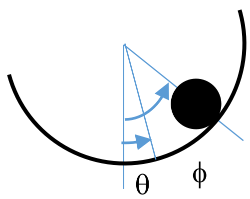
100 Midterm Exam Points

There is a disk of radius r and mass m rotating inside half of a cylinder of radius R and mass M.



M = 578 grams

m = 162 grams

R = 15.625 cm

r = 3.8 cm

1. Determine the center of mass and mass moment of inertia of the half of a cylinder ANALYTICALLY. Show your work!
2. Determine the Lagrangian of the half of a cylinder ANALYTICALLY or in MATLAB. fprintf
3. Determine the Lagrangian of the disk ANALYTICALLY or in MATLAB. fprintf
4. Determine the EOMs for the system.
5. Simplify the θ Lagrangian EOM (equation 1) such that the disk does not exist. Does this make sense? Does it match your ANALYTICALLY determined EOM using Lagrangian Dynamics?
    a. What is the natural frequency of this system? $\theta(0) = \pi/4\,\text{rad}, \dot{\theta} = 0\,\text{rad/sec}$ fprintf
6. Simplify the ϕ Lagrangian EOM (equation 2) such that the half of a cylinder does not move. Does this make sense? Does it match your ANALYTICALLY determined EOM using Lagrangian Dynamics?
    a. What is the natural frequency of this system? $\phi(0) = \pi/4\,\text{rad}, \dot{\phi} = 0\,\text{rad/sec}$ fprintf
7. Integrate the full EOMs for the system using various initial conditions to show the different styles of motion that the system exhibits.
    a. Initial conditions: $\theta = 0\,\text{rad}, \dot{\theta} = 0\,\text{rad/sec}, \phi = \frac{\pi}{18}\,\text{rad}, \dot{\phi} = 0\,\text{rad/sec}$

       Plot the response for 6 seconds.
    b. Initial conditions: $\theta = \frac{\pi}{18}\,\text{rad}, \dot{\theta} = 0\,\text{rad/sec}, \phi = \frac{\pi}{9}\,\text{rad}, \dot{\phi} = 0\,\text{rad/sec}$

       Plot the response for 6 seconds.
    c. Initial conditions: $\theta = \frac{\pi}{9}\,\text{rad}, \dot{\theta} = 0\,\text{rad/sec}, \phi = -\frac{\pi}{9}\,\text{rad}, \dot{\phi} = 0\,\text{rad/sec}$

       Plot the response for 6 seconds.