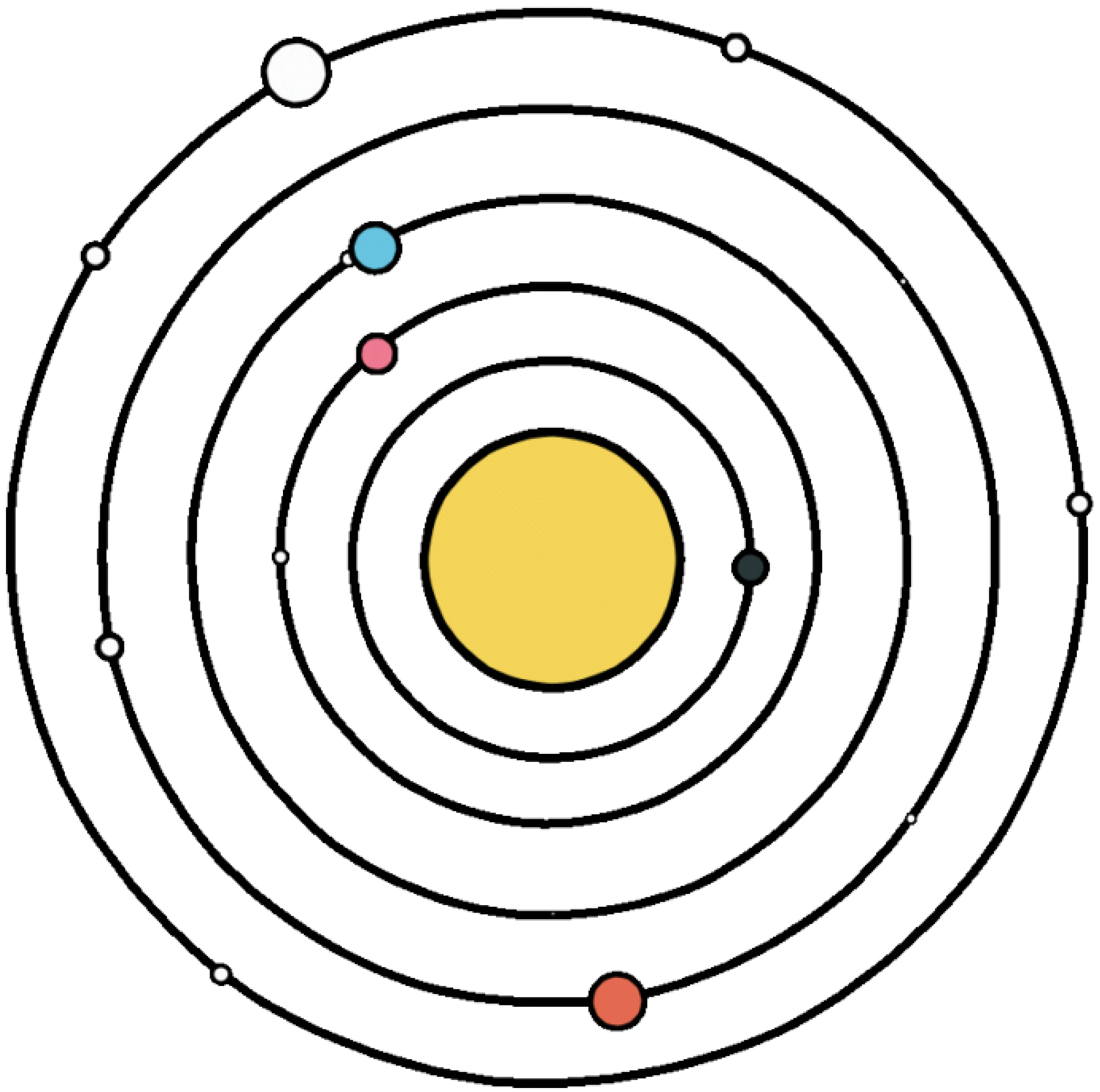
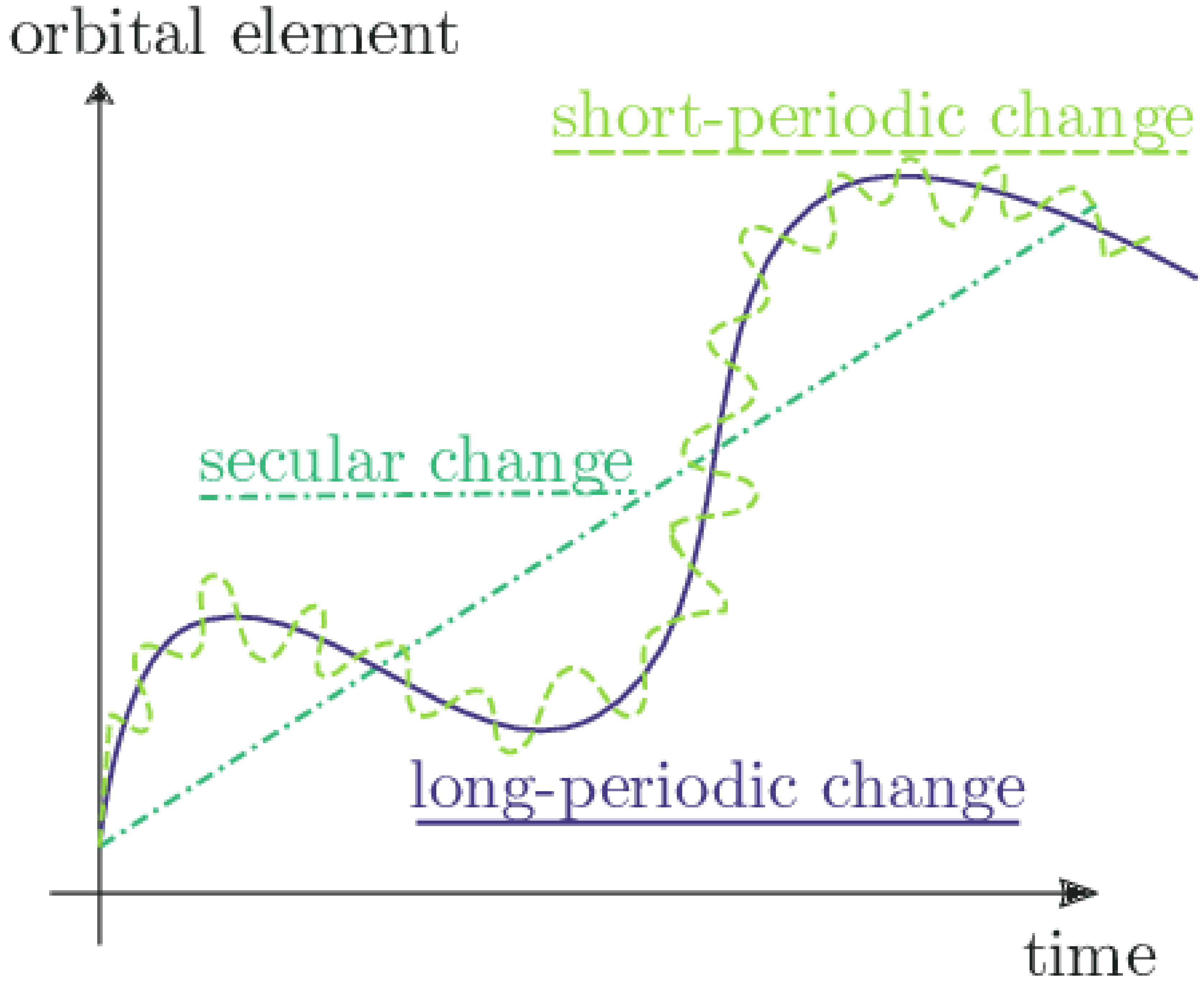
Perturbaciones Seculares

Daniel Niño Villegas
Astronomía UdeA



¿Qué son?

Las perturbaciones seculares son variaciones no periódicas en los elementos orbitales que se determinan a periodos de largo plazo.



Crédito: Suchantke, Isabell. (2019).

Planteamiento

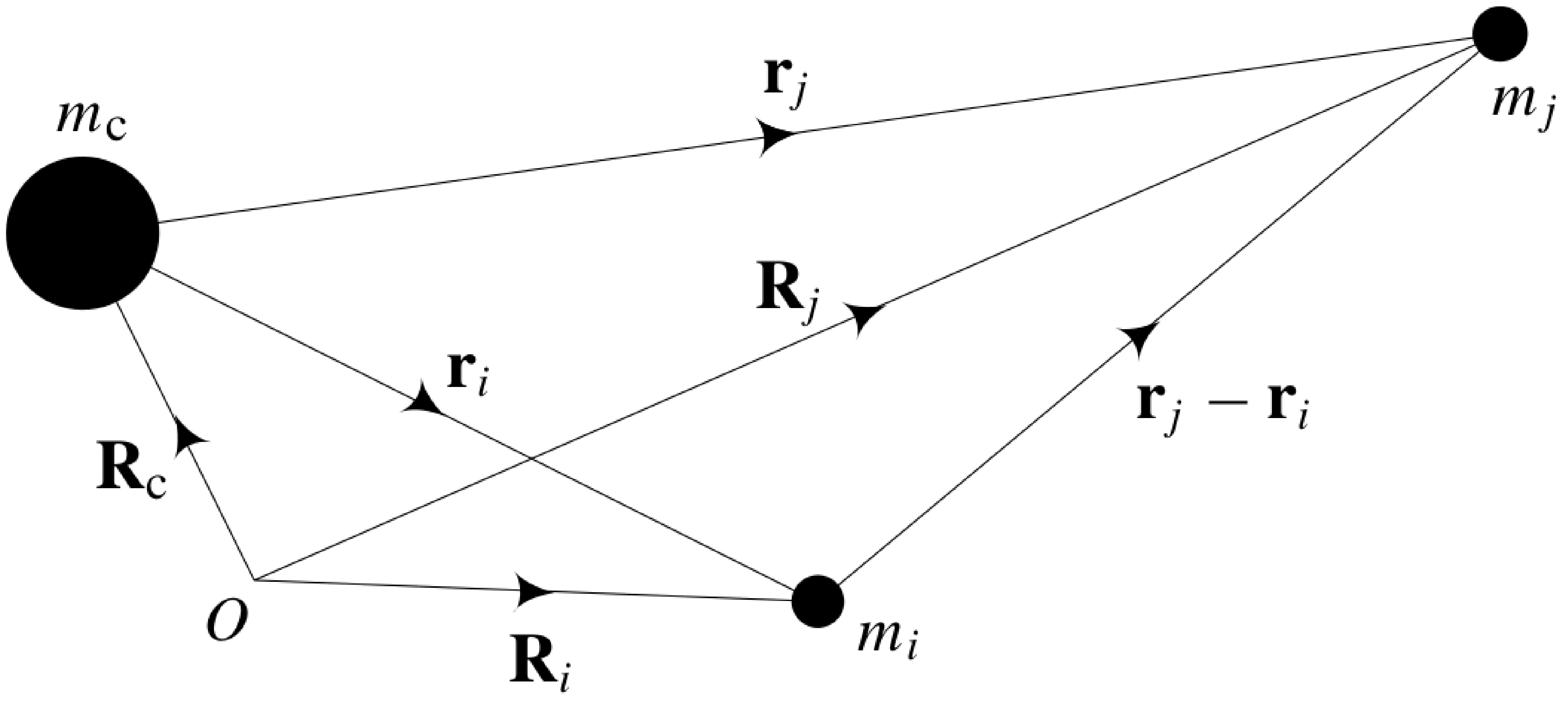


Fig. 6.1. The position vectors $\mathbf{r}_i$ and $\mathbf{r}_j$, of two masses m_i and m_j, with respect to the central mass m_{c}. The three masses have position vectors $\mathbf{R}$, $\mathbf{R}'$, and $\mathbf{R}_{\mathrm{c}}$ with respect to an arbitrary, fixed origin O.

Crédito: Murray & Dermott (2000).

Ecuaciones de movimiento

$$m_c\ddot{\mathbf{R}}_c = \mathcal{G}m_c m_i \frac{\mathbf{r}_i}{r_i^3} + \mathcal{G}m_c m_j \frac{\mathbf{r}_j}{r_j^3}$$

$$m_i\ddot{\mathbf{R}}_i = \mathcal{G}m_i m_j \frac{(\mathbf{r}_j - \mathbf{r}_i)}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3} - \mathcal{G}m_i m_c \frac{\mathbf{r}_i}{r_i^3}$$

$$m_j\ddot{\mathbf{R}}_j = \mathcal{G}m_j m_i \frac{(\mathbf{r}_i - \mathbf{r}_j)}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3} - \mathcal{G}m_j m_c \frac{\mathbf{r}_j}{r_j^3}$$

$$\mathbf{r}_i = \mathbf{R}_i - \mathbf{R}_c$$

$$\mathbf{r}_j = \mathbf{R}_j - \mathbf{R}_c$$

$$\ddot{\mathbf{r}}_i = \ddot{\mathbf{R}}_i - \ddot{\mathbf{R}}_c$$

$$\ddot{\mathbf{r}}_j = \ddot{\mathbf{R}}_j - \ddot{\mathbf{R}}_c$$

$$\ddot{\mathbf{r}}_i + \mathcal{G}\left(m_{\mathrm{c}} + m_i\right)\frac{\mathbf{r}_i}{r_i^3} = \mathcal{G}m_j\left(\frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3} - \frac{\mathbf{r}_j}{r_j^3}\right)$$

$$\ddot{\mathbf{r}}_j + \mathcal{G}\left(m_{\mathrm{c}} + m_j\right)\frac{\mathbf{r}_j}{r_j^3} = \mathcal{G}m_i\left(\frac{\mathbf{r}_i - \mathbf{r}_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3} - \frac{\mathbf{r}_i}{r_i^3}\right)$$

¿Gradiente de
un potencial?

$$\ddot{\mathbf{r}}_i = \nabla_i\left(U_i + \mathcal{R}_i\right)$$

$$\ddot{\mathbf{r}}_j = \nabla_j\left(U_j + \mathcal{R}_j\right)$$

¡Sí se puede!

$$U_i = \mathcal{G}\frac{(m_c + m_i)}{r_i}$$

$$U_j = \mathcal{G}\frac{(m_c + m_j)}{r_j}$$

Potenciales originales

$$\mathcal{R}_i = \overbrace{\frac{\mathcal{G}m_j}{|\mathbf{r}_j - \mathbf{r}_i|}}^{\text{Término directo}} - \overbrace{\mathcal{G}m_j\frac{\mathbf{r}_i \cdot \mathbf{r}_j}{r_j^3}}^{\text{Término indirecto}},$$

$$\mathcal{R}_j = \frac{\mathcal{G}m_i}{|\mathbf{r}_i - \mathbf{r}_j|} - \mathcal{G}m_i\frac{\mathbf{r}_i \cdot \mathbf{r}_j}{r_i^3}.$$

Funciones perturbadoras

Partícula de interés

$$\ddot{\mathbf{r}} + \mathcal{G}\left(m_\mathrm{c} + m\right)\frac{\mathbf{r}}{r^3} = \mathcal{G}m'\left(\frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3} - \frac{\mathbf{r}'}{r'^3}\right)$$

$$\mathcal{R} = \underbrace{\frac{\mu'}{|\mathbf{r}' - \mathbf{r}|}}_{\text{¿Trauma?}} - \mu'\frac{\mathbf{r}\cdot\mathbf{r}'}{r'^3},$$

Expansión en polinomios de Legendre

$$\left|\mathbf{r}' - \mathbf{r}\right|^2 = r^2 + r'^2 - 2rr'\cos\psi,$$

$$\frac{1}{|\mathbf{r}' - \mathbf{r}|} = \frac{1}{r'}\left[1 - 2\frac{r}{r'}\cos\psi + \left(\frac{r}{r'}\right)^2\right]^{-\frac{1}{2}}$$

$$\frac{1}{|\mathbf{r}' - \mathbf{r}|} = \frac{1}{r'}\sum_{l=0}^{\infty}\left(\frac{r}{r'}\right)^l P_l(\cos\psi),$$

Expansión en polinomios de Legendre

$$\mathcal{R} = \frac{\mu'}{r'} \sum_{l=2}^{\infty} \left(\frac{r}{r'} \right)^l P_l(\cos \psi)$$

Expansión en polinomios de Legendre

$$\mathcal{R} = \mu' \sum S(a, a', e, e', I, I') \cos\varphi$$

$$\varphi = j_1\lambda' + j_2\lambda + j_3\varpi' + j_4\varpi + j_5\Omega' + j_6\Omega$$

$$\mathcal{R} = \frac{\mu'}{a'}\mathcal{R}_{\mathrm{D}} + \frac{\mu'}{a'}\alpha\mathcal{R}_{\mathrm{E}}$$

$$
\begin{aligned}
\mathcal{R}_{\mathrm{D}} = &\left(\frac{1}{2}b_{\frac{1}{2}}^{(j)} + \frac{1}{8}\left(e^2+e'^2\right)\left[-4j^2+2\alpha D+\alpha^2D^2\right]b_{\frac{1}{2}}^{(j)}\right.\\
&\quad\left.+\frac{1}{4}\left(s^2+s'^2\right)\left(\left[-\alpha\right]b_{\frac{3}{2}}^{(j-1)}+\left[-\alpha\right]b_{\frac{3}{2}}^{(j+1)}\right)\right)\\
&\quad\times\cos[j\lambda'-j\lambda]\\
&+\left(\frac{1}{4}ee'\left[2+6j+4j^2-2\alpha D-\alpha^2D^2\right]b_{\frac{1}{2}}^{(j+1)}\right)\\
&\quad\times\cos[j\lambda'-j\lambda+\varpi'-\varpi]\\
&+\left(ss'\left[\alpha\right]b_{\frac{3}{2}}^{(j+1)}\right)\cos[j\lambda'-j\lambda+\Omega'-\Omega]\\
&+\left(\frac{1}{2}e\left[-2j-\alpha D\right]b_{\frac{1}{2}}^{(j)}\right)\cos[j\lambda'+(1-j)\lambda-\varpi]\\
&+\left(\frac{1}{2}e'\left[-1+2j+\alpha D\right]b_{\frac{1}{2}}^{(j-1)}\right)\cos[j\lambda'+(1-j)\lambda-\varpi']\\
&+\left(\frac{1}{8}e^2\left[-5j+4j^2-2\alpha D+4j\alpha D+\alpha^2D^2\right]b_{\frac{1}{2}}^{(j)}\right)\\
&\quad\times\cos[j\lambda'+(2-j)\lambda-2\varpi]\\
&+\left(\frac{1}{4}ee'\left[-2+6j-4j^2+2\alpha D-4j\alpha D-\alpha^2D^2\right]b_{\frac{1}{2}}^{(j-1)}\right)\\
&\quad\times\cos[j\lambda'+(2-j)\lambda-\varpi'-\varpi]\\
&+\left(\frac{1}{8}e'^2\left[2-7j+4j^2-2\alpha D+4j\alpha D+\alpha^2D^2\right]b_{\frac{1}{2}}^{(j-2)}\right)\\
&\quad\times\cos[j\lambda'+(2-j)\lambda-2\varpi']\\
&+\left(\frac{1}{2}s^2\left[\alpha\right]b_{\frac{3}{2}}^{(j-1)}\right)\\
&\quad\times\cos[j\lambda'+(2-j)\lambda-2\Omega]\\
&+\left(ss'\left[-\alpha\right]b_{\frac{3}{2}}^{(j-1)}\right)\cos[j\lambda'+(2-j)\lambda-\Omega'-\Omega]\\
&+\left(\frac{1}{2}s'^2\left[\alpha\right]b_{\frac{3}{2}}^{(j-1)}\right)\cos[j\lambda'+(2-j)\lambda-2\Omega'].
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_{\mathrm{E}} &= -\frac{r}{a}\left(\frac{a'}{r'}\right)^2\cos\psi\\
&\approx\left(-1+\frac{1}{2}e^2+\frac{1}{2}e'^2+s^2+s'^2\right)\cos[\lambda'-\lambda]\\
&\quad-ee'\cos[2\lambda'-2\lambda-\varpi'+\varpi]-2ss'\cos[\lambda'-\lambda-\Omega'+\Omega]\\
&\quad-\frac{1}{2}e\cos[\lambda'-2\lambda+\varpi]+\frac{3}{2}e\cos[\lambda'-\varpi]-2e'\cos[2\lambda'-\lambda-\varpi']\\
&\quad-\frac{3}{8}e^2\cos[\lambda'-3\lambda+2\varpi]-\frac{1}{8}e^2\cos[\lambda'+\lambda-2\varpi]\\
&\quad+3ee'\cos[2\lambda-\varpi'-\varpi]-\frac{1}{8}e'^2\cos[\lambda'+\lambda-2\varpi']\\
&\quad-\frac{27}{8}e'^2\cos[3\lambda'-\lambda-2\varpi']-s^2\cos[\lambda'+\lambda-2\Omega]\\
&\quad+2ss'\cos[\lambda'+\lambda-\Omega'-\Omega]-s'^2\cos[\lambda'+\lambda-2\Omega'],
\end{aligned}
$$

$$
\mathcal{R} = \frac{\mu'}{a'}\mathcal{R}_{\mathrm{D}} + \frac{\mu'}{a'}\alpha\mathcal{R}_{\mathrm{E}}
$$