Stabilizing neural networks using iterated graph Laplacian

A seismic impedance example

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joint work with Davide Bianchi[2], Wang Wenlong[1], Mingming Liu[1].

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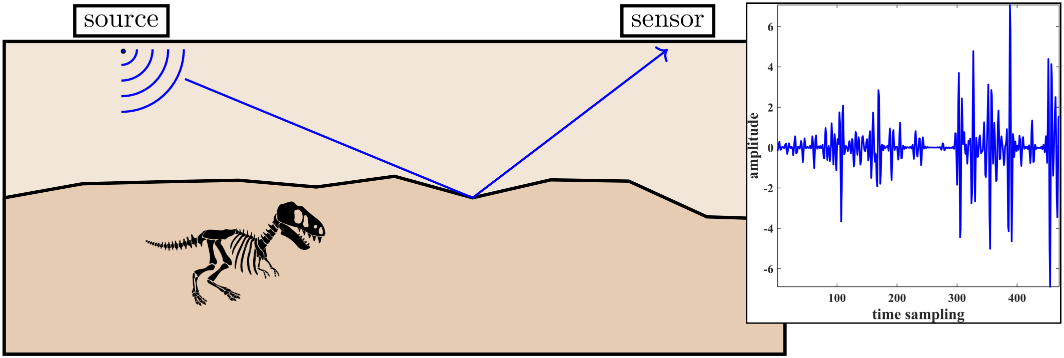
Seismic measurement

1 Seismic Impedance

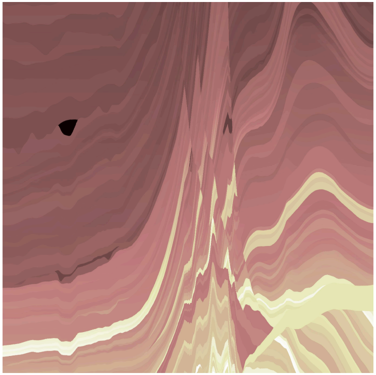


Data shows seismic waves reflected from subsurface boundaries:

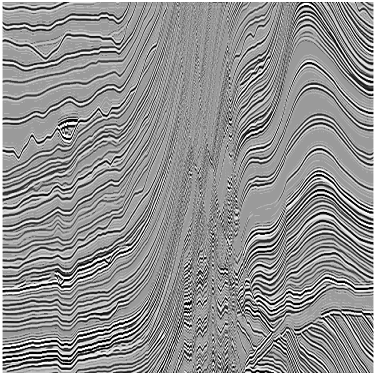
$$d(t) = w(t) * r(t)$$

- $w(t)$ seismic wavelet,
- $r(t)$ reflection coefficient.

Seismic impedance



impedance
(earth layers)



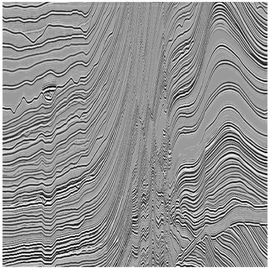
seismic data
(multiple sensors)

- Impedance z: seismic velocity $\times$ material density
- Reflection coefficient: $r = \frac{z_2 - z_1}{z_2 + z_1}$

$\Rightarrow$ (mostly) linear model

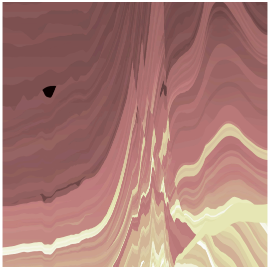
Impedance inversion

1 Seismic Impedance



Given seismic data

? $\Rightarrow$ find



impedance profile

This problem is ill-posed:

- not invertible,
- undersampling / incomplete data,
- noise,
- unknown parameters (e.g., seismic wavelet).

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Loss function

Train forward and inverse model simultaneously:

$$L(\Theta_1, \Theta_2) = \frac{0.2}{N_p} \left\| X_{\mathrm{gt}} - \Psi_{\Theta_1}(Y^\delta) \right\|_{F,\Omega}^2 + \frac{1}{N_s} \left\| Y^\delta - \Psi_{\Theta_2}(\Psi_{\Theta_1}(Y^\delta)) \right\|_F^2$$

- Y, Y^δ: (noisy) seismic data ($\|Y - Y^\delta\|_F^2 \leq \delta$),
- $X, X_{\mathrm{rec}}, X_{\mathrm{gt}}$: impedance profile (reconstructed, ground truth),
- Ψ_{Θ_1}: inverse model DNN,
- Ψ_{Θ_2}: forward model DNN,
- N_s: number of seismic traces,
- N_p, Ω: ground truth given (number and subset of columns).

  (typically obtained via well log samples)

DNN architecture

Forward model: Seismic impedance measurement is (mostly) linear

⇒ A simple CNN[1] suffices:

- 4 convolutional layers
- activation layer after first and second conv. layer

⇒ automatically learns seismic wavelet.

Inverse model AA: proposed by Alfarraj and AlRegib[1]

This model is unstable under noisy data.

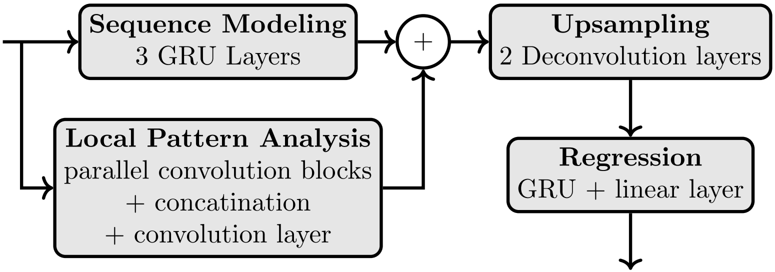
Inverse model LW: proposed by Liu and Wang.

More stable approach under noise.

[1] M. Alfarraj and G. AlRegib. "Semi-supervised learning for acoustic impedance inversion". SEG Technical program Expanded Abstracts. Society of Exploration Geophysicists, 2298-2302, 2019.

Inverse model AA

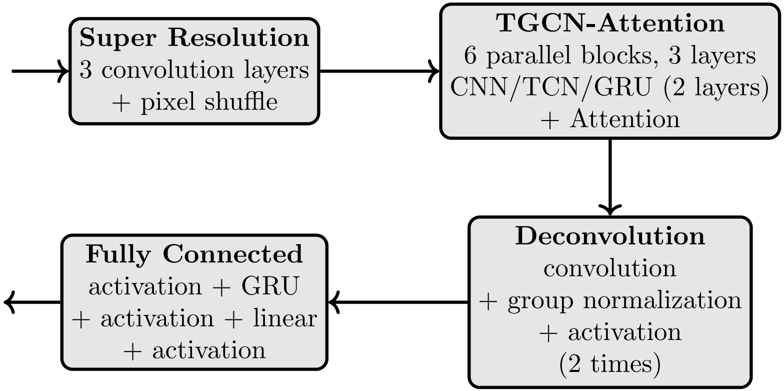
2 Impedance inversion using DNN



[1] M. Alfarraj and G. AlRegib. "Semi-supervised learning for acoustic impedance inversion". SEG Technical program Expanded Abstracts. Society of Exploration Geophysicists, 2298-2302, 2019.

Inverse model [LW]

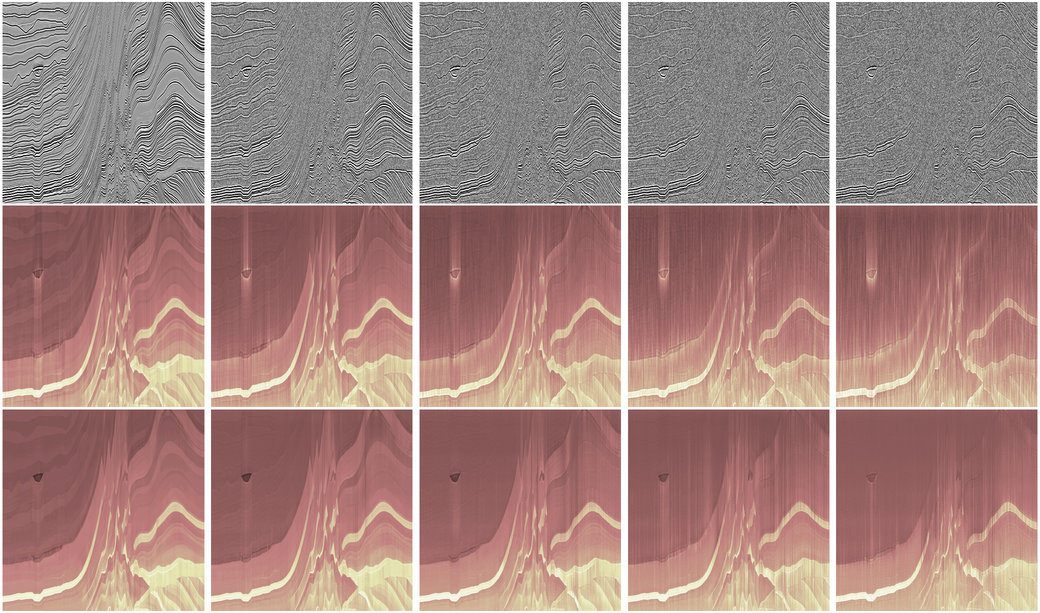
2 Impedance inversion using DNN



[2] W. Wang and M. Liu. to be published soon.

Reconstruction results

2 Impedance inversion using DNN



Reconstruction from noisy data (PSNR $\approx$ 40, 34, 31, 30) with AA (middle row) and LW (bottom row).

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3 Iterated graph Laplacian

Tikhonov-like regularization

$$X_{\text{rec}} = \arg\min_X \frac{1}{2} \left\| KX - Y^\delta \right\|_F^2 + \alpha \mathfrak{R}(X)$$

- K: linear forward model ($KX \approx \Psi_{\Theta_2}(x)$),
- $\mathfrak{R}$: some regularizer (e.g., $\mathfrak{R}(X) = \|X\|_1$),
- α: balances trade-off between data fidelity and regularization

best choice:

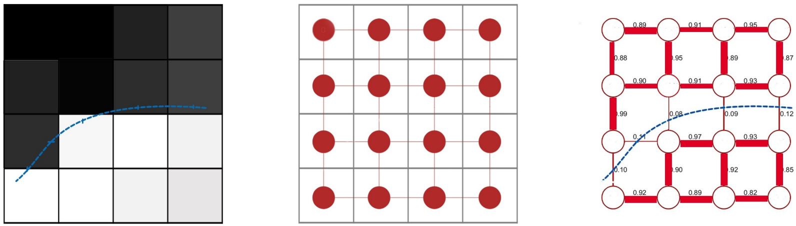
$$\mathfrak{R}_{X_{\text{gt}}}(X) = \begin{cases} 0 & , X = X_{\text{gt}} \\ \infty & , \text{otherwise} \end{cases}$$

Graph Laplacian:

$$\mathfrak{R}_{\Psi_{\Theta_1}(Y^\delta)}(X) = \left\| \Delta_{\Psi_{\Theta_1}(Y^\delta)} X \right\|_1$$

Graph Laplacian

Weighted graph:
- Connect neighboring pixels (R-neighborhood),
- Weights depend on pixel value difference.



Graph Laplacian: For $X \in \mathbb{R}^{M \times N}$ define $\Delta_X \in \mathbb{R}^{MN \times MN}$ as:

$$\Delta_{X,(j,k),(j',k')} = \begin{cases} \texttt{sum}\left|\Delta_{X,(j,k),:}\right| & , (j,k) = (j',k') \\ -\texttt{exp}\left(-\frac{1}{\sigma}|X_{j,k} - X_{j',k'}|^2\right) & , |j-j'|, |k-k'| \leq R \\ 0 & , \text{otherwise} \end{cases}$$

Intent: $\left\|\Delta_{\Psi_{\Theta_1}(Y^\delta)} X\right\|_1$ small $\Leftrightarrow$ $\Psi_{\Theta_1}(Y^\delta)$ and X have similar structure

Iterated graph Laplacian

Algorithm

Input: *data* Y^δ, *parameters* R, σ, n

1: $X_0 = \Psi_{\Theta_1}(Y^\delta)$ *// Initialization*

2: **for** $k = 1, \ldots, n$ **do**

3: $\Delta = \Delta_{X_{k-1}}$ *// Graph Laplacian*

4: $X_k = \arg\min_X \frac{1}{2} \left\| KX - Y^\delta \right\|_F^2 + \|\Delta X\|_1$ *// Update*

5: ***end for***

Output: X_n

Parameter choice: $R = 7$, $\sigma = 0.001$, $n \leq 10$

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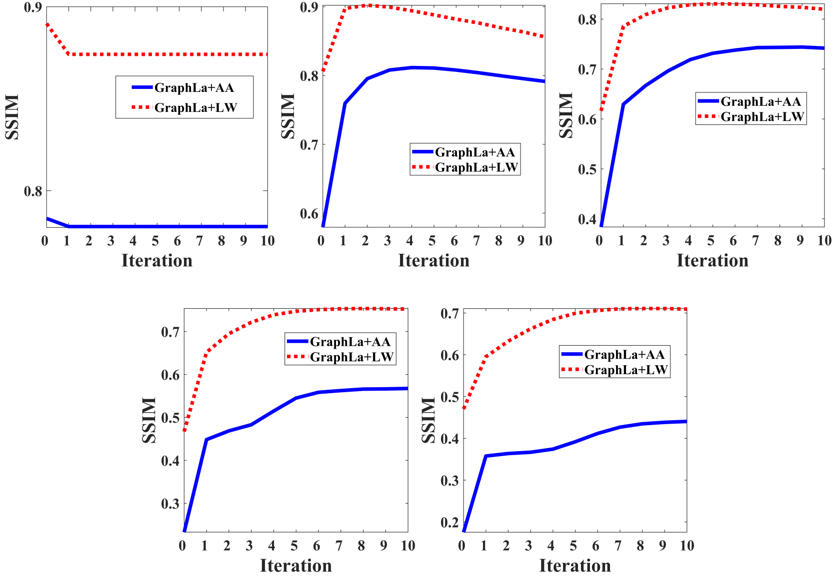
SSIM / Iteration

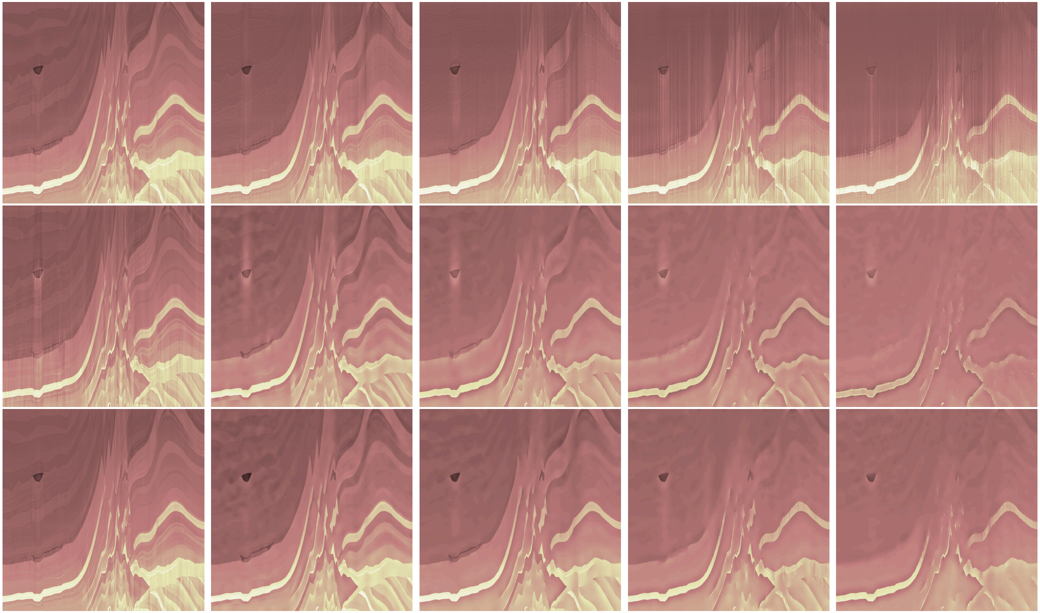
4 Experiments



Structural similarity (SSIM) over $n = 10$ iterations for different levels of noise.

Reconstruction

4 Experiments



Top: Reconstruction from noisy data using LW,
Middle/bottom: iterated graph Laplacian with AA / LW initialization.

Thank you!

[1] D. Bianchi, M. Donatelli, D. Evangelista, W. Li, and E. L. Piccolomini. “Graph Laplacian and Neural Networks for Inverse Problems in Imaging: GraphLaNet”. International Conference on Scale Space and Variational Methods in Computer Vision. 175-186, 2023.

[2] D. Bianchi, D. Evangelista, S. Aleotti, M. Donatelli, E. L. Piccolomini, and W. Li. “A data-dependent regularization method based on the graph Laplacian“. arXiv:2312.16936. 2023.

[3] D. Bianchi, F. Boßmann, W. Wang, and M. Liu. “Improved impedance inversion by deep learning and iterated graph Laplacian“. arXiv, 2024.