CS 613 - Assignment 2

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PART-1 Theory-1

Data =

$$\begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix}$$

X =

$$\begin{bmatrix} -2 \\ -5 \\ -3 \\ s0 \\ -8 \\ -2 \\ 1 \\ 5 \\ -1 \\ 6 \end{bmatrix}$$

Y =

$$\begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

Mean,

$$\mu = -0.9$$

Standard Deviation,

$$\sigma = 4.2282$$

X - Standardized =

$$\begin{bmatrix} -0.2602 \\ -0.9697 \\ -0.4967 \\ 0.2129 \\ -1.6792 \\ -0.2602 \\ 0.44937 \\ 1.3954 \\ -0.0237 \\ 1.6319 \end{bmatrix}$$

Adding Bias Feature to X =

$$\begin{bmatrix} 1 & -0.2602 \\ 1 & -0.9697 \\ 1 & -0.4967 \\ 1 & 0.2129 \\ 1 & -1.6792 \\ 1 & -0.2602 \\ 1 & 0.44937 \\ 1 & 1.3954 \\ 1 & -0.0237 \\ 1 & 1.6319 \end{bmatrix}$$

$X^T * X =$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -0.2602 & -0.9697 & -0.4967 & 0.2129 & -1.6792 & -0.2602 & 0.44937 & 1.3954 & -0.0237 & 1.6319 \end{bmatrix} \times \begin{bmatrix} 1 & -0.2602 \\ 1 & -0.9697 \\ 1 & -0.4967 \\ 1 & 0.2129 \\ 1 & -1.6792 \\ 1 & -0.2602 \\ 1 & 0.44937 \\ 1 & 1.3954 \\ 1 & -0.0237 \\ 1 & 1.6319 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 9 \end{bmatrix}$$

$(X^T * X)^{-1} =$

$$\begin{bmatrix} 0.1 & 0 \\ -0 & 0.1111 \end{bmatrix}$$

$(\mathrm{X}^{\mathrm{T}} * \mathrm{X})^{-1} * \mathrm{X}^{\mathrm{T}} =$

$$\begin{bmatrix} 0.10 & 0 \\ 0 & 0.11 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -0.2602 & -0.9697 & -0.4967 & 0.2129 & -1.6792 & -0.2602 & 0.44937 & 1.3954 & -0.0237 & 1.6319 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ -0.0289 & -0.1077 & -0.0552 & 0.0237 & -0.1866 & -0.0289 & 0.0499 & 0.1550 & -0.0026 & 0.1813 \end{bmatrix}$$

$$Parameters, \theta = ((X^{\mathrm{T}} * X)^{-1} * X^{\mathrm{T}}) * Y$$

$$= \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ -0.0289 & -0.1077 & -0.0552 & 0.0237 & -0.1866 & -0.0289 & 0.0499 & 0.1550 & -0.0026 & 0.1813 \end{bmatrix} \times \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.4000 \\ -1.7449 \end{bmatrix}$$

Final Model:

$$\hat{Y} = \theta_0 + \theta_1 x_{:,1}$$

$$\hat{Y} = 1.4 - 1.7449 * x_{:,1}$$

PART-1 Theory-2

(a). The gradient with respect to x: g(x)'= 4*(x-1)^3 $* (x-1)' = 4 * (x-1)^3 * 1 = 4 * (x-1)^3$
(b). Let g(x)'=0, so 4*(x-1)$^3 = 0. Itmeansat : x = 1, g(x)min = g(1) = 0.$
(c). Plot x vs g(x):

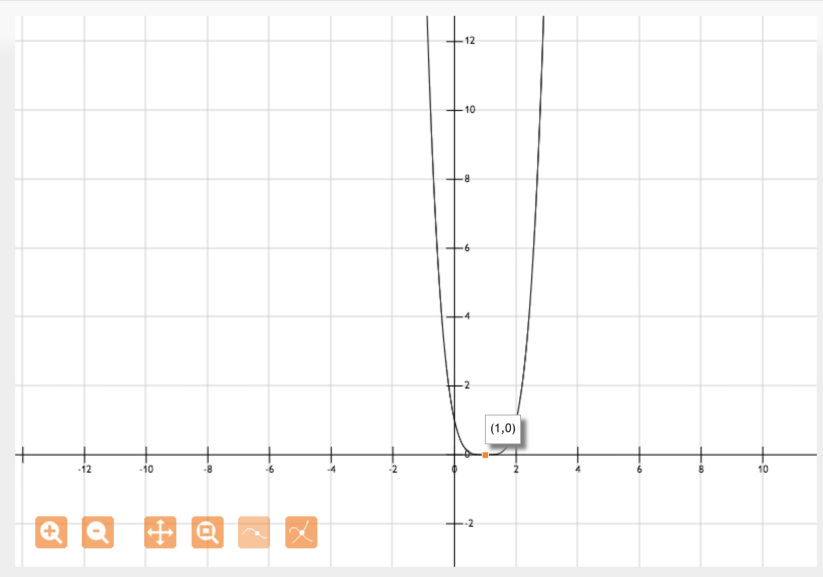

Figure 1: x vs g(x)

PART-2 Closed Form Linear Regression

The final model:

$$\hat{Y} = \theta_0 + \theta_1 x_{:,1}$$

$$\hat{Y} = 3425.6 + 846.9 * x_{:,1} + -369.2 * x_{:,2}$$

The root mean squared error:

$$RMSE = 853.3806$$

PART-3 S-Folds Closed Form Linear Regression, S=3,5,20,N

Mean and Standard Deviations of RMSE for S=3:

$$Mean = 653.0247$$

$$StandardDeviation = 33.7290$$

Mean and Standard Deviations of RMSE for S=5:

$$Mean = 635.6337$$

$$StandardDeviation = 23.9552$$

Mean and Standard Deviations of RMSE for S=20:

$$Mean = 626.6830$$

$$StandardDeviation = 12.7335$$

Mean and Standard Deviations of RMSE for S=N=44:

$$Mean = 623.4051$$

$$StandardDeviation = 2.4880e - 13$$

PART-4 Locally-Weighted Closed-Form Linear Regression

The root mean squared error:

$$RMSE = 466.8133$$

PART-5 Gradient Descent

Final Model:

$$\hat{Y} = \theta_0 + \theta_1 x_{:,1}$$

$$\hat{Y} = 3425.6 + 846.9 * x_{:,1} + -369.2 * x_{:,2}$$

The final RMSE testing error:

$$RMSE = 853.3806$$

The final RMSE training error:

$$RMSE = 548.9724$$

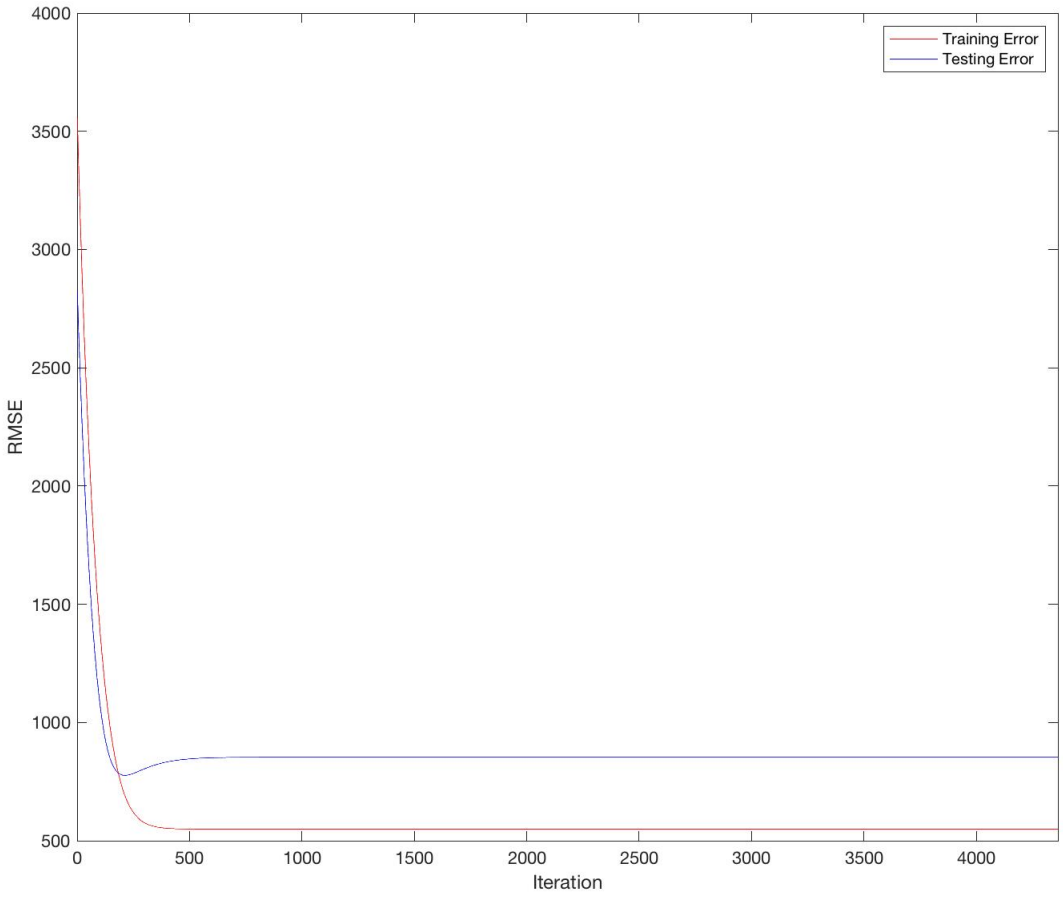

Figure 2: Plot the RMSE as a function of the iteration