Delft University of Technology

Project: Enhancing Sea Ice Dynamics in the Climate System by Machine Learning

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1 Motivation

Sea ice is a thin layer between the atmosphere and the ocean. Snow covered sea ice reflects a significant amount of solar radiation, while open water absorbs most of it. A shrinking ice layer will reduce the reflectivity and thus, reinforce the reduction, the so-called ice-albedo feedback. Therefore, **sea ice is very sensitive to global warming and plays a critical role in the climate system**. In almost all climate models, sea ice is represented as a continuous viscous-plastic (VP) material. The model under consideration goes back to Hibler [3] and is based on a viscous-plastic description of the ice.

Due to large variations in the viscosities of the viscous-plastic material law, **the resulting nonlinear problem is very difficult to solve**. Currently used methods tend to have problems to converge. The development of fast and robust solvers is subject to present research.

Figure 1: Sea ice in Antarctic Ocean.

2 Problem description

A model for three prognostic variables, the sea ice concentration A, the mean sea ice thickness H, and the sea ice velocity $\mathbf{v}$, is presented here. The sea ice dynamics are described by a system of coupled partial differential equations:

$$\rho_{\text{ice}} H \partial_t \mathbf{v} + f_c \boldsymbol{e}_z \times \mathbf{v} = \operatorname{div} \boldsymbol{\sigma} + A\boldsymbol{\tau}(\mathbf{v}) - \rho_{\text{ice}} H g \nabla \tilde{H}_g, \tag{1}$$

$$\partial_t A + \operatorname{div}(\mathbf{v}A) = 0, \quad A \leq 1, \tag{2}$$

$$\partial_t H + \operatorname{div}(\mathbf{v}H) = 0, \tag{3}$$

where ρ_{ice} is the ice density, f_c is the Coriolis parameter, g is the gravitational acceleration, $\tilde{H}_g$ is the sea surface height, and $\boldsymbol{e}_z$ is the vertical (z-direction) unit vector. The forcing term $\boldsymbol{\tau}(\mathbf{v})$ is the sum of the ocean and atmospheric surface stresses. The internal stresses in the ice $\boldsymbol{\sigma}$ are modeled by the viscous-plastic (VP) sea ice rheology; cf. [3]. In particular, the nonlinear viscous-plastic rheology relates the strain rate tensor

$$\dot{\boldsymbol{\epsilon}} = \frac{1}{2}\Big(\nabla\mathbf{v} + \nabla\mathbf{v}^T\Big), \quad \dot{\boldsymbol{\epsilon}}' := \dot{\boldsymbol{\epsilon}} - \frac{1}{2}\operatorname{tr}(\dot{\boldsymbol{\epsilon}})I,$$

where $\operatorname{tr}(\cdot)$ is the trace, to the stress tensor $\boldsymbol{\sigma}$. The relationship is given by

$$\boldsymbol{\sigma} = 2\eta\dot{\boldsymbol{\epsilon}}' + \zeta \operatorname{tr}(\dot{\boldsymbol{\epsilon}})I - \frac{P}{2}I,$$

with the viscosities η and ζ, given by $\eta = e^{-2}\zeta$ and P the sea ice strength.

The standard approach for solving the coupled sea ice system in (1)–(3) is based on a time splitting method. First, the solution of the sea ice momentum equation (1) is computed, followed by the solution of the transport equations (2) and (3). The momentum equation is then solved implicitly in time, for instance, using Newton's method; see, e.g., [4]. With an increasing mesh resolution, this becomes extremely numerically expensive due to the arising strong local nonlinearities as well as large and ill-conditioned sparse linear equation systems.

Instead of using classical numerical techniques to improve the efficiency and robustness of the Newton solver, **this project aims at using surrogate models based on deep learning [2] within the numerical simulation framework**. In particular, the approach from [1], which uses convolutional neural networks (CNNs) and has been shown to be both robust and efficient for steady flow problems, should be extended to sea ice dynamics and used to improve the solver performance.

During the project, a close **collaboration with Carolin Mehlmann[1] from the Max-Planck-Institute of Meteorology** is aspired.

[1] Url: https://mpimet.mpg.de/en/staff/carolin-mehlmann; Email: carolin.mehlmann@mpimet.mpg.de

Tasks

- Install and familiarize with the software packages:
  - There is the option of using an existing implementation of the sea ice problem based on Gascoigne[2] (C++) or FEniCS[3]/Firedrake[4] (C++ with Python interface).
  - The Python machine learning libraries TensorFlow[5] and Keras[6] or PyTorch[7].
- Implement and train a CNN-based autoencoder for simple image data sets.
- Set up a software pipeline based on the implementation of the sea ice problem to automatically generate sea ice dynamics simulation results based on varying input parameters.
- Based on the previous tasks, train a surrogate model for predicting flow fields based on certain input parameters and optimize the model.
- Use the surrogate model to enhance efficiency and/or robustness of the numerical sea ice simulations.

Contact

If you are interested in this project and/or have further questions, please contact Alexander Heinlein, a.heinlein@tudelft.nl.

References

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