effort) hypothesis. This approach has significant challenges, these include the potential for the harvesting rate to become infinite when the harvesting stock or effort is infinite (Figure 1). To address this issue, the paper I am studying uses an alternative harvesting rate by considering a non-linear saturated type of harvesting effort. Furthermore, my paper addresses another flaw in the previous study related to the consideration of a linear demand function. While this approach is commonly used, it has its constraints, particularly as it fails to account for demand saturation as prices rise (i.e demand doesn't increase linearly as price increases but instead as prices rise higher and higher the demand should saturate). To overcome this limitation, the paper adopts a non-linear demand function, which more accurately reflects real-world market dynamics, with demand saturating as prices increases. Additionally, the paper revisits the assumption made in previous studies regarding the selling price healthy and infected fish. In previous models, both healthy and infected fish were considered to be sold for the same price. However, this paper introduces a more accurate approach by assigning a variable price to healthy fish based on demand and harvesting rate, while infected fish are sold at a constant price. This adjustment better aligns with economic realities and market conditions. Moreover, the paper introduces a tax rate as a control mechanism to address the issue of sustainability in fisheries management.Through these modifications and enhancements, the paper improves the previous bio-economic model.

While previously over-fishing and climate change have been cited as the main reasons for major change in the fish population (regime-shift). This model gives us the ability to analyse how the fish population changes with taxation ($\tau$), the infection rate ($\lambda$), environmental carrying capacity ($K$) and the maximum demand ($A$). With this analysis we will see if these parameters cause catastrophic changes in the fish population (regime shift) and are therefore a reason to change our fish management policies.

# 3 Methods

## 3.1 The model

The fish population grows logistically in the absence of infection and harvesting.

$$\frac{d\Delta}{dt} = r\Delta\left(1 - \frac{\Delta}{K}\right)$$

$\Delta$ is the fish population (biomass) at time t. $K$ is the environmental carrying capacity.(maximum population size of a fish species that can be sustained over the long term within a given ecosystem). $r$ is the intrinsic growth rate ( birth rate).

Now including infection and harvesting in the model. We divide the fish population ($\Delta$) into healthy ($X$) and Infected fish ($Y$). ($\Delta = X + Y$)

$$\frac{dX}{dt} = rX\left(1 - \frac{X+Y}{K}\right) - \lambda XY - Q_1(X,H)$$

$$\frac{dY}{dt} = \lambda XY - \mu Y - Q_2(Y,H)$$

$\lambda$ is the transmission rate of the infection. So $\lambda XY$ is the number of fish that are transferred from the healthy class ($X$) to the infected class ($Y$) at time $t$. $H$ is the harvesting effort and $Q_1(X,H)$ and $Q_2(Y,H)$ are the harvesting rate of the healthy and infected fish. In the study that introduced a bioeconomic model for modelling fish populations they use a linear harvesting rate.[10]

$$Q_1(X,H) = q_1XH, \quad Q_2(Y,H) = q_2YH$$

Here $q_1, q_2$ are catchability coefficients. This harvesting scheme follows the CPUE (Catch per unit effort) hypothesis which has serious issues when either the fish population or harvesting effort is large. This paper introduces saturation constants ($D_1, D_2$)to remove these unrealistic features

$$Q_1(X,H) = \frac{q_1XH}{X+D_1}, \quad Q_2(Y,H) = \frac{q_2YH}{Y+D_2}$$

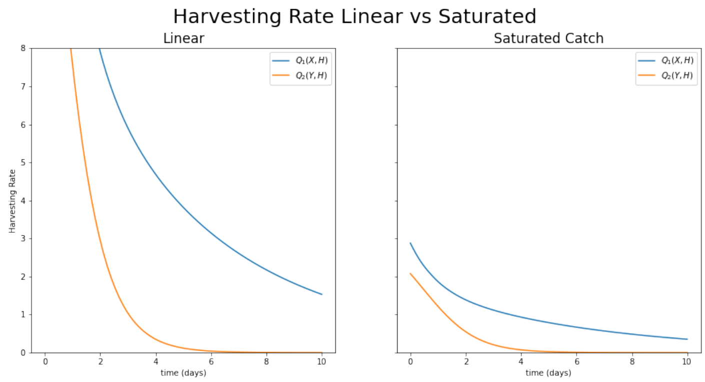



Figure 1: This illustrates how the linear catch becomes unbounded when the harvesting effort and fish population is high

Now since this is a bio-economic model harvesting effort and price are included as state variables. The price fish sell for depends on the demand for the fish at that time and the harvesting effort. Again study [10] is updated here. The price of fish did not differ between healthy and infected fish in this study. In this paper the price variable $(P)$ is the price of healthy fish at time t and infected fish are sold at a constant price p. This paper also added a tax on fish harvested $(\tau)$. $c$ is the cost per unit of fishing effort.

$$\frac{dH}{dt} = \phi_1 H \left( \left( \frac{q_1 (P - \tau) X}{X + D_1} + \frac{q_2 (p - \tau) Y}{Y + D_2} \right) - c \right)$$

Here the Income earned from healthy fish by the fishers is $\frac{q_1(P-\tau)XH}{X+D_1}$. The Income earned from the infected fish by the fishers is $\frac{q_2(p-\tau)YH}{Y+D_2}$.The total income for fishers is then $\left(\frac{q_1(P-\tau)XH}{X+D_1} + \frac{q_2(p-\tau)YH}{Y+D_2}\right)$. The total tax earnings are $\left(\frac{q_1\tau XH}{X+D_1} + \frac{q_2\tau YH}{Y+D_2}\right)$. Therefore the total societal revenue generated from the fishing industry is the fishers income plus the revenue generated from tax. $\left(\frac{q_1PXH}{X+D_1} + \frac{q_2PYH}{Y+D_2}\right)$ (Figure 5).

The Price of healthy fish varies with demand and harvesting effort. The demand function used in study [10] is linear and decreased linearly with price $D(p) = A - \alpha P$. This study introduces a saturated demand so demand doesn't increase linearly with price. $D(P) = \frac{A}{1+BP}$. $A$ is the maximum demand and $B$ is the demand sensitivity parameter

$$\frac{dP}{dt} = \phi_2 P \left( \frac{A}{1 + BP} - \frac{q_1 XH}{X + D_1} \right)$$

## The final model

$$\frac{dX}{dt} = rX\left(1 - \frac{X+Y}{K}\right) - \lambda XY - \frac{q_1 XH}{X + D_1}$$
$$\frac{dY}{dt} = \lambda XY - \mu Y - \frac{q_2 YH}{Y + D_2}$$
$$\frac{dH}{dt} = \phi_1 H \left( \left( \frac{q_1 (P - \tau) X}{X + D_1} + \frac{q_2 (p - \tau) Y}{Y + D_2} \right) - c \right)$$
$$\frac{dP}{dt} = \phi_2 P \left( \frac{A}{1 + BP} - \frac{q_1 XH}{X + D_1} \right)$$

$X$ =densities of Susceptible Fish
$Y$ =densities of Infected Fish
$H$ =harvesting effort
$P$ =Market price of fish at time t

### 3.2 Equilibrium Points

There exists five equilibrium points. The trivial equilibrium $\epsilon_0 = (0, 0, 0, 0)$,the only healthy fish equilibrium $\epsilon_1 = (K, 0, 0, 0)$. The healthy and infected fish equilibrium $\epsilon_2 = \left(\frac{\mu}{\lambda}, \frac{r(\lambda K - \mu)}{\lambda(\lambda K + r)}, 0, 0\right)$. The disease free equilibrium $\epsilon_3$ and the endemic equilibrium $\epsilon_4$. $\epsilon_0, \epsilon_1$ and $\epsilon_2$ are trivial to calculate and are always unstable.For the disease free equilibrium $(\epsilon_3)$ we have.

$$r\left(1-\frac{X}{K}\right)-\frac{q_1H}{X+D_1}=0$$
$$Y=0$$
$$\frac{q1(P-\tau)X}{X+D_1}-c=0$$
$$\frac{A}{1+BP}-\frac{q_1XH}{X+D_1}=0$$

Using Mathematica this is quite easily solved to attain the disease free fixed point (submitted the mathematica file).For the endemic fixed point $\epsilon_4$ the complexity of the equation forces us to use a numerical solver.I did this in python using fsolve.(the function used is in the appendix, the actual python code also submitted).Figure 2 shows how the system tends to each fixed point except the trivial fixed point ($\epsilon_0$).

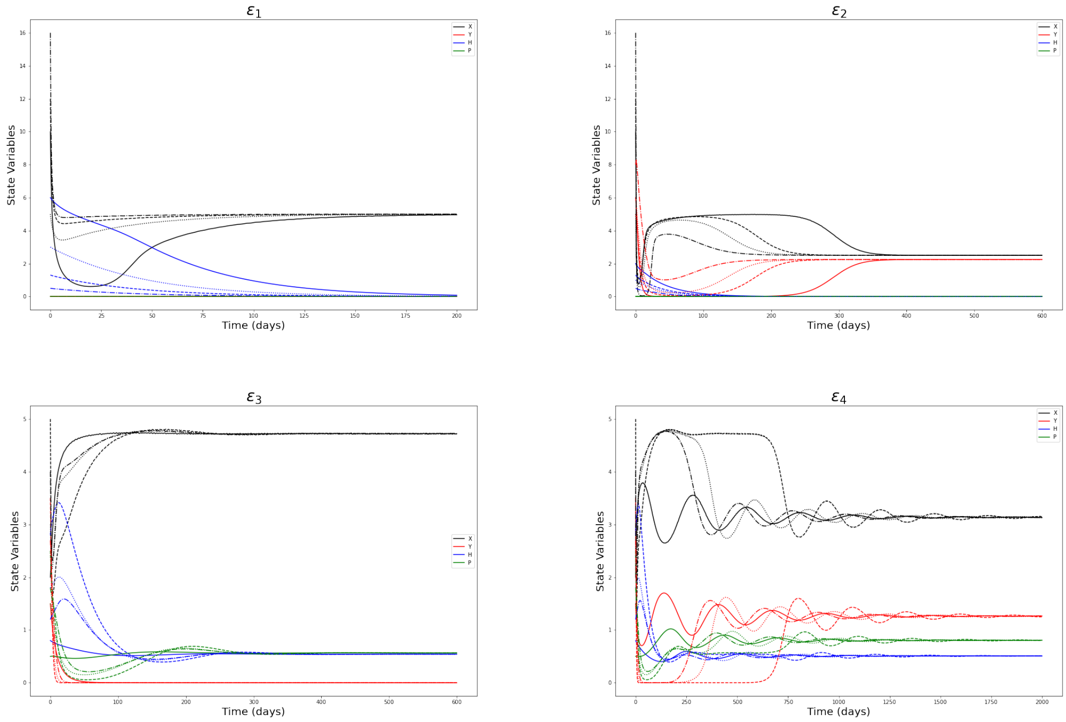



Figure 2: For $\epsilon_1$ the initial conditions used are con1=$(10, 0, 6, 0)$,con2=$(5, 0, 3, 0)$,con3=$(12, 0, 1.3, 0)$,con4=$(16, 0, 0.5, 0)$, for $\epsilon_2$ the initial conditions used were con1=$(10, 6, 2, 0)$,con2=$(5, 6, 1, 0)$,con3=$(12, 5, 1.3, 0)$,con4=$(16, 8, 0.5, 0)$ ,for $\epsilon_3$ the initial conditions were con1=$(2, 1.5, 0.8, 0.5)$,$(3, 2.5, 1.8, 1.5)$,$(5, 3.5, 2.8, 2.5)$,$(4, 2.7, 1.2, 1.8)$, $epsilon_4$ uses the same initial conditions as $\epsilon_3$. For $\epsilon_1, \epsilon_2$ and $\epsilon_3$ $K = 5$,$A = 0.9$,$\tau = 0.45$ and $\lambda = 0.02$ for $\epsilon_4$ all parameters were the same except $\lambda = 0.04$

## 3.3 Calculating $R_0$

We make the next generation matrix.

$$\frac{dY}{dt}=\lambda XY-\mu Y-\frac{q_2YH}{Y+D_2}$$

$f=\lambda XY$ (coming into infection class) and $v=\left(\mu Y+\frac{q_2HY}{Y+D_2}\right)$ (leaving the infection class)

$F=\left|\frac{\partial f}{\partial Y}\right|_{\epsilon_3}=\lambda X_3 \quad V=\left|\frac{\partial v}{\partial Y}\right|_{\epsilon_3}=\mu+\frac{q_2H_3}{D_2}$ where $\epsilon_3=(X_3,0,H_3,P_3)$

$\kappa=FV^{-1}=\frac{\lambda X_3D_2}{\mu D_2+q_2H_3}$

$$R_0=\frac{\lambda X_3D_2}{\mu D_2+q_2H_3}$$

In figure 13. We show that the region where $R_0<1$ for each parameter coincides with the stability region of the disease free fixed point for each parameter as would be expected.

# 4 Results

## 4.1 Jacobian and defining parameters and variables

Defining all the parameters and state variables and giving values for constants used.

| State variables and parameters with their description and default values and units | | |
|---|---|---|
| Variable | Description | Unit |
| $X(t)$ | Healthy fish biomass at time t | metric tonnes (MT) |
| $Y(t)$ | Infected fish biomass at time t | metric tonnes (MT) |
| $H(t)$ | Fishing effort at time t | SFU* |
| $P(t)$ | Market price per unit of biomass at time t | M$**/metric tonne. |
| Parameter | Description | Default value |
| $r$ | Intrinsic growth rate of healthy fish | 0.9/year |
| $K$ | Environmental carrying capacity | (varied) metric tonnes |
| $D_1$ | Half-saturation level of susceptible fish | 4 metric tonnes |
| $D_2$ | Half-saturation level of infected fish | 4.8 metric tonnes |
| $\lambda$ | Transmission rate | (varied) metric tonnes/year |
| $q_1$ | Catchability coefficient of susceptible fish | 0.8 metric tonnes/SFU/year |
| $\mu$ | Total death (natural + virulence) rate of infected fish | 0.05/year |
| $q_2$ | Catchability coefficient of infected fish | 0.9 metric tonnes/SFU/year |
| $c$ | cost per unit of fishing effort | 0.05 $/SFU/year |
| $A$ | Maximum demand | (varied) metric tonnes/year |
| $B$ | Demand sensitivity parameter | 5 metric tonnes/m$ |
| $\Phi_1$ | Stiffness parameter | 0.1 SFU/M$ |
| $\Phi_2$ | Proportionality constant | 0.15 /metric tonne |
| $p$ | Fixed market price of infected fish | 0.05$/metric tonne |
| $\tau$ | Tax per unit biomass of harvesting | (varied) M$/metric tonne |

*SFU stands for Standard Fishing Unit and ** M$ indicates million USD.
The Jacobian calculated using mathematica:

$$
\boldsymbol{J} = \begin{pmatrix}
r\left(1-\frac{2X+Y}{K}\right)-\lambda Y-\frac{q_1 H}{X+D_1}+\frac{q_1 X H}{(X+D_1)^2} & -\left(\frac{r}{K}+\lambda\right)X & -\frac{q_1 X}{X+D_1} & 0 \\
\lambda Y & \lambda X-\mu-\frac{q_2 H}{Y+D_2}+\frac{q_2 Y H}{(Y+D_2)^2} & -\frac{q_2 Y}{Y+D_2} & 0 \\
\frac{\phi_1 q_1 (P-\tau) H}{X+D_1}-\frac{\phi_1 q_1 (P-\tau) X H}{(X+D_1)^2} & \frac{\phi_1 q_2 (p-\tau) H}{Y+D_2}-\frac{\phi_1 q_2 (p-\tau) Y H}{(Y+D_2)^2} & \phi_1\left[\frac{q_1 (P-\tau) X}{X+D_1}+\frac{q_2 (p-\tau) Y}{Y+D_2}-c\right] & \frac{\phi_1 q_1 X H}{X+D_1} \\
\frac{\phi_2 q_1 H P}{X+D_1}-\frac{\phi_2 q_1 X H P}{(X+D_1)^2} & 0 & \frac{\phi_2 q_1 X P}{X+D_1} & \phi_2\left(\frac{A}{1+BP}-\frac{ABP}{(1+BP)^2}-\frac{q_1 X H}{X+D_1}\right)
\end{pmatrix}_{(\hat{X},\hat{Y},\hat{H},\hat{P})}
$$

## 4.2 Analysing how the taxation ($\tau$) effects the system

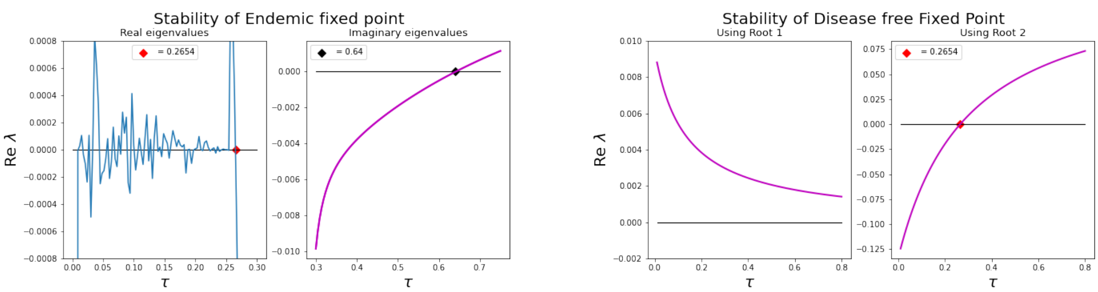
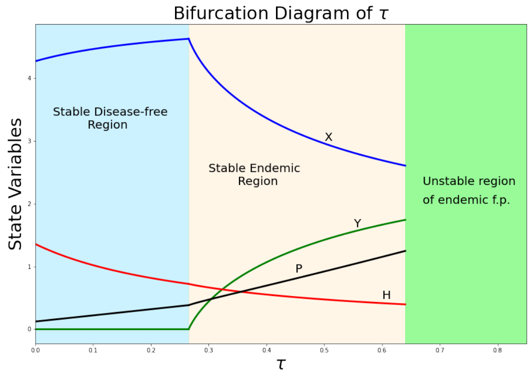



Figure 3: Equilibrium Stability depending on $\tau . A = 0.9, K = 5, \lambda = 0.04$

Using the Jacobian to analyse the stability of the endemic and disease free fixed points. In Figure 3 ,the real part of the Imaginary eigenvalues and the real eigenvalues are plotted. We gather that the endemic fixed point

is unstable when $\tau < 0.2654$ because the real eigenvalue oscillates between being negative and positive. When $\tau > 0.2654$ the real eigenvalues are less than zero and the real part of the imaginary eigenvalues are also less than zero. Therefore the endemic fixed point is stable until $\tau = 0.64$ when the real part of the imaginary eigenvalue is greater than zero.So we have that the Endemic equilibrium is stable in the region $\tau \in [0.2654, 0.64]$.
For the disease free fixed point we have two possible values for X so we examine the stability of both. In Fig 3, we plot the real eigenvalues corresponding to the different X values. We determine that root 2 is stable when $\tau < 0.2654$ from Fig 3. So we have that the disease free equilibrium is stable in the region $\tau \in [0, 0.2654]$



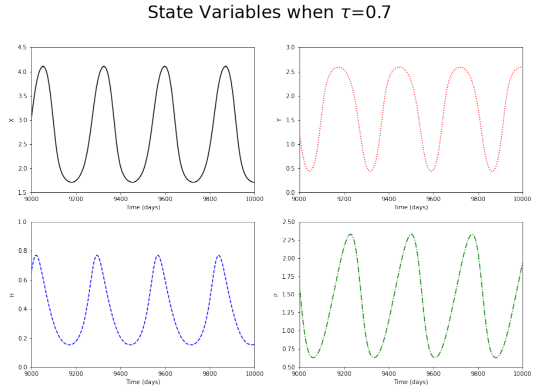




Figure 4: In the bifurcation diagram we Vary $\tau$ in the stable disease free region where $\tau \in [0, 0.2654]$.And vary $\tau$ in the stable endemic region where $\tau \in [0.2654, 0.64]$.The right graph shows the periodic unstable behaviour when $\tau = 0.7$. We use the parameters $A = 0.9, K = 5, \lambda = 0.04$.

Taxation is introduced into the fishery system as a regulatory measure.From our analysis we know there are three main regions where the taxation has a noticeable effect on the fish population dynamics.The disease free steady state is stable in the region when $\tau \in [0, 0.2654]$.This implies that a low tax ($\tau < 0.2654$) removes disease from the population. Low tax results in high harvesting effort ($H$) and a huge amount of healthy fish ($X$) to harvest. Since there is so much fish and harvesting effort is high the price for healthy fish is at its minimum when no tax is imposed.Once $\tau > 0.2654$ the disease free state becomes unstable and the endemic steady state becomes stable. The endemic state remains stable in the region $\tau \in [0.02654, 0.64]$. When the taxation is in this region it allows disease to invade the population. Increasing disease in the population means that the number of healthy fish dramatically reduces. This reduction in healthy fish reduces the harvesting effort as there's less fish to harvest. Then from figure 6 we can see that when the taxation is increased the demand isn't majorly effected but the supply of healthy fish decreases massively due to reduction in the population of healthy fish and the reduction in harvesting rate as tax increases. Therefore with far less supply and similar demand the price of healthy fish will increase. When the endemic region becomes unstable in the taxation region where $\tau > 0.64$. The whole system becomes unstable and periodic. The taxation rate is so high that it destroys the system as a taxation rate ¿ 64% makes no economic sense. The taxation rate destroys any incentive for fishers to harvest fish which reduces supply massively and causes the price to sky rocket. But once demand is met the price reduces again and fishers stop fishing. This becomes an unstable periodic orbit.

| How $\tau$ effects the system | | |
|---|---|---|
| $\tau \in [0, 0.2654]$ | $\tau \in [0.2654, 0.64]$ | $\tau > 0.64$ |
| Removes Disease from Population<br><br>High Harvesting Effort (H)<br><br>Huge amount of Healthy Fish (X)<br><br>Low Price (P) | Disease Invades Population<br><br>Reducing Harvesting Effort (H)<br><br>Healthy Fish reduce rapidly from infection (X)<br><br>Growing Price (P) | Unstable<br><br>and Periodic |

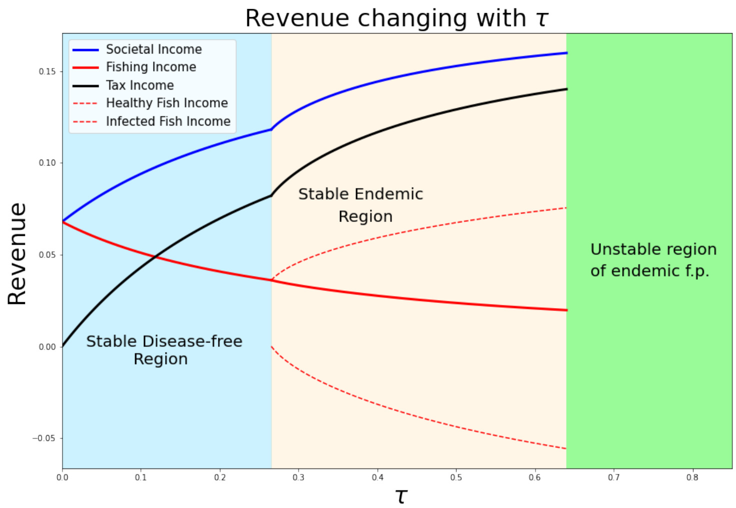
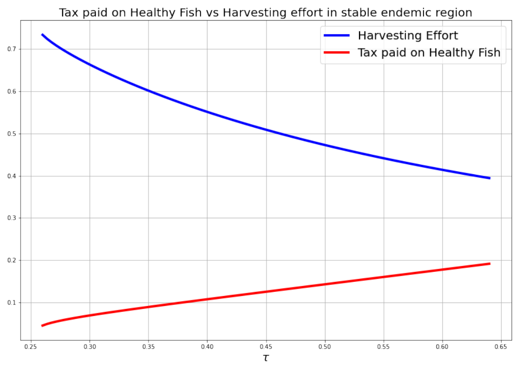



Figure 5: How $\tau$ effects revenue.The blue line shows the societal income. This sums the income from taxation and the sale of fish. The black line is the revenue earned from fish tax. The red line is the revenue from fishing (i.e from selling the healthy and infected fish). The dotted red line that is increasing with taxation is the revenue earned from the sale of healthy fish. The dotted red line decreasing with tax indicates the revenue from the sale of infected fish which is sold at a constant price p. The parameters are $\lambda = 0.04, K = 5, A = 0.9$

The societal income grows with $\tau$ where the maximum income in the stable disease free case occurs at $\tau = 0.2654$ and the maximum income in the endemic case occurs at $\tau = 0.64$. Although the societal income is maximum the fishers income decreases as tax increases.This supports the fact that harvesting effort decreases with increased tax as there is less incentive for fishers(as seen in figure 6) .We see that in the endemic stable case the importance of harvesting healthy fish rather than infected fish increases with taxation. Harvesting infected fish results in negative income in this case. Then as discussed previously once $\tau = 0.64$ the fish market collapses as harvesting fish with that high of a rate of tax becomes infeasible.



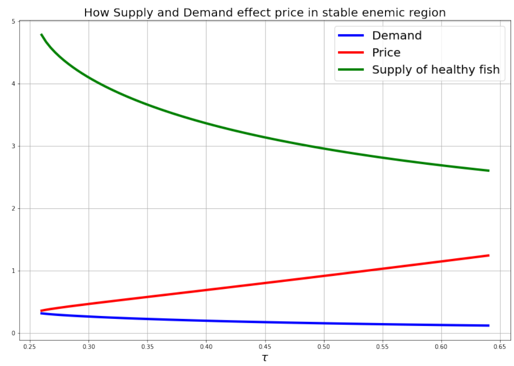



Figure 6: The harvesting effort in the system plotted with the tax paid on healthy fish in the stable endemic region where $\tau$ is varied.We vary the demand and supply with $\tau$ in the endemic stable region and sees how it effects the price.The parameters are as in figure 4.

## 4.3 Analysing how the transmission rate ($\lambda$) and the environmental carrying capacity ($K$) effect the system

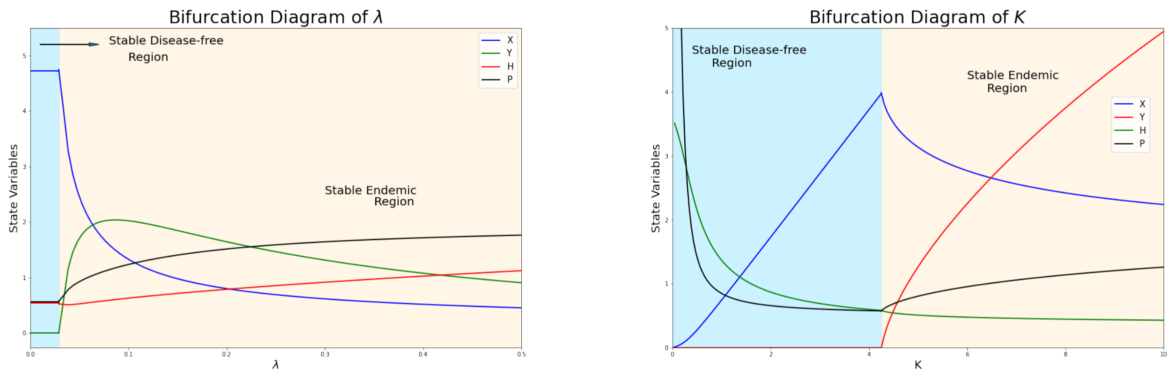



Figure 7: Bifurcation for $\lambda$ (left side) with parameters $A = 0.9, K = 5, \tau = 0.45$ and $K$ (right side) with parameters $A = 0.9, \lambda = 0.04, \tau = 0.045$

Again using the jacobian and the examination of the real part of the eigenvalues we find that the disease free equilibrium is stable for $\lambda \in [0, 0.031]$. When the transmission rate is in this region the system remains disease free. There's high numbers of healthy fish (large supply)which results in a lower demand for fish and therefore a relatively low price. Once the disease free steady state becomes unstable and the endemic steady state becomes stable in the region $\lambda \in [0.031, 0.5]$ the infection spreads rapidly and the number of healthy fish reduces quickly resulting in less supply price. The demand remains fairly constant but with less supply to meet this demand the price increases quickly and therefore the harvesting effort skyrockets because of the extra incentive for fishers. (all demonstrated in figure 8)

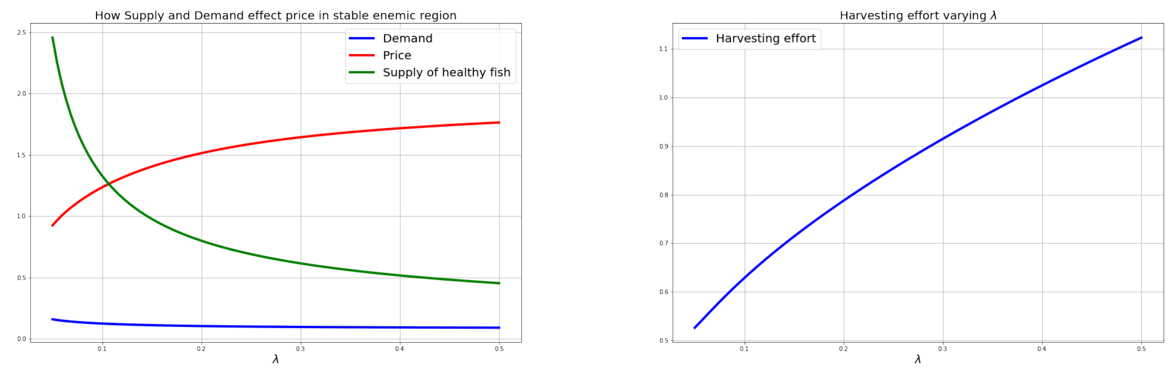



Figure 8: Demand plotted with the Healthy Fish Biomass ($X$) and Price in the stable endemic region where $\lambda$ is varied (left side), the harvesting effort is plotted for varying $\lambda$ (right side). Parameters as in bifurcation diagram of $\lambda$ in figure 7

Figure 8 illustrates the inverse relationship between supply of healthy fish and demand in this model.It also demonstrates the linear relationship between $\lambda$ and harvesting effort.
Very similar analyses with similar conclusions can be performed for $K$. The system remains disease free for low values of $K$ and then the disease invades for higher values of $K$ as seen in figure 7.

### 4.4 Analysing how the maximum demand $(A)$ effects the system

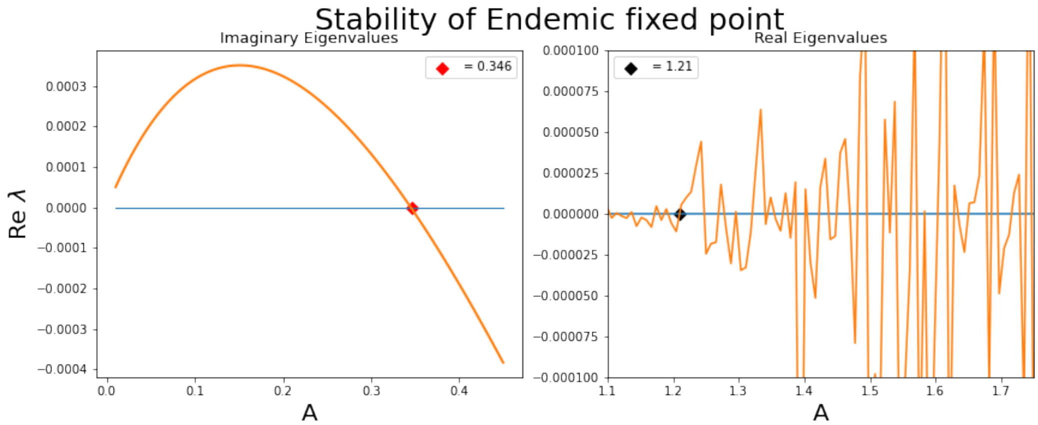



Figure 9: We plot the eigenvalues of the Jacobian in the endemic region depending on A as we have done previously to find the region in which the endemic fixed point is stable. This region is $A \in [0.346, 1.21]$.The parameters used are $K = 5, \lambda = 0.04, \tau = 0.45$

Using very similar analysis using the jacobian we find that the disease free fixed point becomes stable for one root at $A \approx 1.21$ and then doesn't exist when $A > 4.594$. Therefore we have that one root of the disease free state is stable in the region $A \in [1.21, 4.594]$

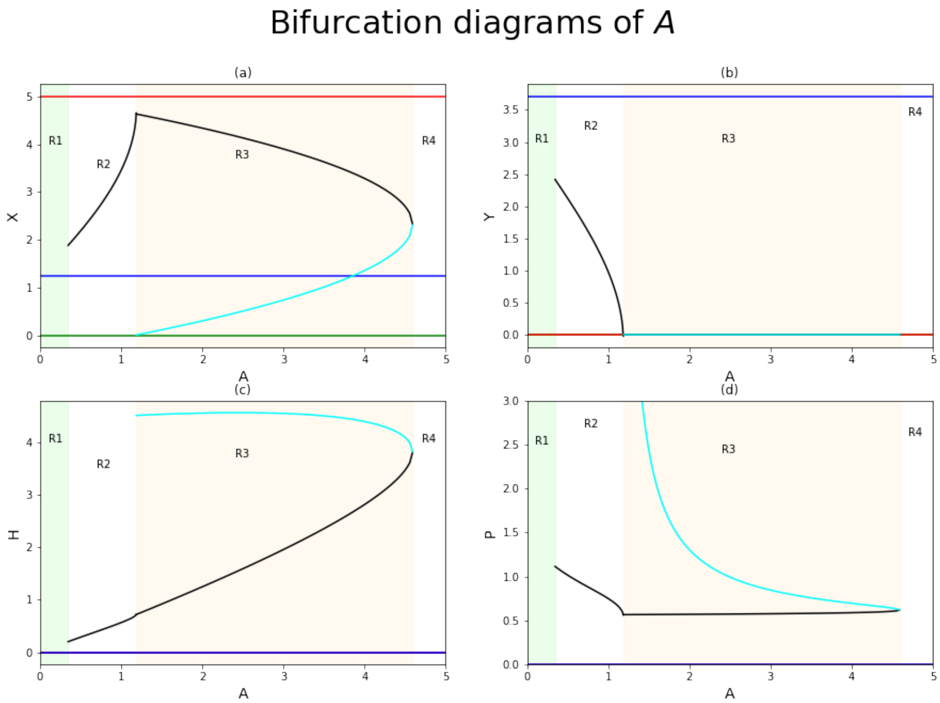



Figure 10: We have bifurcation diagrams where we vary $A$ in the region $A \in [0, 5]$ . The green,red,blue and cyan lines indicate the unstable equilibrium $\epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3$.The endemic equilibrium $\epsilon_4$ is stable in $R2$ where $A \in [0.346, 1.21]$. One disease free equilibrium (the positive root) equilibrium is stable in $R3$ where $A \in [1.21, 4.59]$ . The stable equilibrium in each $A$ range is indicated by a black line.The parameters used are $K = 5, \lambda = 0.04, \tau = 0.45$

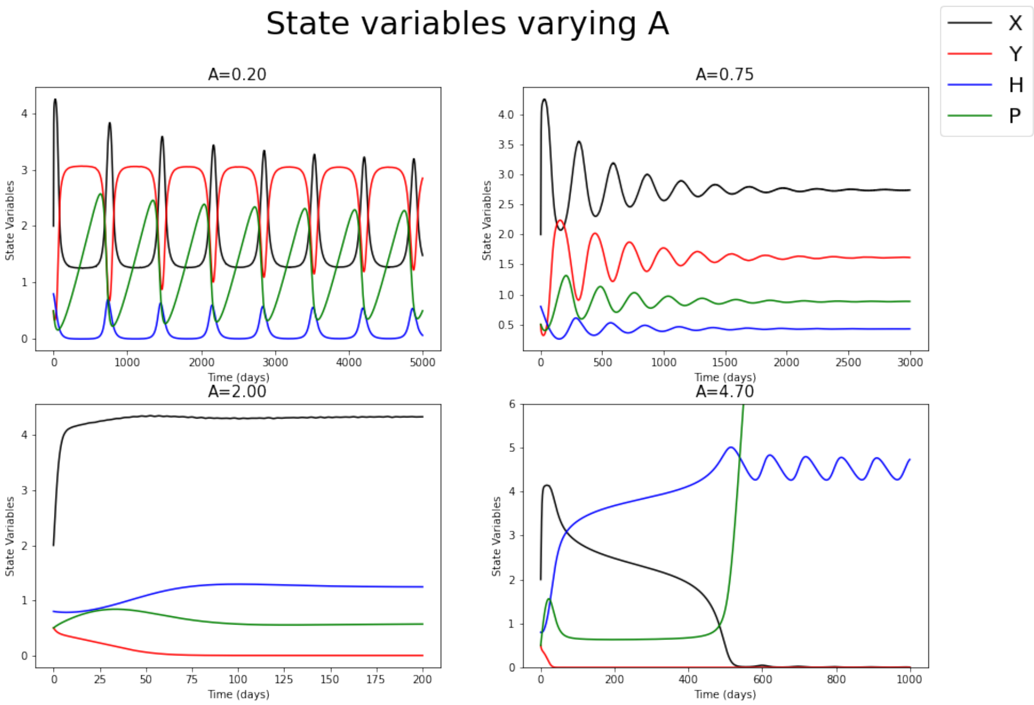



Figure 11: Time evolutions of the system in different zones from figure 10. The endemic equilibrium $\epsilon_4$ is unstable for $A = 0.2$ A periodic orbit exists. When $A = 0.75$ the endemic equilibrium is stable. When $A = 2$ the disease free steady state is stable. When $A = 4.7$ the price becomes unbounded this is because the cap on demand is too high so when demand increases to 4.7 the price increase with it.The parameters used are $K = 5, \lambda = 0.04, \tau = 0.45$.

In the region $A \in [0.345, 1.191]$ the endemic steady state is stable. The infection population reduces till eventually at $A = 1.21$ the infected population is zero and the endemic steady state swaps stability with the disease free steady state. Two equilibrium exist in the disease free steady state region. The stable equilibrium and the unstable equilibrium attract and meet at $A = 4.59$. In the disease free steady state the as the maximum demand increases the harvesting rate increases linearly.Increasing the harvesting rate results in more fish being caught and the so population of healthy fish therefore reduces. The increasing demand is met with increased harvesting rate and supply of fish so the price remains constant. This system remains stable until the maximum demand is increased so much that the its impossible to fulfill. The point where the disease free steady state becomes unstable is when $A = 4.59$. At this point the system becomes chaotic as seen in figure 11. The price becomes unbounded resulting in the harvesting effort increasing but the population of healthy fish collapses because of over-harvesting (Over-fishing).A change in the management practices of the fishery would be required in this scenario as having unbounded price of healthy fish will cause the whole fish market to collapse.

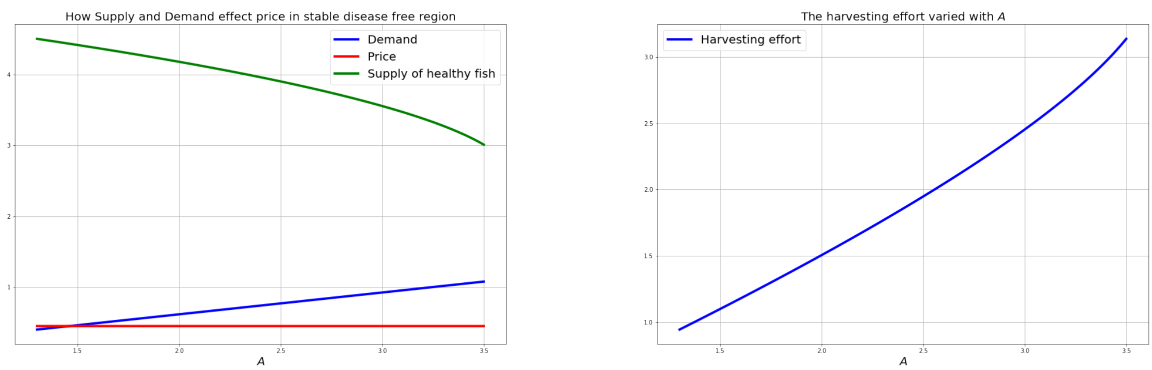



Figure 12: Demand plotted with the Healthy Fish Biomass ($X$) and Price in the stable disease free region where $A$ is varied (left side), the harvesting effort is plotted for varying $A$ (right side). Parameters as in bifurcation diagram of $A$ in figure 10

### 4.5 Analysing $\lambda, \tau, K$ and $A$ together

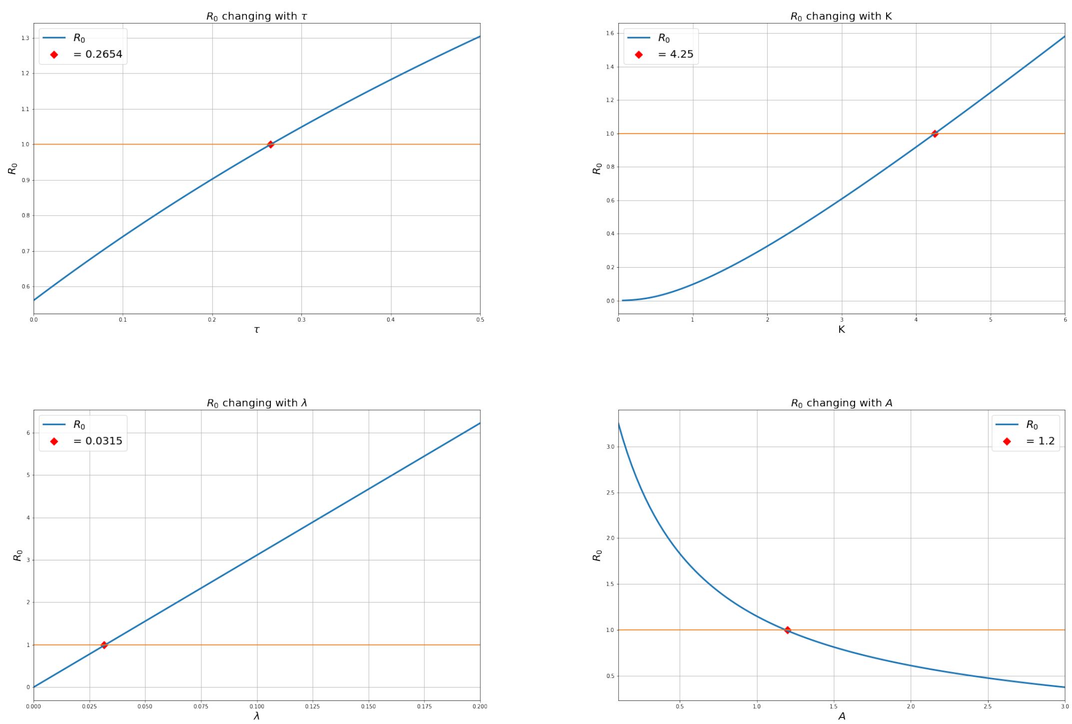



Figure 13: $R_0$ changing with $\tau, \lambda, A, K$, in each case one parameter is varied and the other parameters are $K = 5, \lambda = 0.04, \tau = 0.45, A = 0.9$

We see that the values of the parameters where $R_0 < 1$ correspond to the values of the parameters where the disease free steady state is stable and the population is disease free.
In figure 14 we have animations of the system that shows how the system varies over time while the parameters are varied . Our analysis of when the disease free stable point and the endemic fixed point are stable are supported by the animations.
Figure 14: https://github.com/seanwhite674/Math-Biology-Project/blob/main/README.md

## 5 Conclusion

When modelling fish populations, we must consider the economic and ecological factors effecting the population. Modelling a fish population using purely a biological or economic model doesn't provide an accurate depiction of the population dynamics. This paper utilises an updated bio-economic model from a previous study which introduced the concept of introducing a demand function and a varying price in a fish population model[10] . Using a bio-economic model allows us to examine how a fish population is effected by demand, taxation, supply, fish price ,disease and the environmental carrying capacity. The parameters investigated in this paper were the maximum demand ($A$), the transmission rate ($\lambda$) , the environmental carrying capacity ($K$) and the taxation imposed on caught fish ($\tau$).The main improvements this paper makes from the previous study [10] is that the demand function incorporates a saturated demand. From figure (6 and 8) we see that our new saturated demand improves the economics of our model. When the supply of healthy fish reduces either the price or the demand for healthy fish increases.The change to a saturated harvesting rate also improves the model as seen in figure (1). Taxation and a variable price only for healthy fish was also introduced into the model which improved the model's realism. Stability analysis on this improved model provided us with the stable ranges of different parameters for the disease free and endemic equilibrium. These were:

| Stability of the endemic and disease free fixed points | | | | |
|---|---|---|---|---|
| | $\tau$ | $K$ | $A$ | $\lambda$ |
| Endemic Stable fixed point | $\in [0.2654, 0.64]$ | $\in [4.25, 10[$ [ | $\in [0.346, 1.186]$ | $\in [0.029, 0.5]$ |
| Disease free Stable fixed point | $\in [0, 0.2654]$ | $\in [0, 4.25]$ | $\in [1.21, 4.59]$ | $\in [0, 0.029]$ |

From our bifurcation analysis we find that when taxation is less than 0.2654 ($\tau < 0.2654$) a disease-free population exists. When the maximum demand ($A$) increases past 1.21 metric tonnes a disease free population also exists. We see that when we create an environment where the fishers are incentivised to harvest by introducing a low tax rate and/or high demand the fishers harvesting rate increases and this high harvesting rate then doesn't allow disease to spread in the fish population. However it is not a straight-forward solution to disease since if you increase the maximum demand ($A$)to more than 4.59 metric tonnes the price of healthy fish becomes unbounded and the whole fish market collapses. This paper has developed a bio-economic model which successfully models how the fish population changes with demand. The major reason for changing management systems for fisheries previously has been as a result of over-fishing. By introducing demand into a bio-economic model this paper shows that demand can cause major change in a fish population (regime-shift) and is therefore another factor to account for in fishery management systems . This model improves a previous existing bio-economic model successfully however further research could be conducted into how harvesting practices not only are effected by demand and taxation but also by the fish species being harvested. This paper deals with just one fish population but adding different fish species into the model and seeing how these species interact could change harvesting patterns. In [10] a model is built where the harvesting pattern of fishers changes purely based off of which fish species provides more profit margin. Introducing demand,a variable price,infection and taxation into this model and seeing how the harvesting patterns change would be interesting and would provide useful information for fishery management.

# 6 Appendix

## 6.1 Fsolve function

Function used with Fsolve to find endemic fixed point.

```
def endemicfixed_point(u,K,lam,tau,A):
    X = u[0]
    Y = u[1]
    H = u[2]
    P = u[3]

    eq_X = r*(1-(X+Y)/K)-lam*Y-q1*H/(X+D1)
    eq_Y = lam*X - mu - q2*H/(Y+D2)
    eq_H = q1*(P-tau)*X/(X+D1) + q2*(p-tau)*Y/(Y+D2) - c
    eq_P = A/(1+B*P) - q1*X*H/(X+D1)
    return [eq_X, eq_Y, eq_H, eq_P]
```

## 6.2 Jacobian for endemic fixed point

The jacobian for calculating the stability of the endemic fixed point

```
def J(X,Y,H,P,K,lam,tau,A):
    y=np.zeros((4,4))
    y[0,0]= -r*X/K + q1*X*H/(X+D1)**2
    y[0,1]= -r*X/K - X*lam
    y[0,2]= -q1*X/(X+D1)
    y[0,3]= 0
    y[1,0]= lam*Y
    y[1,1]= q2*Y*H/(Y+D2)**2
    y[1,2]= - q2*Y/(Y+D2)
    y[1,3]= 0
    y[2,0]= phi1*H*((q1*(P-tau))/(D1+X)-(q1*X*(P-tau))/(D1+X)**2)
    y[2,1]= phi1*H*((q2*(p-tau))/(D2+Y)-(q2*Y*(p-tau))/(D2+Y)**2)
    y[2,2]= 0
    y[2,3]= phi1*q1*H*X/(X+D1)
    y[3,0]= P*phi2*((H*q1*X)/(X+D1)**2-H*q1/(D1+X))
    y[3,1]= 0
    y[3,2]= -phi2*q1*P*X/(X+D1)
    y[3,3]= -phi2*A*B*P/(1+B*P)**2
    return y
```

## 6.3 Disease free fixed point

The Jacobian for calculating the stability of the disdease free fixed point and the functions to find both disease free fixed points.

```
def J(X,Y,H,P,K,lam,tau,A):
    y=np.zeros((4,4))
    y[0,0]= -r*X/K + q1*X*H/(X+D1)**2
    y[0,1]= -r*X/K - X*lam
    y[0,2]= -q1*X/(X+D1)
    y[0,3]= 0
    y[1,0]= 0
    y[1,1]= (-H*q2)/D2+lam*X-mu
    y[1,2]= 0
    y[1,3]= 0
    y[2,0]= phi1*H*((q1*(P-tau))/(D1+X)-(q1*X*(P-tau))/(D1+X)**2)
    y[2,1]= phi1*H*((q2*(p-tau))/(D2+Y)-(q2*Y*(p-tau))/(D2+Y)**2)
    y[2,2]= 0
    y[2,3]= phi1*q1*H*X/(X+D1)
    y[3,0]= P*phi2*((H*q1*X)/(X+D1)**2-H*q1/(D1+X))
    y[3,1]= 0
    y[3,2]= -phi2*q1*P*X/(X+D1)
    y[3,3]= -phi2*A*B*P/(1+B*P)**2
    return y
def disfreefixed_point1(K,lam,tau,A):
    X=(K*q1*r + B*r*(c*(-D1+K)+K*q1*tau)-np.sqrt(r*(r*(B*c*(D1 + K) + K*q1 + B*K*q1*tau)**2 -
   4*A*K*q1*(q1 + B*(c + q1*tau)))))/(2*r*(q1 + B*(c + q1*tau)))
    Y=0
    H=r/q1*(1-X/K)*(X+D1)
    P=((A/(r*X*(1-X/K))-1)*1/B)
    return [X,Y,H,P]
```

```
disfreefixed_point1(K,lam,tau,A)
def disfreefixed_point2(K,lam,tau,A):
    X=(K*q1*r + B*r*(c*(-D1+K)+K*q1*tau)+np.sqrt(r*(r*(B*c*(D1 + K) + K*q1 + B*K*q1*tau)**2 -
   4*A*K*q1*(q1 + B*(c + q1*tau)))))/(2*r*(q1 + B*(c + q1*tau)))
    Y=0
    H=r/q1*(1-X/K)*(X+D1)
    P=((A/(r*X*(1-X/K))-1)*1/B)
    return [X,Y,H,P]
```

# 7 References


[1] Employment in fisheries and aquaculture.
https://www.fao.org/3/cc0461en/online/sofia/2022/fisheries-aquaculture-employment.html.

[2]What does the world eat? - Sustainable Fisheries
https://sustainablefisheries-uw.org/seafood-101/what-does-the-world-eat/#:~:text=Though%20fish%20compose%20a%20small,of%20the%20world's%20meat%20consumption

[3]World Economic Forum. which countries consume the most or least fish.
https://www.weforum.org/agenda/2022/11/chart-shows-countries-consume-fish-food-security/#:~:text=Global%20average%20consumption%20of%20fish,person%20per%20year%20on%20average

[4] Did You Know...? Marine Life / Ocean Facts... MarineBio Conservation Society
https://www.marinebio.org/creatures/facts/#:~:text=So%2C%20there%20are%20at%20least,%25%20of%2050%20million%20species

[5] Multispecies exploitation with evolutionary switching of harvesting strategies.
https://onlinelibrary.wiley.com/doi/full/10.1111/nrm.12013

[6] Green Policy Platform
https://www.greenpolicyplatform.org/sites/default/files/Chavez_Using_taxes_to_deter_illegal_fishing_TQ_systems.pdf

[7] A prey–predator fishery model with endogenous switching of harvesting strategy —
https://www.sciencedirect.com/science/article/pii/S0096300313001999

[8] Bioeconomic two predator-prey model of harvesting fishery - IOPscience
https://iopscience.iop.org/article/10.1088/1742-6596/1569/3/032003

[9] Predator-prey model of exploited fish population Arowana
https://iopscience.iop.org/article/10.1088/1755-1315/176/1/012037/pdf

[10] Bioeconomics fishery model in presence of infection: Sustainability and demand-price perspectives
https://www.sciencedirect.com/science/article/pii/S0096300321003155?ref=pdf_download&fr=RR-2&rr=870514872f9b56ed