CS 565 Sample Final

	Your name:
	April 24, 2022
Problem numbers.	1. Give two examples of values in the lambda calculus extended with natural
Problem natural n	2. Give two examples of <u>stuck expressions</u> in the lambda calculus extended with umbers.
with natu	3. Give two examples of reducible expressions in the lambda calculus extended a ral numbers, and show the result of taking a single step using the call-by-value strategy.

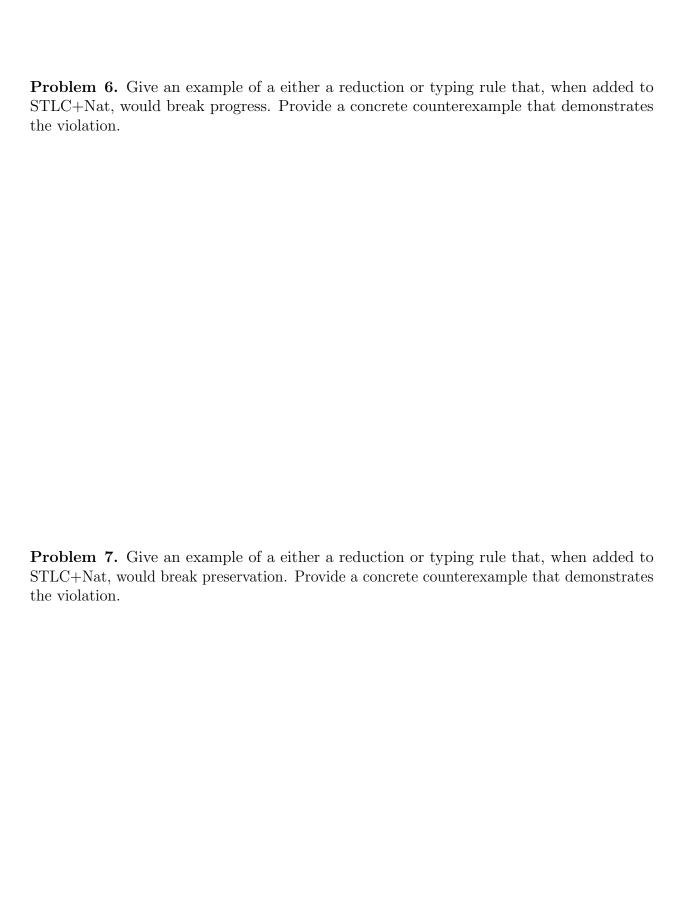
Problem 4. For an arbitrary postcondition and IMP program, is there *always* a precondition that would result in a valid Hoare triple? In other words, for a concrete Q and c is there always a way to fill in the first part of the following triple such that it is provable?

$$|-\{\} c \{Q\}$$

If your answer is "yes", is this assertion always $\underline{\text{unique}}$? If your answer is "no", give an example of a Q and c for which there is no such completion.

Problem 5. Provide typing contexts and annotations that make each of the following be well-typed simply typed lambda calculus with natural number (STLC+Nat) expressions, or state that no such typing context and annotations exist. Give the type for the entire term if it is well-typed.

- $\vdash (\lambda w : ... \lambda z : ... (w z z) + 1) (\lambda y : ... \lambda x : ... y + x) 2 :$
- $\vdash \lambda \times : \quad . \lambda \text{ w} : \quad . \times (1 + \text{w}) :$



Problem 8. Give the most general unifiers for the following sets of constraints, or state that no solution exists.

• $\{Z=Y \rightarrow Q, Q=Nat\}$

 $\bullet \ \, \{W{=}Q{\rightarrow}\; Q,\; Q{=}Nat,\; W{=}U,\; U{=}Nat\}$

• {}(the empty set of constraints)

Problem 9. Suppose we have types Q, U, and W with Q <: U and U <: W. Which of the following subtyping assertions are then true? Write true or false after each one.

•
$$(U \rightarrow Q) \rightarrow U <: (Q \rightarrow U) \rightarrow W$$
.

•
$$\{f: U \rightarrow U, y:U\} <: \{g: Q \rightarrow W, y:W\}.$$

Problem 10. Give the definition of polymorphic function composition \circ in System F. In Coq, the definition of this function is:

$$\label{eq:definition} \mbox{Definition comp } \{A \ B \ C\} \ (f:A \to B) \ (g:B \to C):A \to C \coloneqq \mbox{fun } a \Rightarrow g \ (f \ a).$$

Problem 11. Give an example of a System F term which has no analogue in the simply typed lambda calculus.