

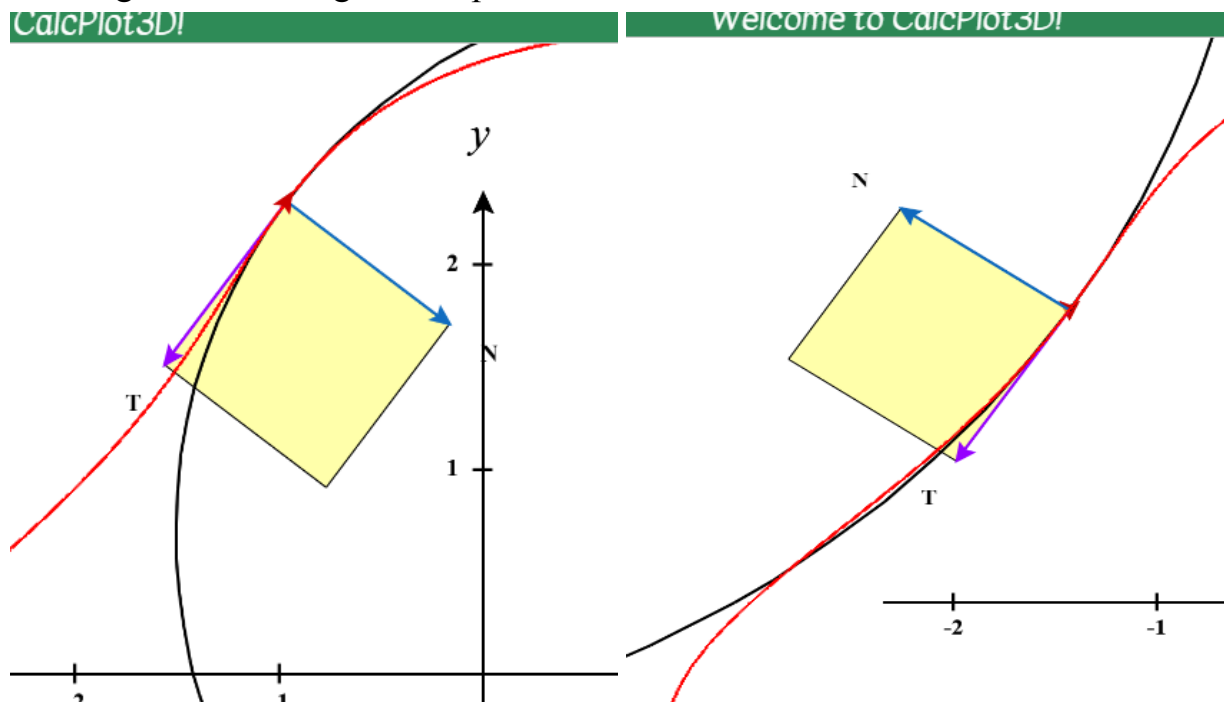
I'm not sure but I believe the functions shown in the video are:

$$x(t) = \cos(t) * (3 + \sin(2t))$$

$$y(t) = \sin(t) * (3 + \cos(t))$$

$$z(t) = 0.1t$$

I think the changing direction of the Bi-Normal vector is caused by the discrepancy between the coefficient of t . If any of the t s in $x(t)$ or $y(t)$ are different from the others, the space curve will appear convex with concave sections (where the coefficient causes a change in the curve's frequency), where the normal changes from pointing inward to pointing outward. This roll effect causes the Bi-Normal to point orthogonal and no longer upwards. It's super easy to visualize this by enabling the osculating circle option in the animation on CalcPlot3D.



To mathematically show this (kinda), we can find the tangent, normal, and bi-normal vectors by finding the 1st, 2nd, and 3rd derivatives of the parametric function at a given point. I'm going to choose two points: $t = 2$ (which is convex) and $t = 2.5$ (which is concave).

The tangent vector can be found by isolating the x portion of the parametric function: $x(t) = \cos(t) * (3 + \sin(2t))$ and finding its derivative:

$x'(t) = 2\cos(t)\cos(2t) - \sin(t)\sin(2t) - 3\sin(t)$, then inputting a value gets you the x-component for the vector at that value. The same is done for the y-component: $y(t) = \sin(t) * (3 + \cos(t))$ and $y'(t) = \cos(2t) + 3\cos(t)$.

The actual tangent vector is defined as: $T(t) = \frac{r'(t)}{\|r'(t)\|}$. For convex, this yields approximately this vector: $\langle -1.496, -1.902 \rangle$, labeled A, and for concave, this yields approximately this vector: $\langle -1.676, -2.120 \rangle$, labeled B.

We can see that the second derivative yields concavity, which the normal vector can be obtained from, and then the third derivative helps with finding the bi-normal vector. Using right-hand-rule, you can find how the normal vector points inward or outward and the binormal vector up or down.

