Physica A 426 (2015) 16–24

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The role of persuasion power on the consensus formation

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HIGHLIGHTS

- An opinion dynamics model where each individual is identified by two parameters, namely, opinion and persuasion is introduced.
- The persuasion parameter is taken as an acceptability measure of the proposed arguments.
- The model has been applied to the Scottish referendum opinion poles data since 2011.

ARTICLE INFO

Article history:
Received 22 June 2014
Received in revised form 4 December 2014
Available online 20 January 2015

Keywords:
Opinion dynamics
Consensus
Sznajd-Weron model
Persuasion ability
Artificial societies
Scottish referendum

ABSTRACT

An opinion dynamics model which is based on a version of two dimensional Sznajd model is introduced. According to this model the dynamics is governed by the interactions between four agents which live on the corners of a plaquette and their neighbors. The distinctive feature of the model is that each individual is identified by two parameters, namely, opinion and persuasion ability. The united group may persuade the individuals living at the neighboring sites according to both the number and their persuasion ability. This form of the model is used to discuss opinion dynamics processes in societies where a campaign is conducted by the principle being united and putting forward arguments which are commonly accepted by the members of the society. It is seen that persuasion parameter plays the major role in the societies where a minority opinion gains ground to be the major opinion of the society. The model has been applied to the Scottish referendum opinion poles data since 2011. The model in its simplicity, predicts that the arguments of the minority opinion ("YES" votes) are more appealing despite the observed win of the "NO" votes. This result may be due to the abundance of the "NO" opinion supporters at the beginning of the campaign.

1. Introduction

Modeling opinion formation is important for both to understand social phenomena and to predict the effects of social elements during the time evolution of social events. Despite its importance, mathematical modeling of social phenomena only recently became possible by using the techniques imported from statistical physics and hence a new area of research has emerged under the general name of sociophysics. The break through in sociophysics came with the observation that in social phenomena, similar to that of statistical mechanics, systems can be formulated in terms of local interactions of the constituents, but the macroscopic changes are the result of collective phenomena rather than the effects of the individual entities. Modeling the social systems in terms of locally interacting constituents together with the techniques imported from statistical physics, and large computer resources made it possible to make ever more realistic models of social events with some prediction power [1].

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http://dx.doi.org/10.1016/j.physa.2015.01.040
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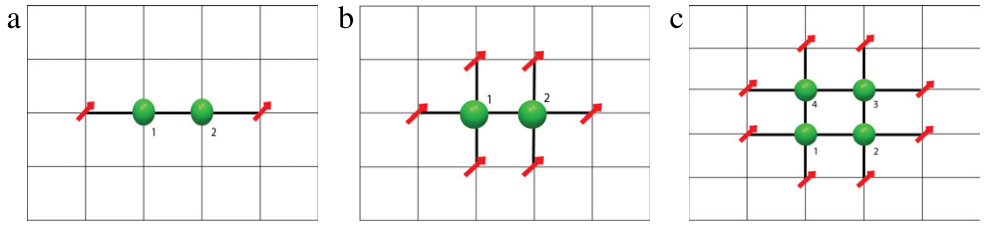


Fig. 1. Basic rules for SM (USDF) in one and two dimensional systems.

The very first models are very simple but equally powerful for describing the social interactions. One of the most acknowledged model of social interactions is Schelling model of segregation [2]. Since then many models on the specific area of dynamics of opinion formation, the Voter model [3–6] is one of the first opinion formation models. Similarly, opinion formation models the majority rule [7,8] and the bounded confidence [9] models are widely considered as representative of many real social phenomena. The interactions between the individuals in these models can be classified in two basic types: In the first type a randomly chosen individual interact with one of its neighbor. In the second, randomly chosen individual changes opinion under the influence of its surrounding neighbors. In this sense Sznajd-Weron model [10–12] has unique feature. Sznajd-Weron model assumes that when a group of individuals unite, they may influence their surrounding neighbors. The Sznajd model [10] is another sociophysics model which considers opinion as Ising type variables. In the original version of this model, which assumes a chain of individuals, a pair of neighboring sites sharing the same opinion may influence the opinions of their nearest neighbors [10,11,13].

The aim of this work is to introduce an opinion formation model in which individuals living on a 2-dimensional regular lattice are identified by two variables, a binary opinion value and a persuasion parameter. The persuasion parameter is directly related with the acceptability of an opinion by the community. In the proposed model opinion formation dynamics is based on a modified Sznajd-Weron type interactions: A group of individuals unite to convert the neighbors of opposite opinion according to a majority rule. This model can be used to study both equally divided societies and societies with some minority supporting one of the opinions.

Sznajd-Weron model [10] (SM) has attracted much attention since it is a model of binary spins with an update mechanism which is unique of its kind. The crucial difference of this model from the previously introduced models of opinion formation or from Ising model of statistical mechanics is that the information flows outward: If two neighboring sites share the same value, they unite and influence the surrounding neighbors. This update mechanism has close resemblance with the trade union idea, hence the model originally called USDF after the slogan "United we Stand, Divided we Fall". SM has been studied as a model of statistical mechanics [14–16].

As far as the phase transitions are concerned, the one dimensional SM shows smooth dependence on the initial conditions. As one type of opinion supporters increase, the winning possibility increases. For two dimensional case, a phase transition was observed as the effect of initial conditions on the time evolution of the system [11].

Apart from the theoretical investigations modified versions of SM has found many applications in marketing, finance, and politics [17].

The most successful application of SM is on the politics. Predictions on Brazilian and Indian elections has been possible by introducing more than two opinion states together with probabilistic external influences of the candidates on the individuals [18–20].

Marketing is an other area of research that SM has found applications. Here in this field, advertisement has been considered as an external field. Initial market distribution, Neighbor–neighbor interactions and advertisement are the modifications to the SM [21–23].

Attempts to extend simple one dimensional SM to two dimensional square lattices necessarily require some modifications or new assumptions which will extend the original model. On a square lattice, four individuals living at the sites which form smallest closed 2×2 surface (plaquette), Fig. 1 are chosen instead of a pair of individuals of one dimensional case. At this point there exist two major interaction algorithms [17].

- If four neighbors living at the corners of a 2×2 square, are sharing the same opinion, they persuade all 8 nearest neighbors to share the same opinion.
- If the chosen pairs living on the sides of the plaquette share the same opinion, they persuade their neighbors to follow orientation of the pair. This process is repeated for all possible nearest neighbor pairs of the chosen plaquette.

The two dimensional version of the Sznajd-Weron model has been extensively studied on simple square lattice [11,13,24–26], on triangular lattice [27], on three-dimensional cubic lattice [19] and also on the dilute [12] systems.

These rules leads to complete consensus as steady state. Moreover, a phase transition is observed—initial densities below 1/2 of up-spins lead to all spins down and densities above 1/2 to all spins up for large enough systems [11].

The proposed model is based on modified Sznajd-Weron type interaction with individuals carrying both opinion and persuasion values. Two new concepts are introduced in this model which to our opinion make the opinion dynamics studies more realistic.

The first new concept in this model is on the interaction dynamics which is different from those described above. The modification comes by considering when a group of individuals come together they neither necessarily always share the same opinion nor they can convinced all of their neighbors. Never the less if the majority share the same opinion and this opinion is more appealing to the society this group can influence at least some of their neighbors. This modified version of the two-dimensional Sznajd-Weron model may be considered as an outflow dynamics with majority rule.

In opinion dynamics models the initial densities of different opinions play a crucial role in the observed phase transition [11]. The second new concept of this model come into play at this point: the persuasion parameter. If a group of people claims to be the defender of the sensitive issues of the society in their propaganda or claims that the others are insensitive on these issues may expect to attract more supporters. This extra power is modeled by introducing a parameter which is called persuasion ability parameter. In the two dimensional extensions of the Sznajd model there is a critical density $(1/2)$ at which a phase transition is observed [11]. Introduction of the persuasion parameter enable one to control the transition point. In this model persuasion ability of one opinion can be larger than the other. If a group has larger persuasion parameter, this group can easily influence the others. In this form, smaller or larger persuasion parameter is an indication how easily one can convinced to the opposite opinion. Even if a minority group pursue an opinion which is acceptable by the majority of the population, hence carry larger persuasion parameter, minority may win at the end. Persuasion parameter acts as an external influence. Best example for this situation may be that a party which uses common values of the society in their propaganda may be in more advantageous position. The sensitivities may vary from society to society but always some sensitive issues such as national identity, religion, human rights, etc. can be detected by the propagandists to attract more supporters.

This model has virtue of studying both equally divided societies and societies with some minority supporting one of the opinions. In this work, both the majority rule and the persuasion power are unified to formulate the consensus dynamics. In this form, it is shown that even a minority with correct persuasion power can convince the others.

In the following section the model in which the proposed interaction rules are described in detail is introduced. The results and the effects of the proposed dynamics together with the persuasion parameter are discussed in Section 3. The work closes with the conclusion section where the main findings of the model is presented.

2. The model

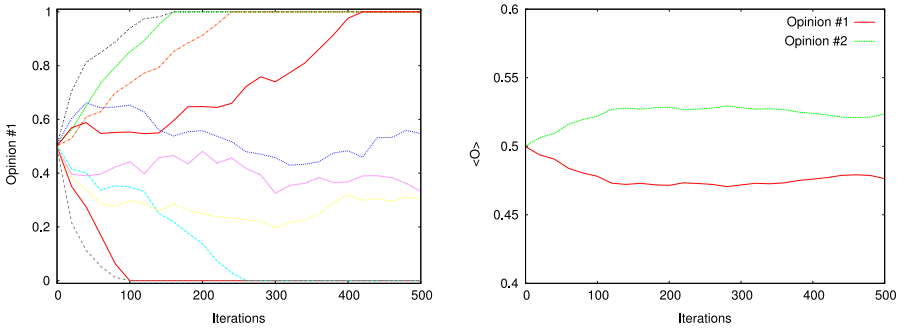
The system is built on a 2-dimensional regular square lattice of spatial extend L in each direction. Each individual interact with only its nearest neighbors. Periodic boundary condition sets the nearest neighbors of the individuals living at the extreme ends of the lattice. Each site on the above described lattice accommodate only one individual. In this model each individual is identified with two variables. The first variable, O, is the opinion of the individual. This variable takes integer values, $O = \pm 1$, to identify the pro and con individuals. The initial opinion values are set randomly for each individual from the set of values, $\{1, -1\}$ according to a given probability distribution. If the probability of any one of the individual having either one of the opinions is equal, $p_i = 0.5,\ i = 1, 2$, both opinions have equal number supporters. p_1 is the probability of any individual having the opinion #1 and p_2 is the probability of any individual having the opinion #2. If $p \neq 0.5$ a minority group supports one opinion while the majority group supports the other. $p_1 < 0.5$ indicate that only a minority is supporting opinion #1.

The second variable is designated to the persuasion ability, PA, of the individuals. The persuasion ability parameter, $PA_i,\ i = 1, 2$ varies in the range $1 < PA_i < 2$. The followers of one of the opinions may have some arguments strong enough that it may be at most twice as much convincing as the opponents. The persuasion parameter enters as a multiplicative factor into the interactions.

The interactions of the individuals are assumed to represent an outflow dynamics. Hence the model is based on a two dimensional version of the Sznajd model: four agents which live on the corners of a plaquette unite to persuade the individuals living on the neighboring sites. Here instead of commonly used alternative extensions (for a review see Ref. [17]) of the one dimensional Sznajd-Weron model, a model based on the majority rule is proposed: If the majority of the group living at the corners of the chosen plaquette supports an opinion, they may influence the neighbors. How influential the group may be is proportional with the persuasion ability parameters. Larger the persuasion ability parameter indicate that larger group of neighbors can be converted. Since both parties have their persuasion ability parameter a more convenient way is to consider the relative persuasion power $R = PA_2/PA_1$.

Initially the society is prepared either as an equally divided or one of the opinions is supported by majority. The group of individuals which live on the corners of a plaquette unite persuade the individuals living in the neighboring sites. The group of individuals, G, can unanimously support the same idea or some of the group members may choose the opposite opinion. Among these individuals, the number of supporters of the first opinion are G_1 while the number of supporters of the second opinion are G_2. The rule of the model can be stated as,

1. If the group is equally divided, they have no power to convince others.
2. If one of the opinion is dominantly supported by the group, their persuasion power increase proportional with the number of individuals supporting the dominant opinion, G_i, and decreases proportional with the persuasion power of the opposition, G_j.
3. The neighboring sites are occupied by the individuals supporting both opinions. Among the nearest neighbors, NN,
   (a) N_i sites are occupied by the individuals which share the same opinion with the majority of the group,
   (b) N_j sites are occupied by the supporters of the opposite opinion.



(a) Ten typical runs starting from a random initial configuration.

(b) Average taken over 500 independent runs.

Fig. 2. Opinion dynamics in an equally divided society.

In this situation only $G_j + N_j$ individuals must be persuaded in order to achieve a local consensus situation.

$$O_i = \begin{cases} O_i & \text{if} \sum_i O_i = 0 \\ O & \text{if } O \times \left[\sum_i PA_i \times O_i + \sum_j PA_j \times O_j \right] > 0 \end{cases} \tag{1}$$

where $O_i = O,\ O_j = O, i = 1 \ldots 4, j = 1 \ldots NN$ is the opinion of the group $O = \text{sign}\left[\sum_{i=1}^{4} O_i\right]$. The persuasion ability comes as a multiplication factor. If both opinions have the same persuasion ability, group of four can only persuade the same number of neighbors. If members of a group have more persuasion power, the individuals carrying this opinion can convert opposition members which may exceed their number. Moreover, an uneven distribution of the opinions in the society case exhibit rich phase space.

In the next section the results of having a persuasion parameter will be presented.

3. Results and discussions

In order to study the effects of the being united and supporting ideas that are more acceptable for the society, an artificial closed society consist of $N = 100\,000$ individuals are considered. Square lattice with periodic boundary conditions is taken as the underlying social network. In these studies two free parameters namely, the probability p_i of an individual carrying an opinion, O_i and the ratio of persuasion abilities R are considered as free parameters. The changes in the society is followed by studying changes in the percentage of the individuals with positive attitude ($O_1 = +1$) (pro individuals). In the simulation work, the averages are taken over 500 independent configurations.

In an voting campaign, conducted on an evenly divided society, the supporters of both opinions have equal opportunity to change the opinion of the opposition. Fig. 2 exhibits time evolution of the opinion formation dynamics during a simulation of an equally divided society. Fig. 2(a) shows ten typical runs of opinion formation in an equally divided society. The lines show the change in the percentage of the individuals with positive attitude ($O_1 = +1$) (pro individuals). Each run starts from an independent random configuration. Both opinion supporters have equally powerful arguments. Hence depending on the initial configuration the society may accept either one of the opinions. Some configurations reach consensus in the observation interval. Some other configurations require longer time to reach consensus. It can be seen from the figure (Fig. 2(a)) that at each run the percentage of the individuals with positive view exhibit very different path to consensus: Consensus on either one of the opinions are equally probable. The figure shows the first 500 iterations, since the model is a consensus model, longer runs end with unanimous agreement on either one of the opinion. When an average behavior of a society where both parties have the same persuasion ability and equal in numbers, it is not possible to predict which one of the opinions will be the winning one. In such societies reaching consensus on either one of the opinions are equally probable. Fig. 2(b), exhibit average behavior of the similar societies.

The configuration average of the individuals exhibit the tendency of the society towards either one of the opinions. At a given time slice different starting configurations may exhibit very different average value. The distribution of these average values are shown in Fig. 3. To show expected tendencies, 5000 runs have been started from random initial configurations with equal probability that each site may take either one of two opinion values. The system is followed 500 time steps. The sharp peak represent the distribution of the initial configurations. The figure indicate that after very short time majority of the starting configurations end up with unanimous agreement between the members of the society. After 500 iterations only a very small percentage of the sample societies remain undecided.

This situation can be completely changed if supporters of one of the opinions can come out with slightly more convincing arguments.

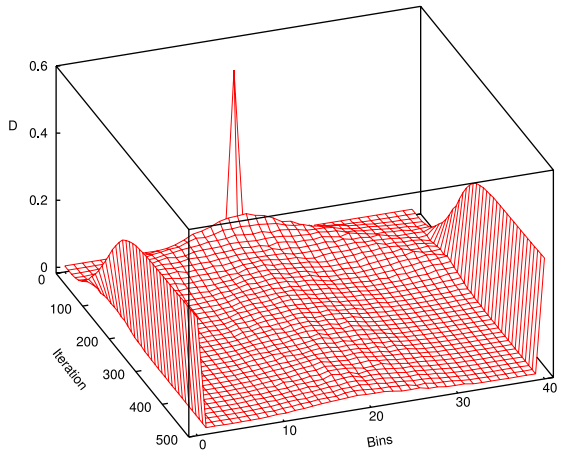


Fig. 3. Time evolution of the distribution of the configuration averaged opinion of the society.

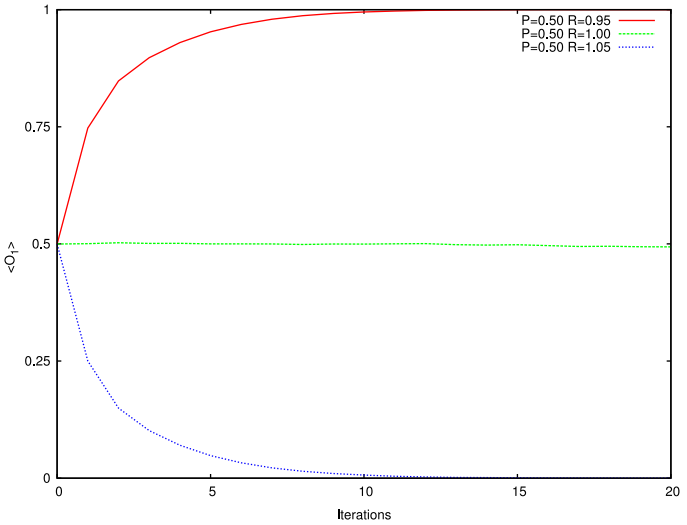


Fig. 4. Effects of the persuasion ability: $R = 0.95$, 1.00 and 1.05 corresponds to the cases where the opposition has less equal and greater persuasion ability. For $R \neq 1.0$.

Fig. 4 show the effect of the persuasion ability. In the study of effects of different persuasion abilities assigned to supporters of different opinions are studied in terms of the ratio of the persuasion values.

The ratio of the persuasion abilities, are denoted by R. The ratio, R, is varied between 0.95 and 1.05 in steps of 0.05. The persuasion variable of the first opinion (positive point of view) is kept as unity while the persuasion value of the second view (opposition) is varied. $R = 0.95$, 1.00 and 1.05 correspond to the cases where the opposition has less, equal and greater persuasion abilities respectively. In the first case, $R = 0.95$, all initial configurations lead to consensus on the positive point of view. When $R = 1$, both opinions have equal chance of winning. $R = 1.05$ corresponds to the case where opposition wins for each starting configuration.

Reaching consensus is also depend on the initial number of supporters. In societies which are constituted with unbiased and equal individuals, the opinion supported by the majority eventually spread out to all population. The chance of winning is proportional with the initial unbalanced distribution of the opinions. If one of the opinions is supported by even a small majority, the chance of winning at the end increases. Fig. 5, shows the effects of difference in both the number supporters and the persuasion parameter. The initial configurations are prepared with 55% of the population supporting the idea while only the rest of the population is in opposition. If both opinions have the same level of persuasion ability ($R = 1.0$), majority opinion wins and consensus is reaches for all initial configurations. If the persuasion ability of the minority is slightly higher than the majority opinion supporters, ($R = 1.05$) the number of majority opinion supporters decrease and consensus is reached on the opposition opinion. Fig. 5 shows the average percentage, $\langle O_1 \rangle$ of the supporters of the first opinion which is prepared initially as $O_1 = 55\%$. The situation where both opinions have the same level of persuasion ability, $R = 1.0$, even in 50 time steps the population end up with consensus on the first opinion. Even a small increase in the persuasion ability of the supporters of the second opinion immediately change the situation, the society unanimously accepts the second opinion.

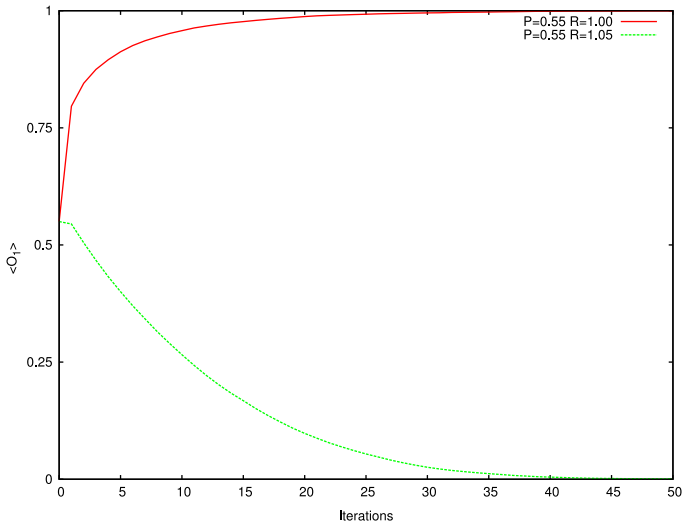


Fig. 5. Opinion formation process in an unevenly divided society. The change of the percentage of the number of supporters of opinion #1 with respect to time steps for persuasion ability ratio $R = 1.00$ (top) and $R = 1.05$ (bottom).

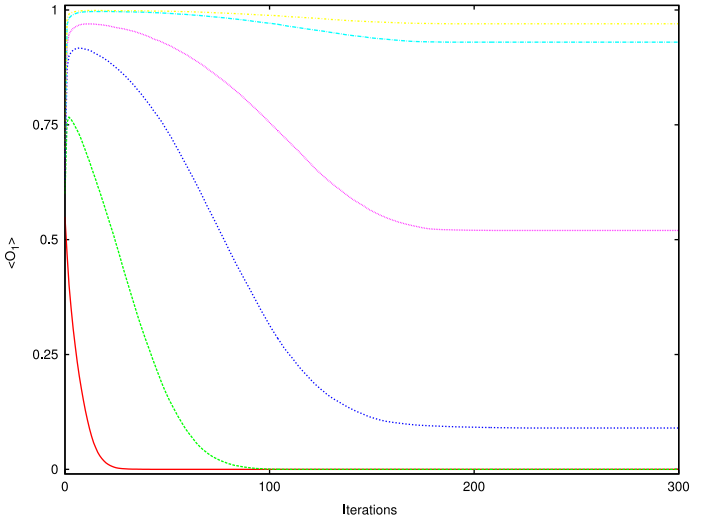


Fig. 6. The role of persuasion ability in consensus formation: for a fixed ratio of persuasion ability $R = 1.25$, increasing percentage of the initial supporters (0.6, 0.63, 0.65, 0.68 and 0.7 from bottom to top) increases the probability of reaching consensus on the opinion of the majority.

Another way of looking the same question is to study the opinion formation process among a group of people who have to vote between two proposition with slightly different appeal. In such an artificial society question may be that if minority has a more appealing opinion (larger persuasion power) for the general public, could minority persuade the majority supporters? What should be the optimum size and persuasion ratio of the minority in order to convince majority? Moreover, what is the relation between the persuasion ability and the population of the supporters. Fig. 6 designated to answer this question. The time evolution of the percentages of the supporters of the first opinion can be seen in this figure. Each line corresponds to a different percentage of the initial population as the supporters of the opinion #1. Since the society has a fixed size the population of the second opinion supporters changes accordingly. Initial configurations are prepared such that the population of the first opinion changed from 60% to 70%. The second opinion supporters on the other hand complemented in number according to the increase of the opponents but their persuasion ability kept constant. The persuading ability of the second opinion is considered to be 25% higher than the majority ($R = 1.25$).

Fig. 6 gives a clear indication that in an unbiased society a well argument campaign can be very successful even when it is conducted against overwhelming majority. The curves from bottom to top show the change of population supporting the first opinion with respect to simulation time (iterations). The bottom curve corresponds to an initial state which is prepared to contain 60% of the population supporting the first opinion. The following curves from bottom to top exhibit the effect of having 63%, 65%, 68% and 70% supporter population. The lowest curve which corresponds to initially 60% of the population support the first opinion, vanish within 100 iterations. Since this is an average value, very small number of

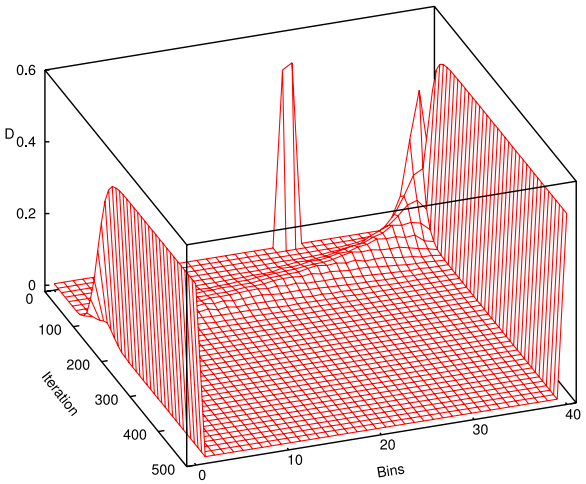


Fig. 7. Time evolution of the distribution of the configuration averaged opinion of the society in which 65% has positive point of view while minority has persuasion power 1.25 times higher than the majority.

initial configurations end with consensus on the first opinion. Increasing the percentage of the supporters of the opinion #1, in the initial configuration, start to increase the chance of having consensus on the first opinion. The percentage of such configurations increase with the increasing population of the supporters of the first opinion. As the initial population increases, the first opinion increases at the initial stages of the simulation due to abundance of like individuals. After initial iterations the dominant effect of the persuasion ability of the minority come into play and considerable number of initial configurations end with success of the minority opinion. This can be best understood by studying distribution of configuration averaged opinion of the society.

Fig. 7 shows time evolution of the distribution of opinions for a society of which 65% has positive point of view. The ratio of the persuasion abilities are given as $R = 1.25$. The figure shows the result of time development of 5000 statistically independent initial configurations. The peak at the beginning of time evolution shows the distribution of the initial configurations. Initially most of the configurations end with consensus on the positive point of view. This is due to large groups supporting the first opinion may persuade the individuals supporting the second idea. Immediately after that persuasion ability ratio come into play and some configurations end with the second opinion. In the mean time some configurations start to develop some regions of opposition groups due to high persuasion ability of the negative attitude pursuing minority. These groups develop consensus after some iterations.

Probability of having consensus decreases with the increasing majority. In Fig. 6, it can be seen that less and less configurations end with consensus on the minority opinion as the difference between majority and minority increases. After 70% limit no configuration ends with unanimous agreement on the minority opinion.

Scottish referendum [28] is one of the recent social events that is very important with political, sociological and economical consequences. Since its implications are very important for both United Kingdom (UK) and European Union (EU) it has been well studied, well documented and considerable welt of data are available. The referendum was on the question "Do you want an independent Scotland?" Each of the participants have to vote either yes or no for independent Scotland. Both the independence and continuation of the current position are very appealing arguments from different points of view.

Opinion pole results [28] are available starting from May 2011. The data contained information on three groups. The first group is the group supporting the independence idea, ("YES" votes). The second group is the group who preferred the current position of Scotland in UK ("NO" votes). "YES" votes fluctuates between 30% and 45% while the NO" votes fluctuated between 40% and 60% during a period of over 2 years. Almost 20% of the population seemed "undecided" or "declined" to declare their choices until the final weeks of the campaign. Among the people who openly declared their vote, the group supporting independence seemed in minority at the beginning. In time this group of voters have gained ground and at the final weeks race became neck to neck.

As an application area Scottish referendum is a very good laboratory for the proposed model. Since it is a referendum only two possible opinion values are sufficient and moreover despite arguments of both parties are very strong and appealing for the society one of them may be more acceptable by the public opinion. Despite the apparent similarities between the Scottish referendum process and proposed model, actual opinion dynamics is much more complicated for such a simple model to represent. Apart from the complicated interaction dynamics which leads the population towards the final result, a simple model cannot account the effects of public debates, radio and television broadcasts, outside pressures which are also important elements of the dynamics. The model in this simple form cannot also accommodate the undecided population.

Here the model is applied to the Scottish referendum opinion pole data without any modification. The public debates, and any other external influences are ignored. Also the undecided individuals are not accounted for. Only initial parameters p_1 and $p_2 = 1 - p_1$ and $R = PA_2/PA_1$ are set. Two different size lattices, namely $L = 100$ and $L = 400$ are used in order

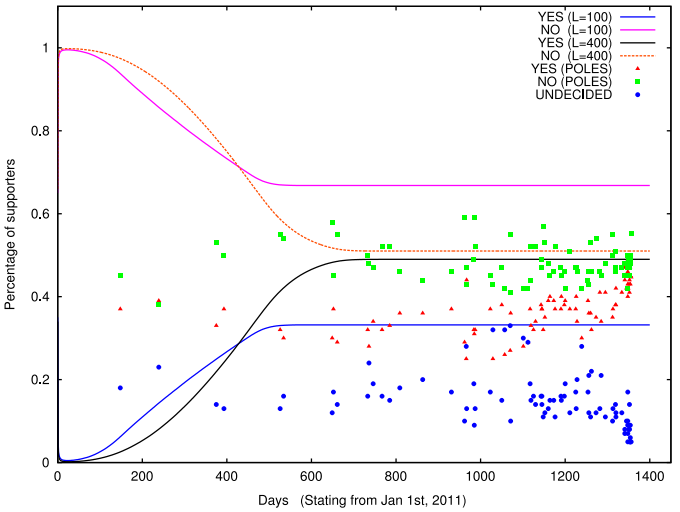


Fig. 8. Scottish referendum opinion poles data and model prediction for $p_1 = 0.35$ $p_2 = 0.65$ $R = 0.80$ (for lattice sizes, $L = 100$ and $L = 400$).

to discuss the finite size effects. Random initial configurations are created in accordance with the initial parameter set. For each size lattice, 500 sample configurations are created. For simulation the system, each sample configuration is followed 1400 time steps which roughly corresponds to the number of days since the beginning of 2011, from which opinion pole data available.

Initial parameters, yes–no percentages are determined from the public opinion pole data. R, the ratio of persuasion abilities is considered as the free parameter and tuned to obtain the final result. Fig. 8 show the opinion pole data and model predictions. Here this figure is obtained by taking initial "YES" and "NO" votes percentages as 35% and 65% respectively. With this initial state, the observed percentages at the end of the referendum is obtained by taking the persuasion ability ratio parameter as $R = 0.80$ which indicates that for Scottish people independence is more favorable argument but still there exist a considerable majority which prefers the current system.

This model is a consensus model hence the observed average behavior is in a delicate balance. Any small change in both percentages or the ratio of persuasion ability results in overwhelming dominance of one or the other opinions. The observed change is so abrupt that the transition seem to be a phase transition rather than a smooth change from one situation to the other. This is expected since Figs. 4 and 5 clearly show strong dependence of the results on the initial parameters. In fact, in a real societies abrupt changes are rather rare. This is a clear indication that the modifications for realistic models are necessary. These modifications must take into account the external influences that are clearly seen from the pole data. Moreover despite the fact that each individual must only choose one of the two alternatives, the opinions cannot be discrete must change continuously from one end to other. This smooth changes will slow down the possibility of the observed abrupt changes.

4. Conclusions

In democratic societies issues of very high importance are resolved through referendums. In referendums the voters have two choices "YES" or "NO". For the unbiased general public their "YES" or "NO" votes are effected by two parameters. The first of these parameters is related with the answer to the question: What is the opinion of the majority? The second parameter is related with the question: Which side of the argument is more appealing for my personal well being?

In this work the aim has been to study the relation between the distribution of different opinions and the comparative advantage or appeal of the arguments on the opinion formation process of the individuals using a simple model.

Opinion formation dynamics is modeled by using a modified version of two dimensional Sznajd-Weron model with an extra parameter which is called the persuasion ability. This parameter enables one to make realistic models of the social systems since all social interactions are influenced by the common values of the society. If both parties have the same persuasion ability, after the start of the simulation one or the other opinion wins and consensus is reached. In fact, Sznajd model is a consensus model and hence this feature is inherited from the underlying model. Never the less, it is impossible to predict the outcome of the interactions from the initial configuration. If both parties have the same tools, any one of the opinions have equal chance to win.

If one of the opinions are supported by less than the half of the population majority opinion wins for every initial configuration. This situation is a trivial case as far as the dynamics of the opinion formation is concerned. Persuasion ability parameter makes it possible to discuss non-trivial situations where one of the opinions are supported by less than the half of the population.

In this work it is shown that a minority opinion can win if the arguments are acceptable by the members of the society.

The model is also applied to one of the recent referendums, namely Scottish independence referendum. Despite its simplicity the model is sufficient to put forward an augment: According to the proposed model independence is more appealing for the general public. The win of the "NO" votes are related with the number of supporters being in majority at the initial stages of the campaign. Surely such a complicated social event cannot be modeled without taking external parameters, debates and global relations. Never the less even this simple model gave an indication about the dynamics of a society who can freely make their choices.

Since the majority of opinion formation studies are based on equally divided societies, a study considering societies with minority opinion is rare and interesting to best of our knowledge.

In a further study the effects of public debates, external influences and connectivity issues among the individuals will be added into the model.

Acknowledgments

We thank one of the referees for suggesting the application of the model to the Scottish referendum. This work is partially supported under the project number 13H4343002 by Ankara University Scientific Research Coordination Unit.

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