# Antiferromagnetic Ising Model in Hierarchical Networks

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# Introduction

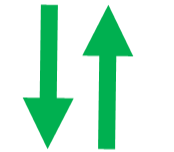- Antiferromagnetic Ising model

  ➢

$$E = -J \sum S_i S_j \ , \quad J < 0$$

$$\boldsymbol{E = -J > 0}$$

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# Introduction

- Antiferromagnetic Ising model

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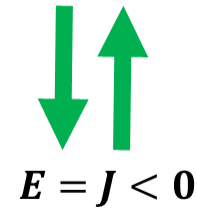
$$E = -J > 0 \qquad E = J < 0$$

  ➢ glassy dynamics

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- Antiferromagnetic Ising model

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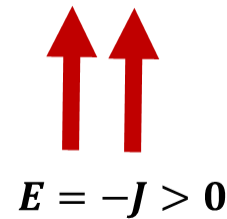
  ➢ glassy dynamics

- Geometric frustration
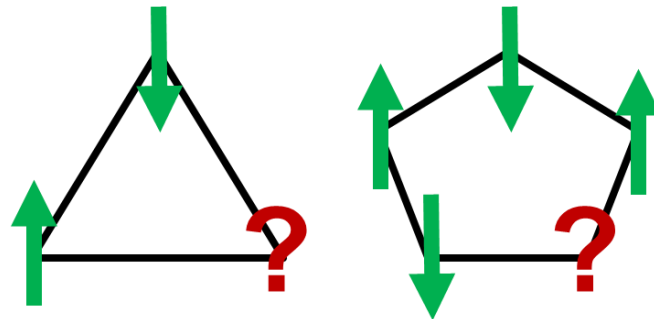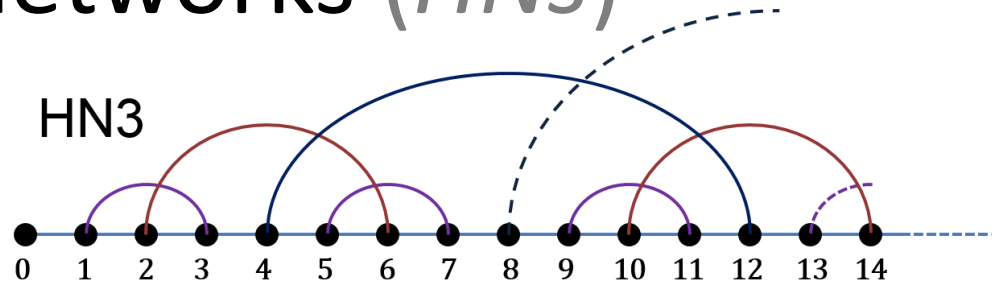
# Research Questions

- Phase transitions?

  - Equilibrium/non-equilibrium transition?
  - Spin glass phase?

- Glassy relaxation?

- Influence of geometry?

- Difference to mean-field models?

# Hierarchical networks (*HNs*)

- HN3:
  - degree 3
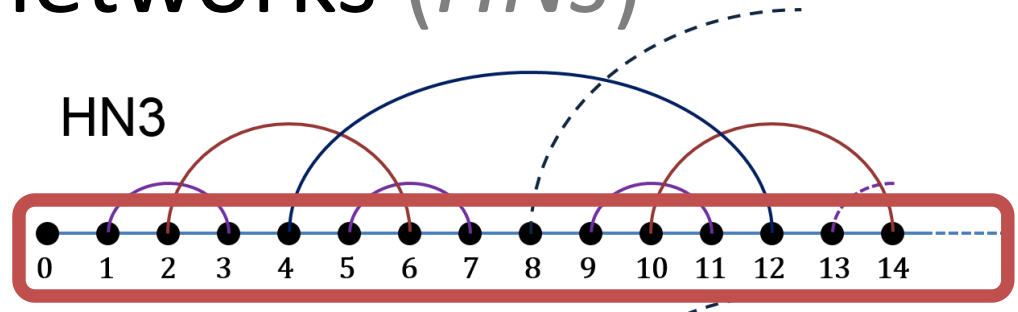
- HN5:
  - average degree 5

- HNNP:
  - average degree 4
  - nonplanar

- HN6
  - average degree 6
  - nonplanar



HN3

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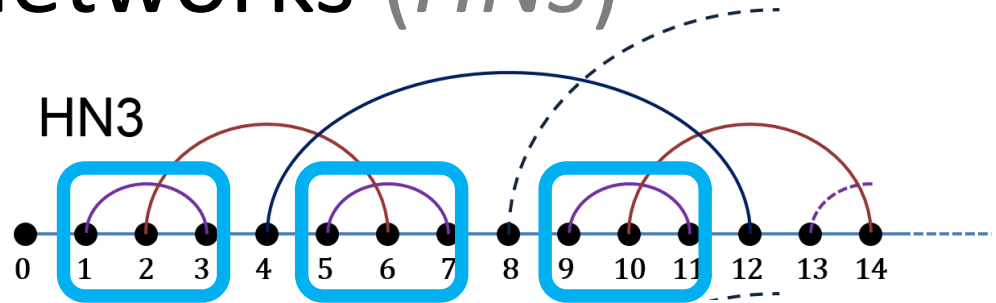
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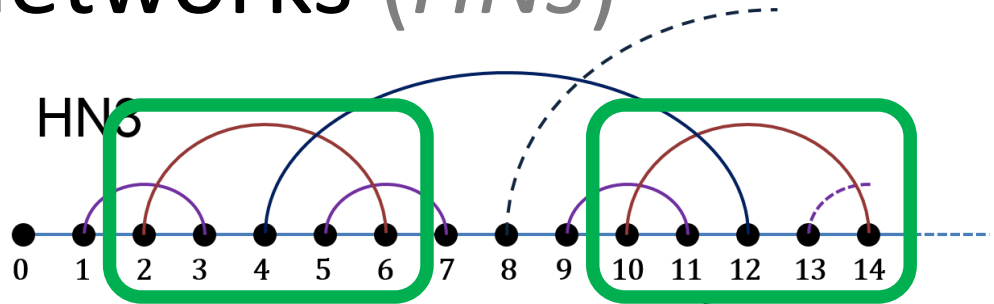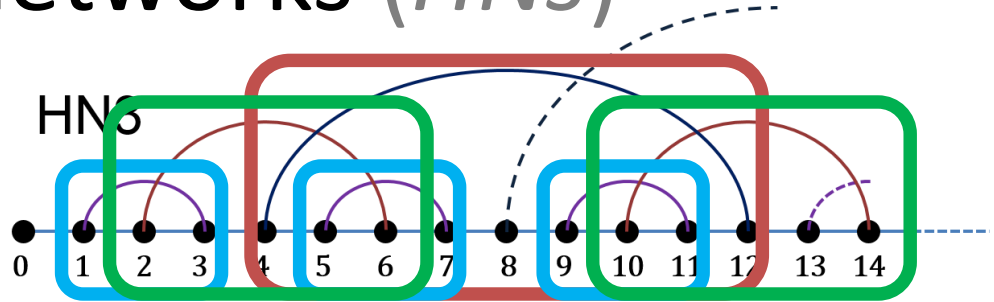  - nonplanar

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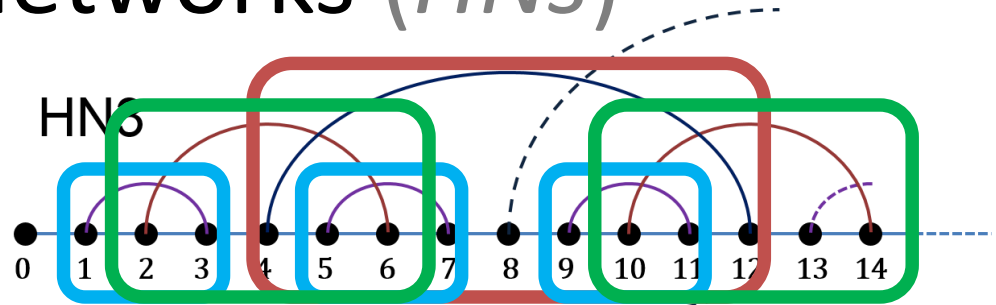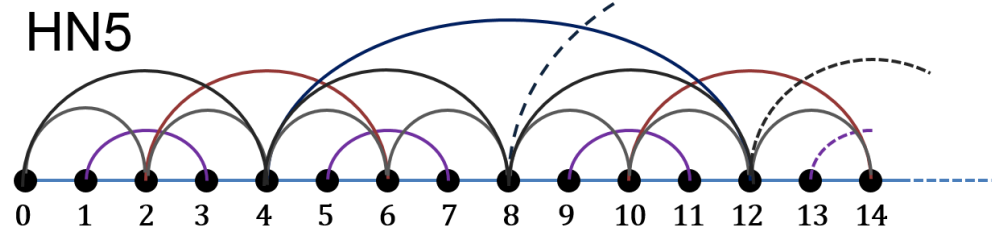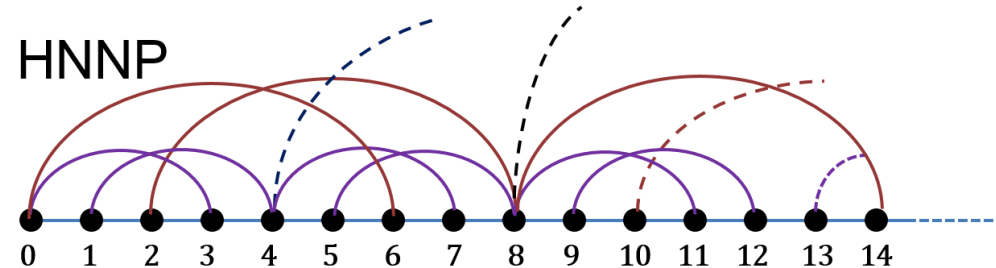# Hierarchical networks (*HNs*)

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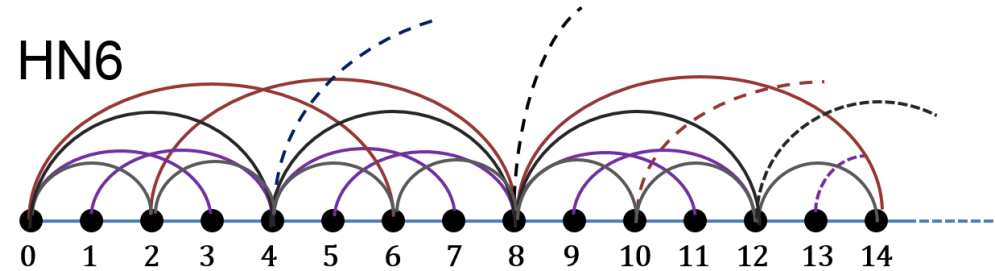
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# Why Hierarchical Networks (HNs)?

- Exactly solvable by **R**enormalization **G**roup (RG)

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    - Mean-Field ➡ HNs ➡ Regular lattice

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- Different structures among HNs
  - Different degrees: 3, 4, 5, 6
  - Planar vs non-planar

# Methods

- **W**ang-**L**andau Sampling (WL)
  - direct access to *Density of States*
  - partition function ➡ equilibrium quantities

- **S**imulated **A**nnealing (SA)
  - probe dynamical behaviors
  - Glassy relaxation

- **R**enormalization **G**roup (RG)
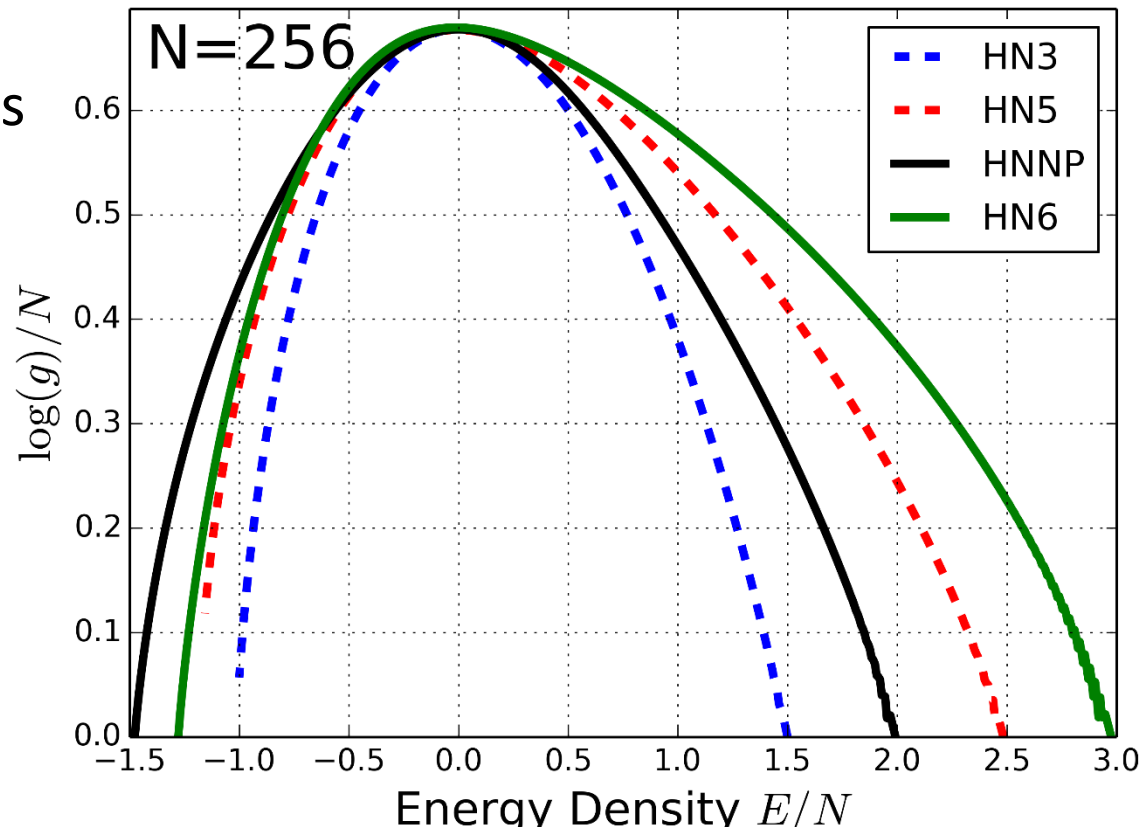  - Exact solutions in the thermodynamic limit

# Density of States (WL)

- <u>Planar</u>: HN3, HN5
  Degenerate ground states

- <u>Non-planar</u>: HNNP, HN6
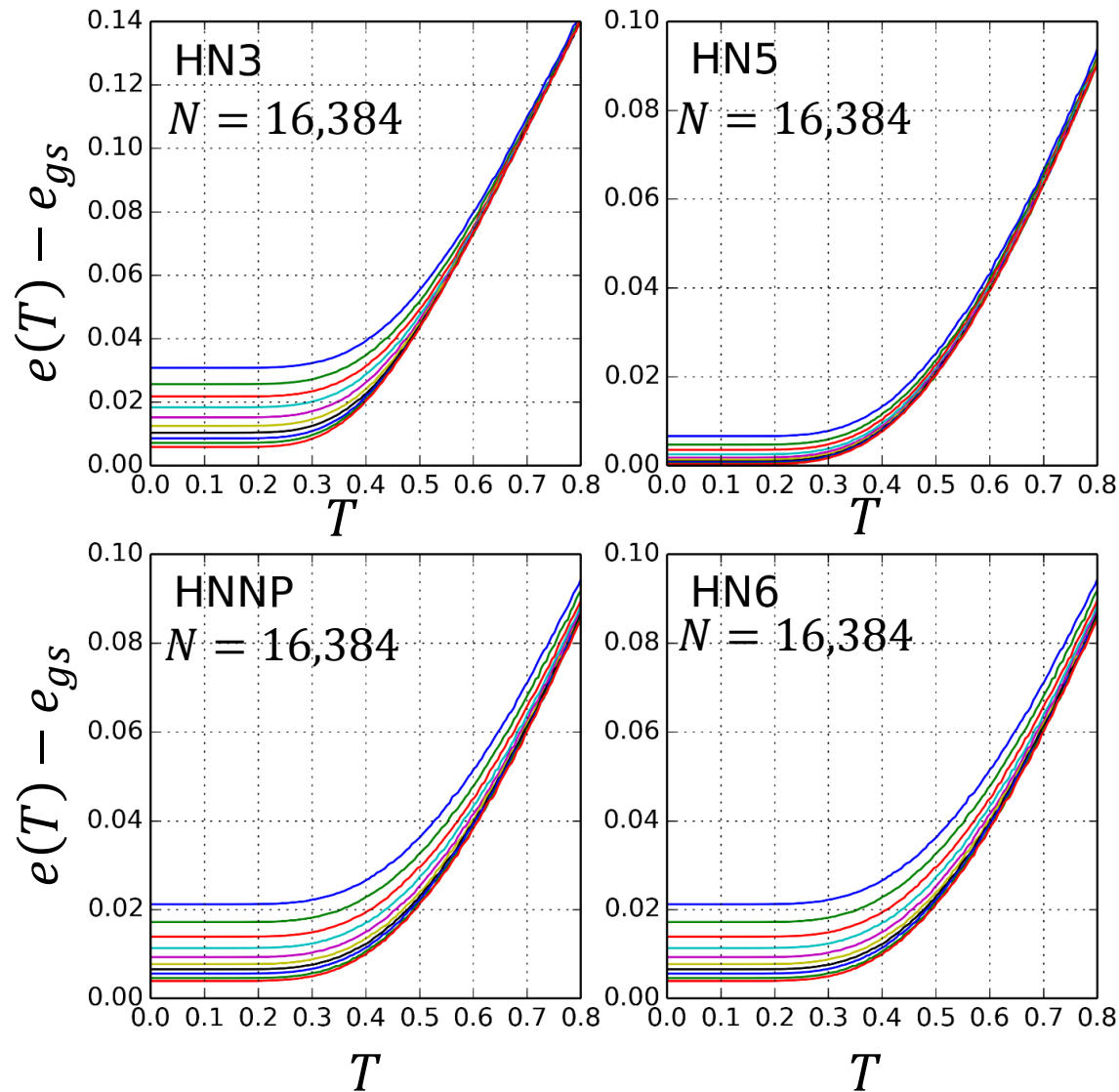  Unique ground states
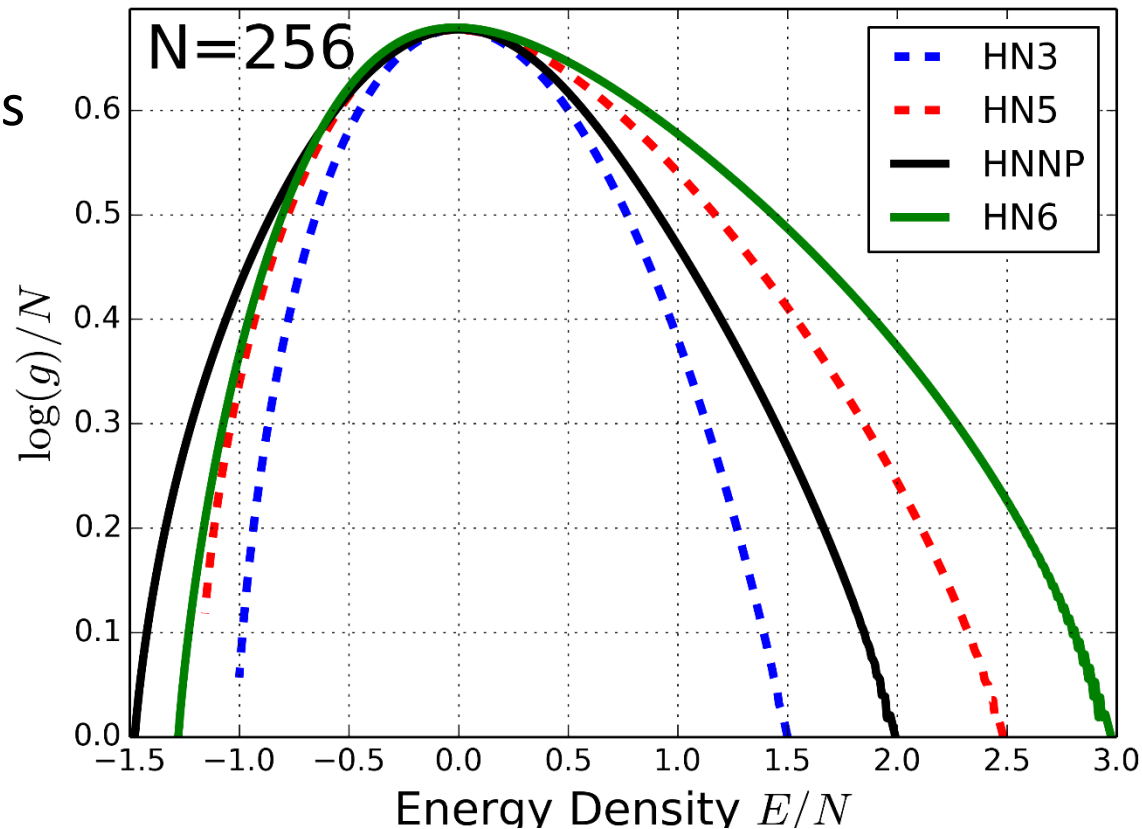
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- <u>Planar</u>: HN3, HN5
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  Unique ground states

- Reference of **SA** & **RG**

- Wang-Landau fails
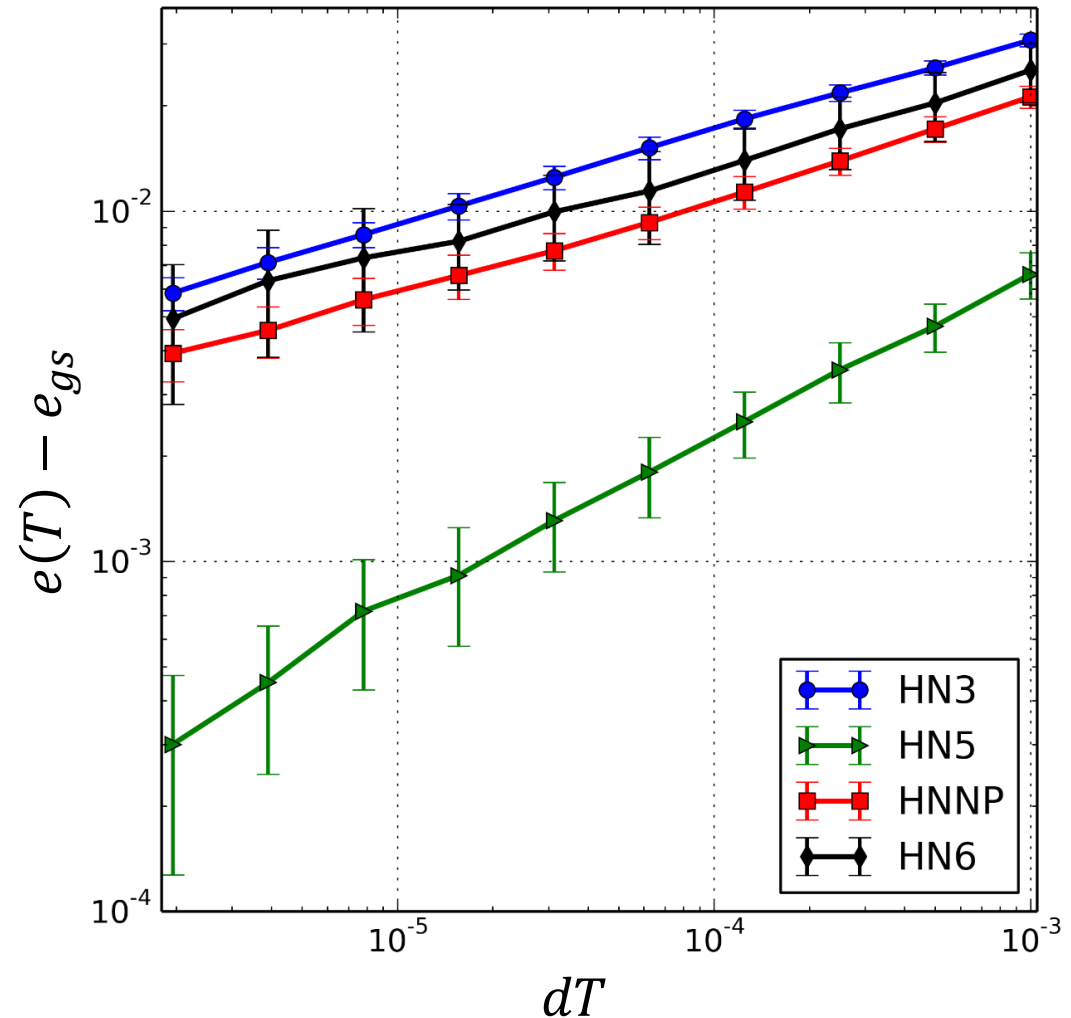  - $N > 1024$
  - Geometric frustration?

# Glassy relaxation (SA)

- $x$ axis: $T$

- $y$ axis: $e(T) - e_{gs}$

- Annealing schedules:

$$\frac{dT}{dt} = \frac{10^{-3}}{1}, \frac{10^{-3}}{2}, \cdots, \frac{10^{-3}}{512}$$

- Out of Equilibrium at low $T$

- Extremely slow relaxation at low $T$
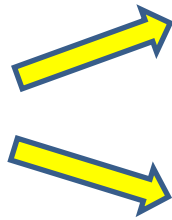
# Power-law relaxation (SA)

- Power-law relaxation

- HN3, HNNP, HN6:
  - Slope = ~0.27

- HN5 may equilibrate gradually

# Spin glass transition (RG)

- Renormalized interaction strength $J$

■ <u>**Recursive equations**</u>

Numerical solution

Analytical solution

- Planar: HN3, HN5
  - stable fixed-point solution
  - no phase transition

McKay and Berker, *Phys. Rev. Lett.* **48**, 11 (1982)

# Spin glass transition (RG)

- Non-Planar: HNNP, HN6
  - partially stable fixed-point solution
  - possible spin glass transition

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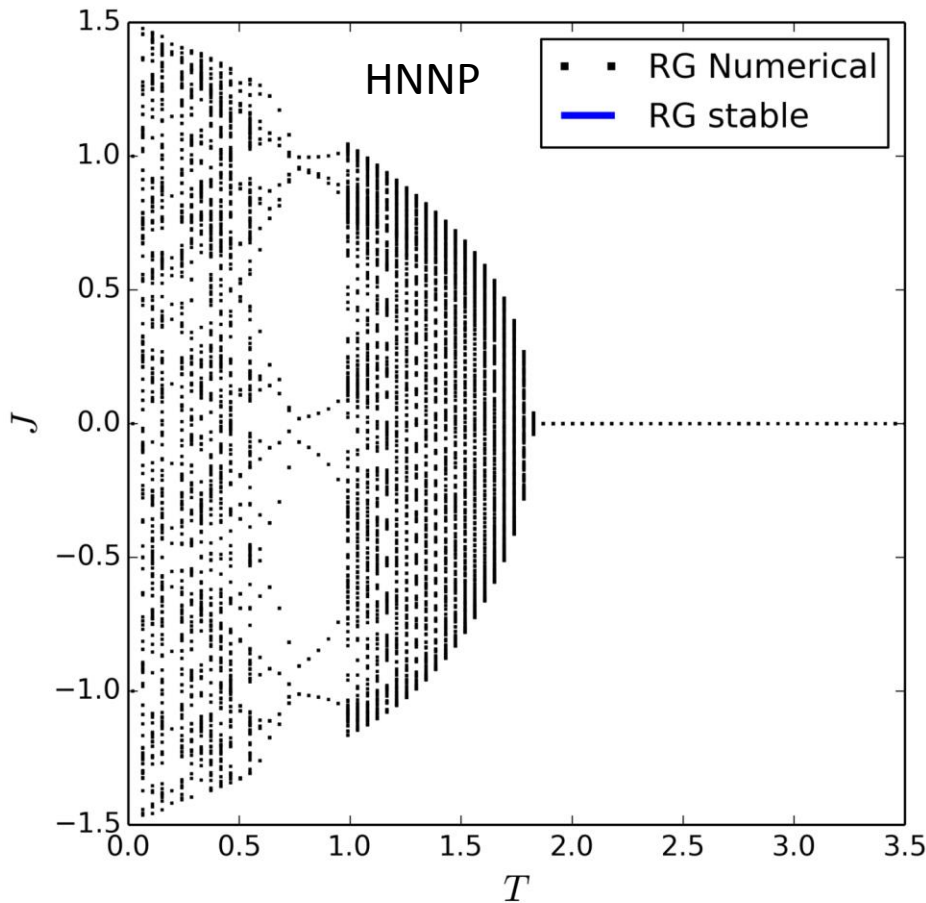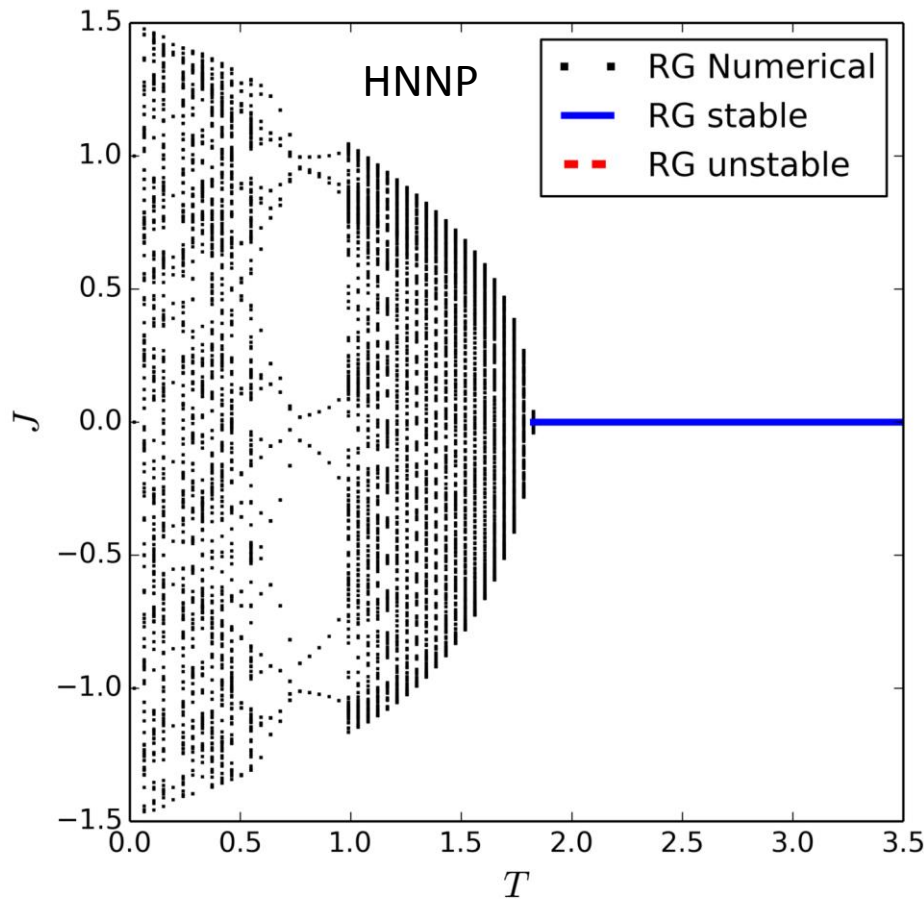  - possible spin glass transition

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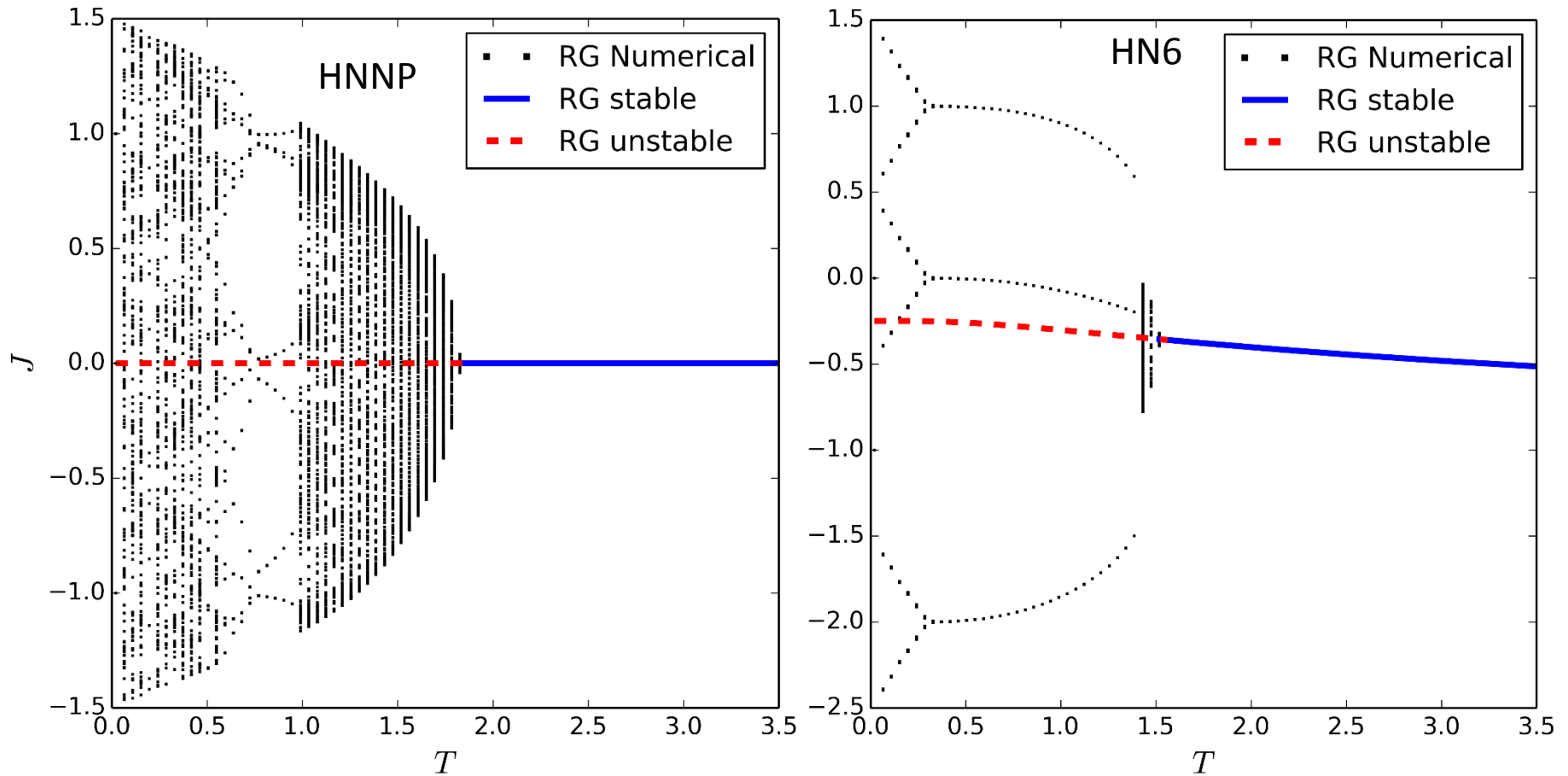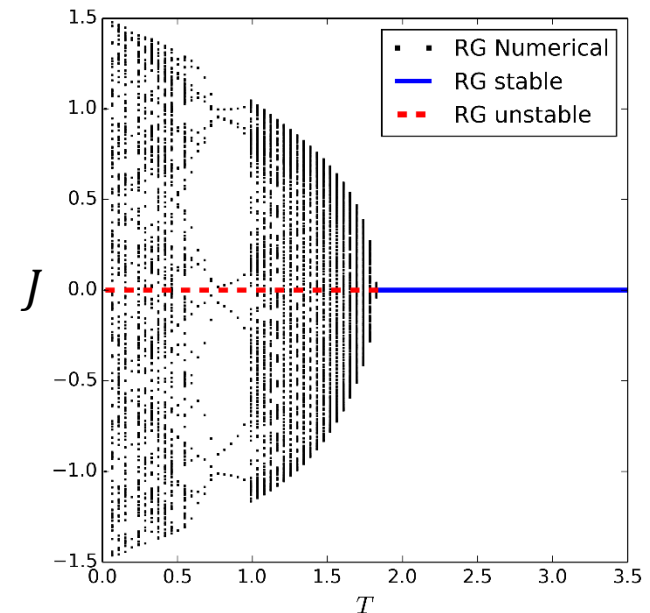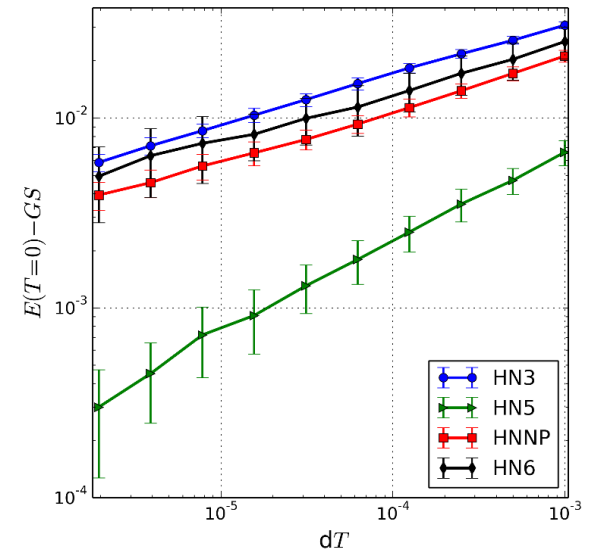  - possible spin glass transition



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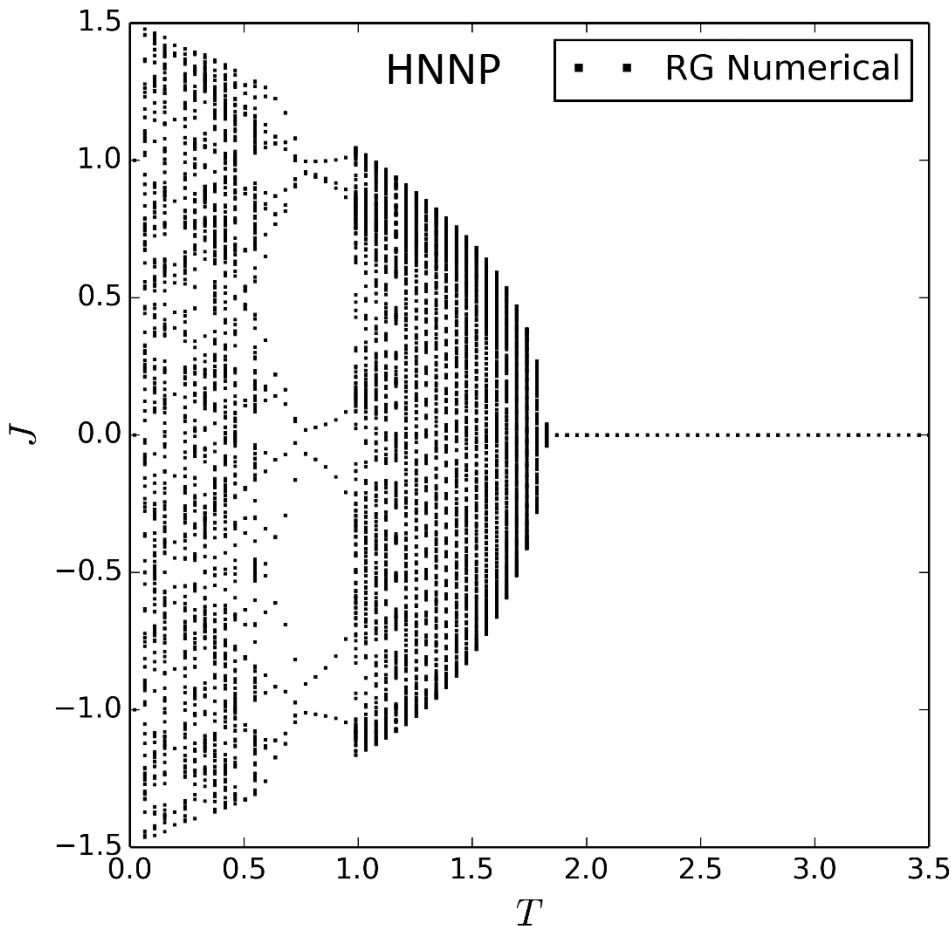
# Summary & future work

- Out of equilibrium at low $T$

- Power-law relaxation

- Spin glass transition

- Future work
  - reconstruct partition function
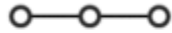  - $C_v$, $m$, $\chi$ using RG
  - Spin glass ⟺ geometry?

# Hierarchical networks (*HNs*)

- HN3:
  - degree 3

    $N = 2^{\mathbf{1}} + 1$   o——o——o

# Outline

- Introduction
  - ➤ Model, research questions
- Hierarchical Networks (HNs)
  - ➤ structure & why HNs

- Methods:
  - ➤ Wang-Landau, simulated annealing
  - ➤ Renormalization Group

- Results
- Future work