CS 565 Sample Final

Your name: ______________________________

April 24, 2022

Problem 1. Give two examples of <u>values</u> in the lambda calculus extended with natural numbers.

Problem 2. Give two examples of <u>stuck expressions</u> in the lambda calculus extended with natural numbers.

Problem 3. Give two examples of <u>reducible expressions</u> in the lambda calculus extended with natural numbers, and show the result of taking a <u>single</u> step using the call-by-value reduction strategy.

Problem 4. For an arbitrary postcondition and IMP program, is there *always* a precondition that would result in a valid Hoare triple? In other words, for a concrete Q and c is there always a way to fill in the first part of the following triple such that it is provable?

$$| - \{\ \}\ \mathsf{c}\ \{\mathsf{Q}\}$$

If your answer is "yes", is this assertion always <u>unique</u>? If your answer is "no", give an example of a Q and c for which there is no such completion.

Problem 5. Provide typing contexts and annotations that make each of the following be well-typed simply typed lambda calculus with natural number (STLC+Nat) expressions, or state that no such typing context and annotations exist. Give the type for the entire term if it is well-typed.

- $\vdash (\lambda\ \mathsf{w} : \quad .\ \lambda\ \mathsf{z} : \quad .\ (\mathsf{w\ z\ z}) + 1)\ (\lambda\ \mathsf{y} : \quad .\ \lambda\ \mathsf{x} : \quad .\ \mathsf{y} + \mathsf{x})\ 2 :$

- $\vdash \lambda\ \mathsf{x} : \quad .\ \lambda\ \mathsf{w} : \quad .\ \mathsf{x}\ (1 + \mathsf{w}) :$

Problem 6. Give an example of a either a reduction or typing rule that, when added to STLC+Nat, would break progress. Provide a concrete counterexample that demonstrates the violation.

Problem 7. Give an example of a either a reduction or typing rule that, when added to STLC+Nat, would break preservation. Provide a concrete counterexample that demonstrates the violation.

Problem 8. Give the most general unifiers for the following sets of constraints, or state that no solution exists.

- $\{\mathsf{Z}=\mathsf{Y}\rightarrow \mathsf{Q},\ \mathsf{Q}=\mathsf{Nat}\}$

- $\{\mathsf{W}=\mathsf{Q}\rightarrow \mathsf{Q},\ \mathsf{Q}=\mathsf{Nat},\ \mathsf{W}=\mathsf{U},\ \mathsf{U}=\mathsf{Nat}\}$

- $\{\}$(the empty set of constraints)

Problem 9. Suppose we have types Q, U, and W with Q <: U and U <: W. Which of the following subtyping assertions are then true? Write true or false after each one.

- (U→ Q)→ U <: (Q→ U)→ W.

- {q:Q, y:Q} <: {y:U, q:W}.

- {f: U → U, y:U} <: {g: Q → W, y:W}.

Problem 10. Give the definition of polymorphic function composition ∘ in System F. In Coq, the definition of this function is:

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Definition comp {A B C} (f : A → B) (g : B → C) : A → C :=
 fun a ⇒ g (f a).
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Problem 11. Give an example of a System F term which has no analogue in the simply typed lambda calculus.