Antiferromagnetic Ising Model in Hierarchical Networks

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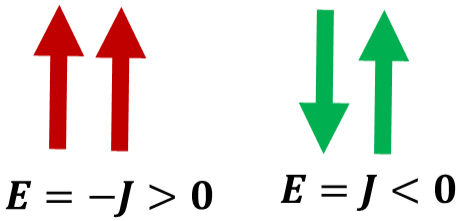
Outline

- Introduction
  - Model, research questions
- Hierarchical Networks (HNs)
  - structure & why HNs
- Methods:
  - Wang-Landau, simulated annealing
  - Renormalization Group
- Results
- Future work

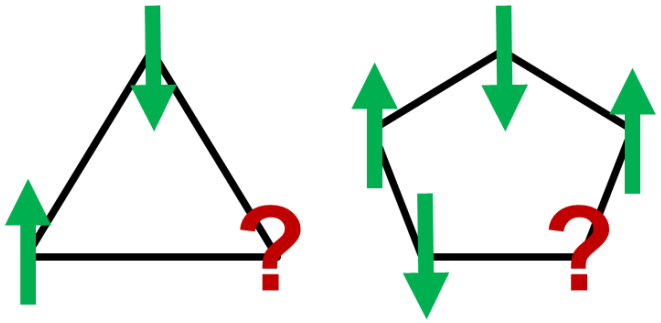
Introduction

- Antiferromagnetic Ising model
  - $$E = -J \sum S_i S_j \ , J < 0$$



  - glassy dynamics

- Geometric frustration

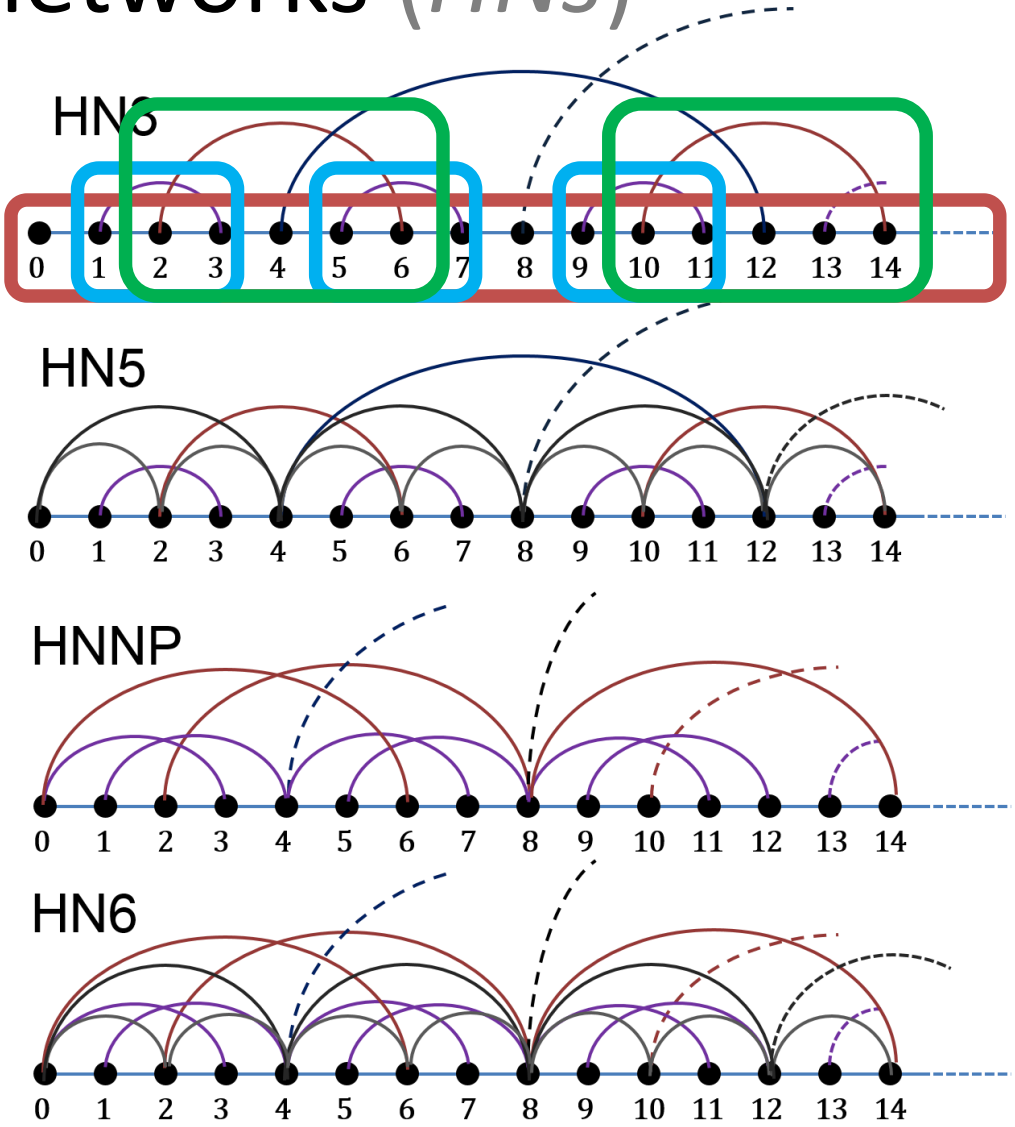


Research Questions

- Phase transitions?
  - Equilibrium/non-equilibrium transition?
  - Spin glass phase?
- Glassy relaxation?
- Influence of geometry?
- Difference to mean-field models?

Hierarchical networks (*HNs*)

- HN3:
  - degree 3
- HN5:
  - average degree 5
- HNNP:
  - average degree 4
  - nonplanar
- HN6
  - average degree 6
  - nonplanar



Why Hierarchical Networks (HNs)?

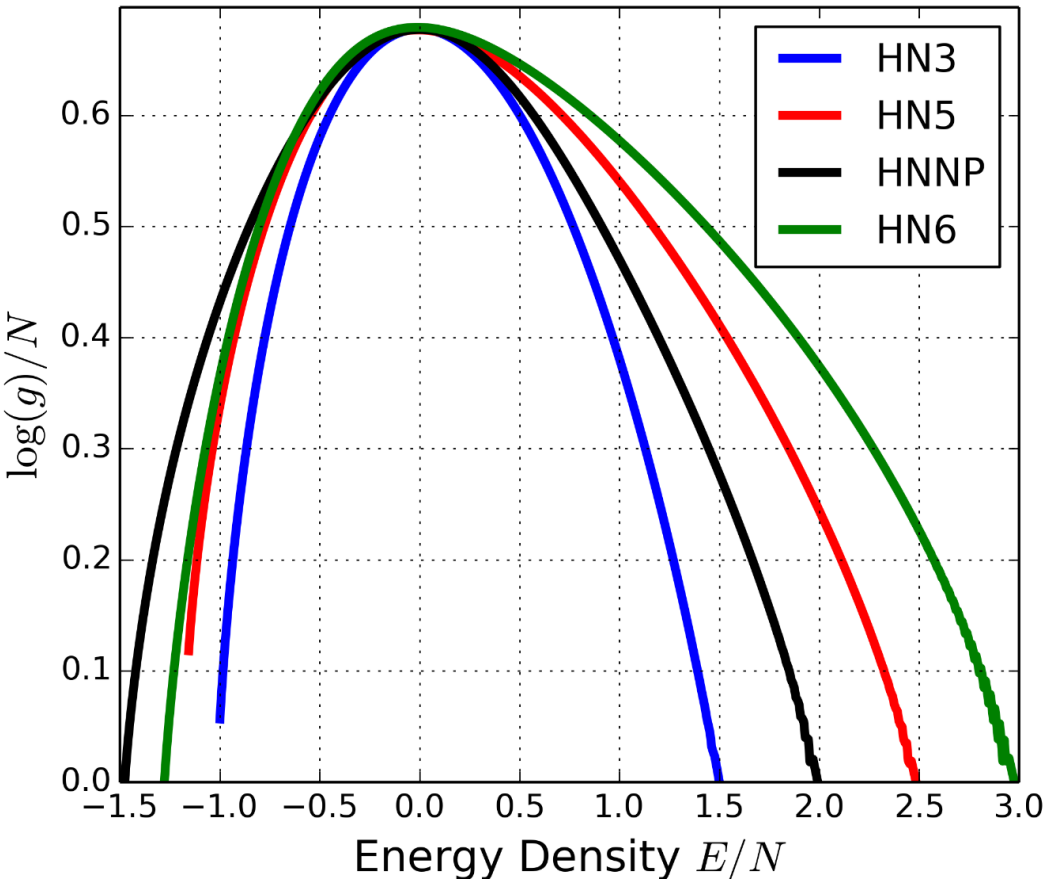
- Exactly solvable by **R**enormalization **G**roup (RG)
- Lattice-like structure
  - Mean-Field ⇨ HNs ⇨ Regular lattice
- Different structures among HNs
  - Different degrees: 3, 4, 5, 6
  - Planar vs non-planar

Methods

- **W**ang-**L**andau Sampling (WL)
  - direct access to *Density of States*
  - partition function ⇨ equilibrium quantities
- **S**imulated **A**nnealing (SA)
  - probe dynamical behaviors
  - Glassy relaxation
- **R**enormalization **G**roup (RG)
  - Exact solutions in the thermodynamic limit

Density of States (WL)

- Planar: HN3, HN5
  Degenerate ground states
- Non-planar: HNNP, HN6
  Unique ground states
- Reference of **SA** & **RG**
- Wang-Landau fails
  - $N > 1024$
  - Geometric frustration?



Glassy relaxation (SA)

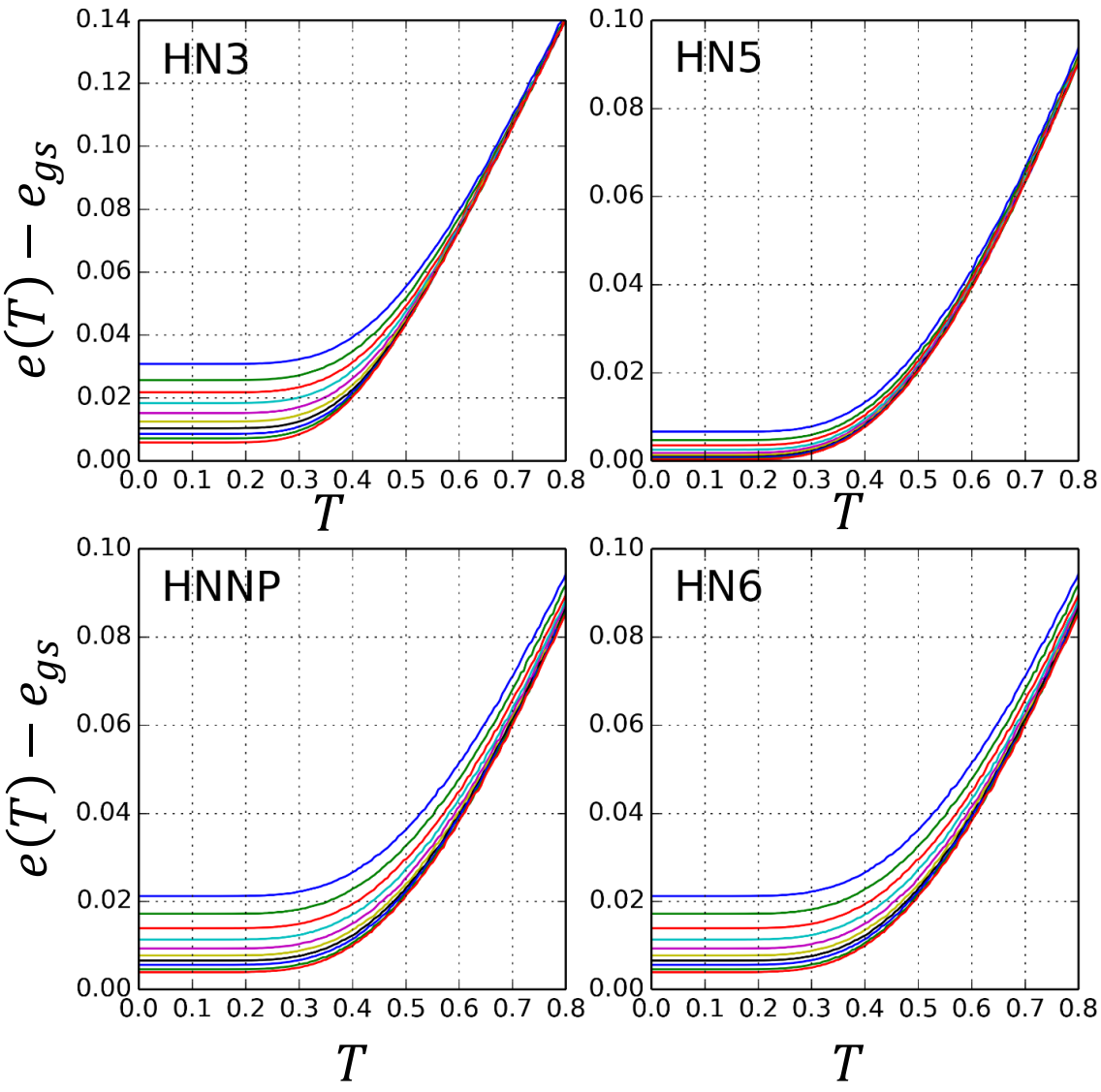
- x axis: T
- y axis: $e(T) - e_{gs}$

- Annealing schedules:

$$\frac{dT}{dt} = \frac{10^{-3}}{1}, \frac{10^{-3}}{2}, \cdots, \frac{10^{-3}}{512}$$

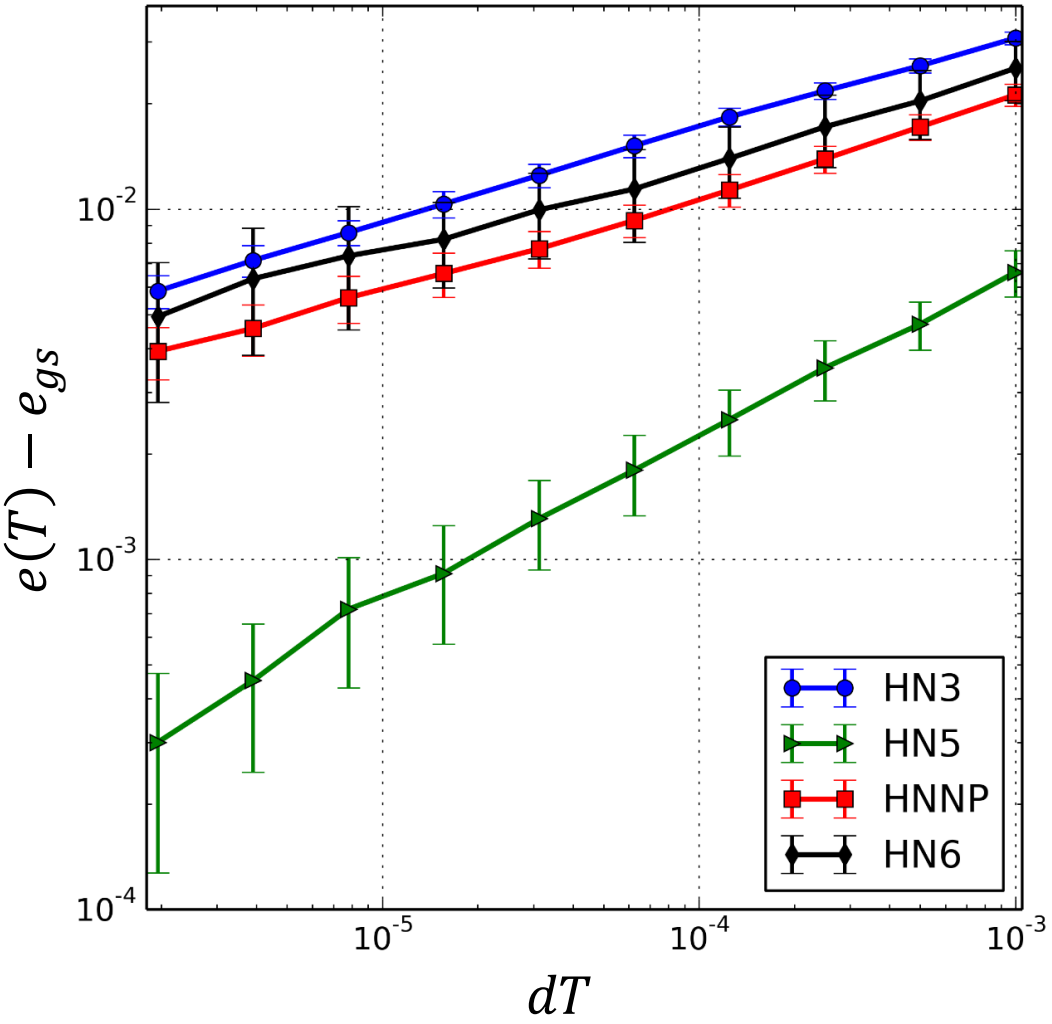
- $N = 16{,}384$

- Extremely slow relaxation at low T



Power-law relaxation (SA)

- Power-law relaxation
- HN3, HNNP, HN6:
  - Slope = ~ 0.27
- HN5 may equilibrate gradually



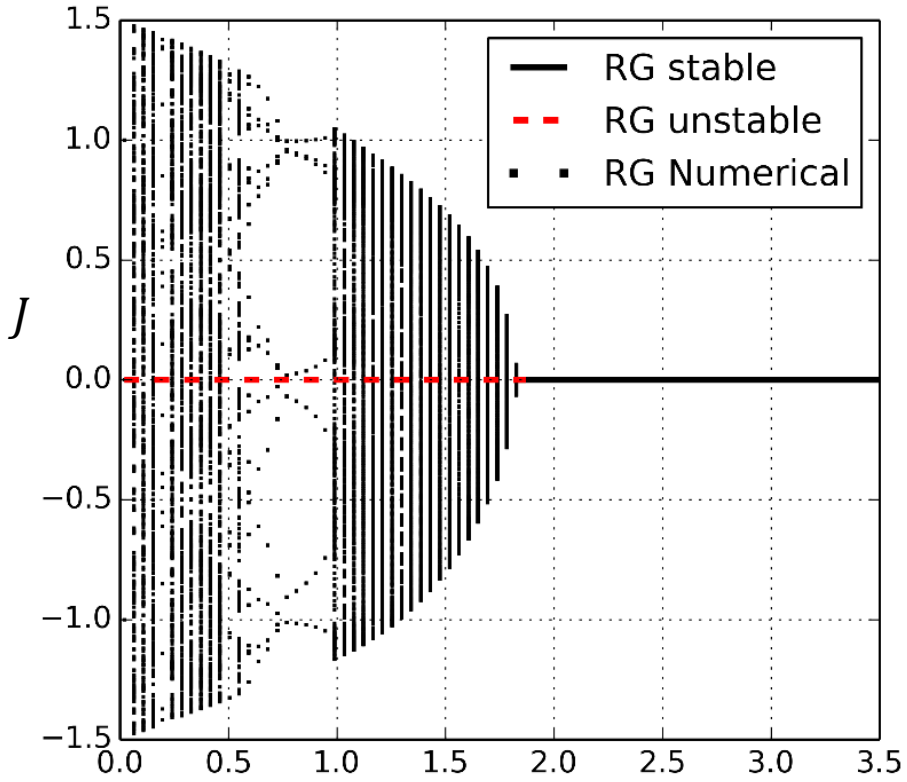
Spin glass transition (RG)

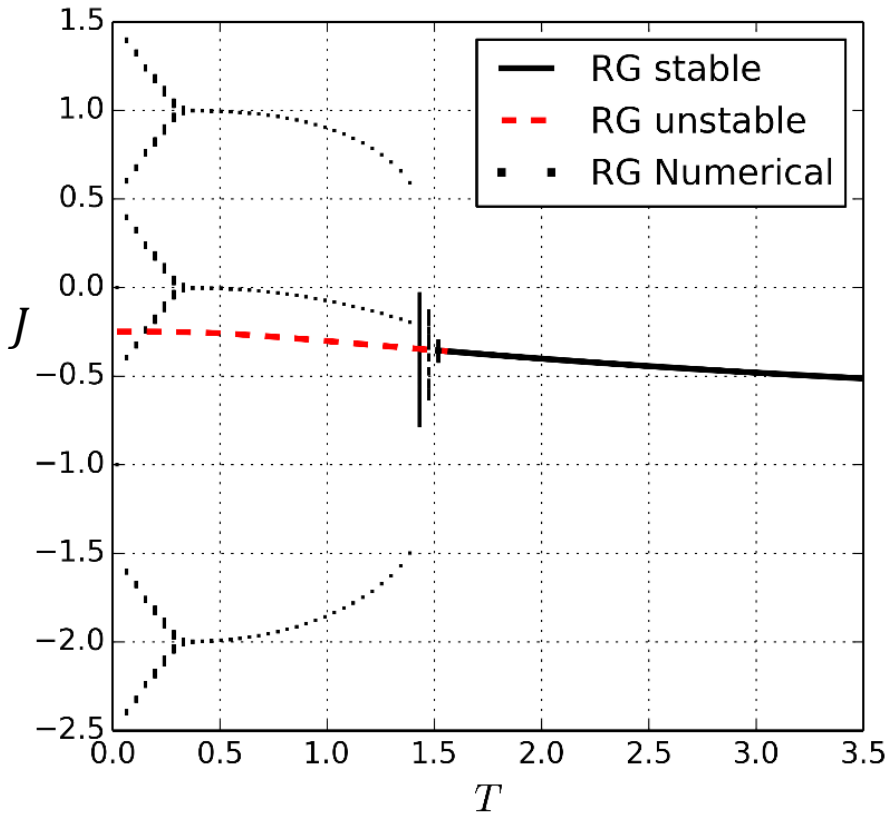
- Renormalized interaction strength J
  - Recursive equations
    - → Numerical solution
    - → Analytical solution
- Planar: HN3, HN5
  - stable fixed-point solution
  - no phase transition

McKay and Berker, *Phys. Rev. Lett.* **48**, 11 (1982)

Spin glass transition (RG)

- Non-Planar: HNNP, HN6
  - partially stable fixed-point solution
  - possible spin glass transition





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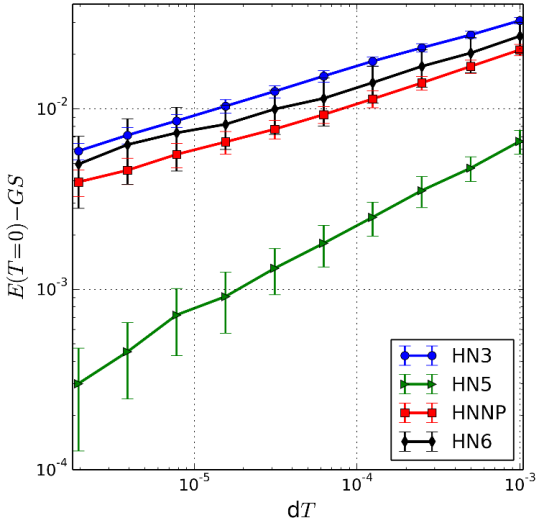
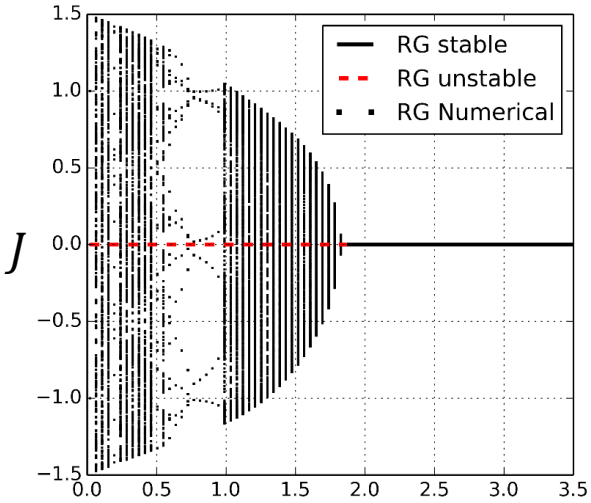
Summary & future work

- Nonequlibrium at low T
- Power-law relaxation
- Spin glass transition
- Future work
  - reconstruct partition function
  - C_v, m, χ using RG
  - Spin glass ⟺ geometry?





Hierarchical networks (*HNs*)

- HN3:
  - degree 3

$N = 2^{1} + 1$