CS 565 Sample Midterm

Your name: ____

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Problem 1 (Inductively Defined Propositions). Figure 1 gives three inference rules for establishing whether a number is odd or even. Using these rules, give a derivation proving the claim *Odd* 3.

$$\frac{}{\mathit{Even}(0)}\ (\textsc{EvenO}) \qquad \frac{\mathit{Even}\ n}{\mathit{Odd}\ (1+n)}\ (\textsc{OddEven}) \qquad \frac{\mathit{Odd}\ n}{\mathit{Even}\ (1+n)}\ (\textsc{EvenOdd})$$

Figure 1: An Exciting Set of Inference Rules!

Problem 2 (Big-Step Operational Semantics). Write down the final states produced by each of the following IMP expressions, or state that the program does not terminate. If the program does produce a final state, also provide the names of the reduction rules needed to reach that final state (you only need to list the names of repeated rules once). The names of big-step rules for Imp can be found on the pen and paper section of Homework 5.

- EMPTY, X := 0; Y := 1; Z := 2 $/\!\Rightarrow$ [X$\mapsto$ 0, Y$\mapsto$ 1, Z$\mapsto$ 2]

- [X$\mapsto$ 2], **if** (X $\leq$ 1)**then** Y := X + 3 **else** Z := 4 **end** $/\!\Rightarrow$ [X $\mapsto$ 2, Z $\mapsto$ 4]

- [X$\mapsto$ 0], **if** (X $\leq$ 1)**then** Y := X + 3 **else** Z := 4 **end** $/\!\Rightarrow$ [X $\mapsto$ 0, Y $\mapsto$ 3]

- [X$\mapsto$ 0], while (X $\leq$ 1)do Y := Y + 1 **end** $/\!\Rightarrow$

- [Z$\mapsto$ 0, Y $\mapsto$ 10], while (Z $\leq$ 3)do Y := Y + 2; Z := Z + 1 **end** $/\!\Rightarrow$ [Z$\mapsto$ 4, Y $\mapsto$ 18]

Suppose we have extended the lambda calculus with syntax for natural numbers:

$$\mathsf{t} ::= \ \mathsf{x} \mid \lambda\, \mathsf{x}.\ \mathsf{t} \mid \mathsf{t}\ \mathsf{t} \mid \mathbb{N} \mid \mathsf{t} + \mathsf{t}$$

where values are either $\lambda\,\mathsf{x}.\ \mathsf{t}$ or numeric constants $\mathbb{N}$ and the following additional reduction rules:

$$\frac{\mathsf{e}_1 \longrightarrow \mathsf{e}_3}{\mathsf{e}_1 + \mathsf{e}_2 \longrightarrow \mathsf{e}_3 + \mathsf{e}_2} \qquad \frac{\mathsf{n} \in \mathbb{N} \qquad \mathsf{e}_2 \longrightarrow \mathsf{e}_3}{\mathsf{n} + \mathsf{e}_2 \longrightarrow \mathsf{n} + \mathsf{e}_3} \qquad \frac{\mathsf{n} \in \mathbb{N} \qquad \mathsf{m} \in \mathbb{N}}{\mathsf{n} + \mathsf{m} \longrightarrow \mathsf{n} +_{\mathbb{N}} \mathsf{m}}$$

The ever-so-slightly larger $+_{\mathbb{N}}$ to the right of $\longrightarrow$ in the third rule is mathematical addition, e.g. $1 + 2 \longrightarrow 3$.

Problem 3 (CBV Semantics of the Lambda Calculus). Using the lambda calculus's call by value small-step semantics and the reduction rule above, show how to reduce the following lambda term to a normal form. $(\lambda\,\mathsf{x}\ \mathsf{y}.\ \mathsf{x} + \mathsf{y})\ (1 + 2)\ 3$

Problem 4 (CBN Semantics of the Lambda Calculus). Using the lambda calculus's call by name small-step semantics and the reduction rule above, show how to reduce the following lambda term to a normal form. $(\lambda\,\mathsf{x}\ \mathsf{y}.\ \mathsf{x} + \mathsf{y})\ 3\ ((\lambda\,\mathsf{x}.\ \mathsf{x})\ 1)$

Problem 5 (Semantics of the Lambda Calculus). Write down a lambda expression that requires more steps to reach a normal form under a call-by-value strategy than under a call by-name evaluation strategy.

Problem 6 (Semantics of the Lambda Calculus). Write down a lambda expression that requires more steps to reach a normal form under a call-by-name strategy than under a call by-value evaluation strategy.

Problem 7 (Semantics of the Lambda Calculus). Using the lambda calculus's call-by-value semantics and the reduction rule above, show how to reduce the following lambda term to a normal form.

$(\lambda\ \mathsf{m\ s\ z.\ s\ (m\ s\ z)})\ ((\lambda\ \mathsf{m\ n\ s\ z.\ m\ s\ (n\ s\ z)})\ (\lambda\ \mathsf{s\ z.\ s\ z})\ (\lambda\ \mathsf{s\ z.\ z}))\ (\lambda\ \mathsf{x.}\ 1 + \mathsf{x})\ 0$

Problem 8 (Semantics of the Lambda Calculus). Using the lambda calculus's call by name small-step semantics and the reduction rule above, show how to reduce the lambda term from above to a normal form.

$(\lambda\ \mathsf{m\ s\ z.\ s\ (m\ s\ z)})\ ((\lambda\ \mathsf{m\ n\ s\ z.\ m\ s\ (n\ s\ z)})\ (\lambda\ \mathsf{s\ z.\ s\ z})\ (\lambda\ \mathsf{s\ z.\ z}))\ (\lambda\ \mathsf{x.}\ 1 + \mathsf{x})\ 0$