CS 565 Spring 2022 Homework 4 (Axiomatic Semantics)

Your name: _____

March 14, 2022

Problem 1 (2 points). Paraprhase each of the following Hoare triples in English.

a. $\vdash \{\mathsf{True}\}\ \mathsf{c}\ \{\mathsf{X} = 5\}$

b. $\vdash \{100 < \mathsf{Z}\}\ \mathsf{c}\ \{\mathsf{Z} = 100\}$

c. $\vdash \{\mathsf{X} \neq \mathsf{Y}\}\ \mathsf{c}\ \{\mathsf{False}\}$

d. $\vdash \{\mathsf{True}\}\ \mathsf{c}\ \{(\mathsf{Z} = \mathsf{Y} \leftrightarrow \mathsf{Y} < \mathsf{X}) \wedge (\mathsf{Z} = \mathsf{X} \leftrightarrow \mathsf{X} \leq \mathsf{Y})\}$

Problem 2 (1 point). Here is the proof rule for conditionals we saw in class:

$$\frac{\vdash \{P \wedge \mathrm{b}\}\ \mathsf{c}_1\ \{Q\} \qquad \vdash \{P \wedge \neg \mathrm{b}\}\ \mathsf{c}_2\ \{Q\}}{\vdash \{P\}\ \textsf{if}\ \mathsf{b}\ \textsf{then}\ \mathsf{c}_1\ \textsf{else}\ \mathsf{c}_2\ \{Q\}} \qquad \text{(HOARE-IF-ALT)}$$

and here is an alternative:

$$\frac{\vdash \{P \wedge \mathrm{b}\}\ \mathsf{c}_1\ \{Q\} \qquad \vdash \{P \wedge \neg \mathrm{b}\}\ \mathsf{c}_2\ \{Q\}}{\vdash \{P \wedge \mathrm{b}\}\ \textsf{if}\ \mathsf{b}\ \textsf{then}\ \mathsf{c}_1\ \textsf{else}\ \mathsf{c}_2\ \{Q\}} \qquad \text{(HOARE-IF-ALT}_1\text{)}$$

If we replace HOARE-IF with HOARE-IF-ALT$_1$, would the resulting program logic be a) sound and b) complete? If not, explain why.

Problem 3 (1 point). Now consider this rule:

$$\frac{\vdash \{P \wedge \mathrm{b}\}\ \mathsf{c}_1\ \{Q\} \qquad \vdash \{P \wedge \neg \mathrm{b}\}\ \mathsf{c}_2\ \{\neg Q\}}{\vdash \{P \wedge \mathrm{b}\}\ \textsf{if}\ \mathsf{b}\ \textsf{then}\ \mathsf{c}_1\ \textsf{else}\ \mathsf{c}_2\ \{Q\}} \qquad \text{(HOARE-IF-ALT}_2\text{)}$$

If we instead replaced HOARE-IF with HOARE-IF-ALT$_2$, would the resulting program logic be a) sound and b) complete? If not, explain why.

Problem 4 (3 points). Give a (sound and complete) proof rule for the repeat c until b command from the third homework[1]

[1]. **Hint**: Use the rule for regular loops, Hoare-While, for inspiration.