Fungal-Inspired Stochastic Equations for the Cosmic Web

Adapting Hyphal Growth Models to Galaxy Filament Formation

1. Introduction: The Fungal-Cosmic Analogy

Fungal mycelium and the cosmic web share remarkable structural similarities: both form vast networks of interconnected filaments, with dense nodes at intersections and sparse voids between structures. This document presents stochastic equations for cosmic filament growth adapted from fungal hyphal models developed by DTU-Denmark (iGEM 2018), combining tip-extension dynamics, branching processes, substrate depletion, and reaction-diffusion PDEs.

1.1 Structural Correspondence

Fungal System	Cosmic Web Analog	Physical Interpretation
Spores	Primordial density peaks	Seeds of structure formation
Hyphal tips	Filament growth fronts	Active matter accretion regions
Hyphae (filaments)	Galaxy filaments	Thread-like matter distributions
Mycelium network	Cosmic web	Large-scale interconnected structure
Branching events	Filament bifurcations	Node formation at intersections
Substrate (nutrients)	Dark matter + gas	Available matter for growth
Mobile biomass	Inflowing baryonic gas	Matter flowing through filaments
Immobile biomass	Collapsed halos/galaxies	Virialized structures
Substrate depletion	Matter accretion	Local density evolution
Colony merger	Filament connections	Network topology formation

2. Filament Tip Extension Model (Microscopic)

Adapting the fungal hyphal tip extension model, we describe how cosmic filament endpoints grow through gravitational accretion of surrounding matter.

2.1 Cosmic Tip Extension Rate

Original fungal equation (Spohr et al.):

$$r\_tip,i = (k\_tip,1 + k\_tip,2 \cdot l\_br,i/(l\_br,i + K\_t)) \cdot (S/(S + K\_s))$$

Cosmic adaptation — Filament extension rate:

$$v\_fil,i(t) = a(t) \cdot H(t) \cdot [v\_0 + v\_max \cdot L\_fil,i/(L\_fil,i + L\_c)] \cdot [\rho\_DM(x)/(\rho\_DM(x) + \rho\_crit)]$$

where:

- a(t) = scale factor (cosmic expansion)
- H(t) = Hubble parameter
- v_0 = initial extension velocity (analog to k_tip,1)
- v_max = maximum extension velocity (analog to k_tip,2)
- L_fil,i = length of filament i (analog to l_br,i)
- L_c = characteristic length scale (analog to K_t)
- ρ_DM(x) = local dark matter density (analog to substrate S)
- ρ_crit = critical density (analog to K_s)

2.2 Stochastic Tip Direction

Filament tips follow the local gravitational gradient with stochastic perturbations:

$$d\theta/dt = -\kappa \cdot \partial\Phi/\partial\theta + \sigma\_\theta \cdot dW\_\theta(t)$$

where Φ is the gravitational potential and dW_θ is a Wiener process representing turbulent perturbations from mergers and feedback.

2.3 Filament Length Evolution

Following the fungal model, filament length obeys:

$$dL\_fil,i/dt = v\_fil,i(t) + \eta\_L(t)$$

$$L\_fil,i(t) = \sqrt{[(x(t) - x\_0)^2 + (y(t) - y\_0)^2 + (z(t) - z\_0)^2]}$$

3. Stochastic Branching Process

Fungal hyphae branch with probability q. In the cosmic web, filaments bifurcate at high-density nodes where multiple matter streams converge.

3.1 Branching Probability

Original fungal branching: P(branch) = q (constant)

Cosmic adaptation — density-dependent branching:

$$P(\text{branch}|x) = q_0 \cdot [1 + \alpha \cdot (\delta(x)/\delta_c)^\beta] \cdot \exp(-L_{fil}/L_{branch})$$

where:

- q_0 = base branching probability
- $\delta(x)$ = local density contrast
- $\delta_c \approx 1.686$ = collapse threshold
- α, β = enhancement parameters for overdense regions
- L_{branch} = characteristic branching length

3.2 Branching Angle Distribution

New branches emerge at angles drawn from a von Mises distribution:

$$P(\theta_{branch}) = \exp(\kappa \cdot \cos(\theta_{branch} - \theta_{parent})) / (2\pi \cdot I_0(\kappa))$$

where I_0 is the modified Bessel function and κ controls concentration around the parent direction ($\kappa \to 0$: isotropic, $\kappa \to \infty$: aligned).

3.3 Node Formation at Branches

When branching occurs, a node (cluster/halo) forms with mass:

$$M_{node} = M_0 \cdot (\rho_{local}/\rho_{mean})^\gamma \cdot (1 + z)^{(-3/2)}$$

4. Reaction-Diffusion PDE Model (Mesoscopic)

The DTU-Denmark team used Falconer's PDE model for fungal biomass evolution. We adapt this to cosmic web dynamics with gravitational effects.

4.1 Original Fungal PDEs (Falconer Model)

$$\partial b_i/\partial t = \zeta \cdot D_b \cdot \nabla^2 b_n \quad \text{(insulated/immobile biomass)}$$

$$\partial b_n/\partial t = (1-\zeta) \cdot D_b \cdot \nabla^2 b_n \quad \text{(non-insulated biomass)}$$

$$\partial n/\partial t = D_n \cdot \nabla^2 n + s \cdot (\lambda\blacksquare b_n + \lambda\blacksquare b_i) \quad \text{(mobile biomass/nutrients)}$$

$$\partial s/\partial t = -(\lambda\blacksquare b_n + \lambda\blacksquare b_i) \cdot s \quad \text{(substrate depletion)}$$

4.2 Cosmic Web PDEs (Adapted)

We map: $b_n \rightarrow \rho_{fil}$ (filament density), $b_i \rightarrow \rho_{halo}$ (collapsed halos), $n \rightarrow \rho_{gas}$ (baryonic gas), $s \rightarrow \rho_{DM}$ (dark matter reservoir)

Collapsed Halo Density (analog to insulated biomass):

$$\partial\rho_{halo}/\partial t = \zeta \cdot D_{eff} \cdot \nabla^2\rho_{fil} + \Gamma_{collapse} \cdot \rho_{fil} \cdot \Theta(\delta - \delta_c)$$

Filament Density (analog to non-insulated biomass):

$$\partial\rho_{fil}/\partial t = (1-\zeta) \cdot D_{eff} \cdot \nabla^2\rho_{fil} - \nabla \cdot (\rho_{fil} \cdot v) + S_{accretion}$$

Baryonic Gas (analog to mobile biomass):

$$\partial\rho_{gas}/\partial t = D_{gas} \cdot \nabla^2\rho_{gas} - \nabla \cdot (\rho_{gas} \cdot v) + \rho_{DM} \cdot (\lambda\blacksquare\rho_{fil} + \lambda\blacksquare\rho_{halo}) - \Lambda_{cool} \cdot \rho_{gas}^2$$

Dark Matter Reservoir (analog to substrate):

$$\partial\rho_{DM}/\partial t = -\nabla \cdot (\rho_{DM} \cdot v_{DM}) - (\lambda\blacksquare\rho_{fil} + \lambda\blacksquare\rho_{halo}) \cdot \rho_{DM} + \eta_{DM}(x,t)$$

4.3 Effective Diffusion Coefficient

Following the fungal model where D_n is reduced when biomass exceeds threshold:

$$D_{eff}(\rho) = D_0 \cdot [1 + (\rho/\rho_{crit})^2]^{(-1)}$$

This captures gravitational binding: high-density regions have suppressed diffusion (matter is trapped in potential wells).

5. Velocity Field and Matter Flow

Unlike fungal growth where transport is purely diffusive, cosmic matter flows under gravity. We add advection terms to the fungal PDEs.

5.1 Gravitational Velocity Field

$$v(x,t) = -\nabla\Phi/(4\pi G\rho_{\blacksquare}a^2) + v_{peculiar}$$

where the potential satisfies Poisson's equation:

$$\nabla^2\Phi = 4\pi G a^2 \rho_{\blacksquare}\cdot\delta(x,t)$$

5.2 Stochastic Velocity Perturbations

Turbulence from mergers and AGN feedback adds stochastic velocities:

$$dv = -\nabla\Phi\cdot dt + \sigma_{turb}\cdot(\rho/\rho_{\blacksquare})^{(1/3)}\cdot dW(t)$$

6. Substrate-Limited Growth Regime

The fungal model shows that limited substrate restricts mycelium growth. Similarly, cosmic filaments in voids have limited matter supply.

6.1 Monod-Type Growth Kinetics

Adapted from the Monod equation used in fungal growth:

$$\text{Growth rate: } \mu(\rho_{DM}) = \mu_{max} \cdot \rho_{DM}/(\rho_{DM} + K_\rho)$$

This saturating function prevents unbounded growth in high-density regions and naturally produces void regions where ρ_{DM} is depleted.

6.2 Accretion Source Term

$$S_{accretion} = \mu(\rho_{DM})\cdot\rho_{fil}\cdot f_b\cdot(1 - \rho_{fil}/\rho_{max})$$

where $f_b = \Omega_b/\Omega_m$ is the baryon fraction and ρ_{max} is the maximum collapsed density (virial density).

7. Filament Connection and Network Topology

The fungal model simulates multiple colonies growing and merging. We adapt this to describe cosmic web connectivity.

7.1 Connection Criterion

Two filament tips connect when:

$$|x_{tip,i} - x_{tip,j}| < r_{connect} \cdot (M_i \cdot M_j / M_*^2)^{(1/3)}$$

and the density bridge between them satisfies:

$$\min(\rho \text{ along path}) > \rho_{threshold}$$

7.2 Network Topology Evolution

The filament network evolves as a stochastic graph G(t) = (V(t), E(t)):

$$dN_{nodes}/dt = \Gamma_{branch} - \Gamma_{merge}$$

$$dN_{edges}/dt = \Gamma_{connection} + 2 \cdot \Gamma_{branch} - \Gamma_{dissolution}$$

8. Complete Stochastic System

Combining all components, the full fungal-inspired cosmic web model is:

8.1 Tip Dynamics (Microscopic)

$$dx_{tip}/dt = v_{fil}\cdot\hat{e}_\theta + \sqrt{(2D_{tip})}\cdot dW_x$$

$$d\theta_{tip}/dt = -\kappa_{grav}\cdot\partial\Phi/\partial\theta + \sigma_\theta\cdot dW_\theta$$

$$v_{fil} = a\cdot H\cdot[v_0 + v_{max}\cdot L/(L+L_c)]\cdot[\rho_{DM}/(\rho_{DM}+\rho_{crit})]$$

8.2 Field Equations (Mesoscopic)

$$\partial\rho_{fil}/\partial t + \nabla\cdot(\rho_{fil}\cdot v) = D_{eff}\cdot\nabla^2\rho_{fil} + \mu(\rho_{DM})\cdot\rho_{fil} - \Gamma_c\cdot\rho_{fil}\cdot\Theta(\delta-\delta_c)$$

$$\partial\rho_{halo}/\partial t = \Gamma_c\cdot\rho_{fil}\cdot\Theta(\delta-\delta_c) + \zeta\cdot D_{eff}\cdot\nabla^2\rho_{fil}$$

$$\partial\rho_{DM}/\partial t + \nabla\cdot(\rho_{DM}\cdot v) = -\lambda_{acc}\cdot(\rho_{fil}+\rho_{halo})\cdot\rho_{DM} + \eta(x,t)$$

8.3 Branching Process

$$P(\text{branch in } dt) = q_0\cdot[1 + \alpha(\delta/\delta_c)^\beta]\cdot\exp(-L/L_b)\cdot dt$$

$$\theta_{new} \sim \text{vonMises}(\theta_{parent}, \kappa_{align})$$

9. Numerical Implementation Notes

Following DTU-Denmark's approach:

• Spatial discretization on N×N grid (finite differences)

• Time integration via SSP-RK2 (Strong Stability Preserving Runge-Kutta)

• No-flux (Neumann) boundary conditions for closed box

• Periodic boundaries for cosmological volume

• Adaptive timestep: $\Delta t = C_{CFL}\cdot\Delta x / \max(|v|)$

10. Parameter Mapping Table

Fungal Parameter	Cosmic Parameter	Value	Units
k_tip,1	v_0 (initial velocity)	50	km/s
k_tip,2	v_max (max velocity)	200	km/s
K_t	L_c (characteristic length)	5	Mpc/h
K_s	ρ_crit (critical density)	2.8×10^11	M_■/Mpc^3
q (branch prob)	q_0	0.05	per timestep
D_b (diffusion)	D_eff	10	Mpc^2/Gyr
D_n (mobile diff)	D_gas	5	Mpc^2/Gyr
ζ (conversion)	ζ (collapse rate)	0.01	dimensionless
λ■ (uptake)	λ_acc (accretion)	0.1	Gyr^(-1)
λ■ (immobile uptake)	λ_halo	0.01	Gyr^(-1)
S_0 (initial substrate)	ρ_DM,0	Ω_m·ρ_crit	M_■/Mpc^3
n_0 (threshold)	ρ_vir (virial)	200·ρ_crit	M_■/Mpc^3

11. Key Insights from the Fungal Analogy

1. **Tip-driven growth**: Both systems grow primarily at active fronts (hyphal tips / filament endpoints), not uniformly throughout the structure.

2. **Substrate limitation**: Growth saturates when local resources (nutrients / dark matter) are depleted, naturally producing voids.

3. **Branching creates topology**: Network connectivity emerges from local branching events, not global optimization.

4. **Two-phase system**: Mobile (gas/nutrients) and immobile (halos/mycelium) components have different dynamics but are coupled.

5. **Colony merger**: Independent structures can merge when they encounter each other, increasing network connectivity.

References

DTU-Denmark iGEM 2018. Growth Modelling. https://2018.igem.org/Team:DTU-Denmark/GrowthModelling

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