# Introduction to Control System Engineering

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## Part I: Introduction to Control System Engineering

# What is feedback control?

## Dynamic System

A *dynamic system* is a system whose behavior *changes over time*, often in response to external stimulation or forcing.

- **Control Input:** $u \in \mathbb{R}^m$
  *Control input* excites a dynamic system, and changes its responses over time.
- **System Output:** $y \in \mathbb{R}^v$
  *System output* is a function of state variables and inputs of the system, and it is of interest.

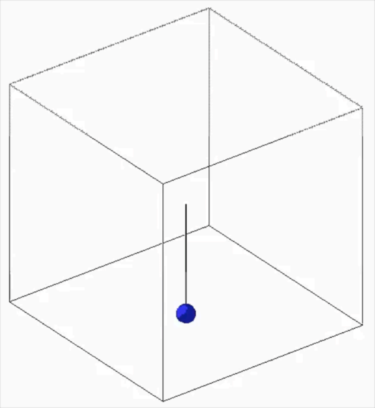
- Automotive Systems

  - Control Input: steering wheel, gas and break pedal, gear shift...
  - System Output: speed, direction, location...

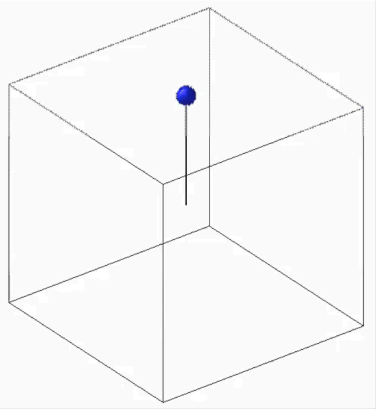
- Spherical Pendulum

## Equilibrium

A body is in an *equilibrium* when it is *at rest* or *in a uniform motion*



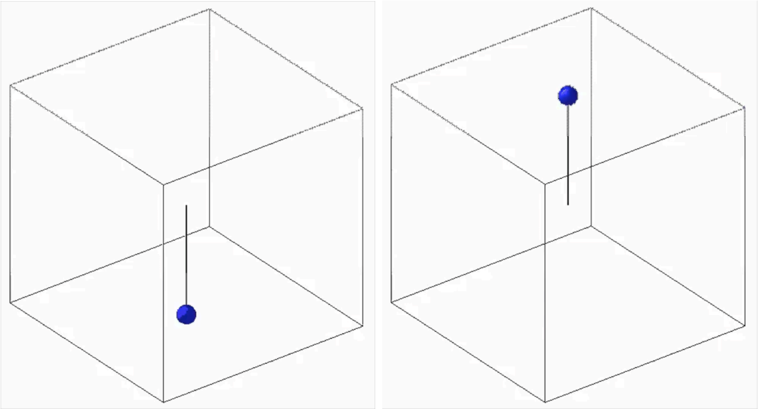
(a) Hanging equilibrium



(b) Inverted equilibrium

### Stability

A property of an equilibrium: an equilibrium is *stable* if, when a body is *slightly perturbed*, it *returns* back to the equilibrium.



(a) Hanging equilibrium: stable

(b) Inverted equilibrium: unstable

### Control Systems

*Control system* is to generate a control input $u$ such that the output $y$ of a dynamic system behaves *in a desired manner*.

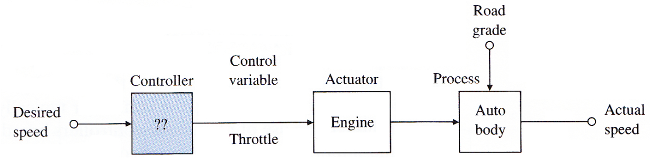
- Improve stability properties of an equilibrium
- Stabilize an unstable equilibrium
- The output is fixed to zero regardless of disturbances (*Regulating control*)
- The output is transferred to a new reference value (*Transition*)
- The output follows a changing reference signal (*Tracking control*)



- Automobile Cruise Control
  - Dynamic system: Engine / Body
  - Control input: Engine throttle
  - System output: Speed

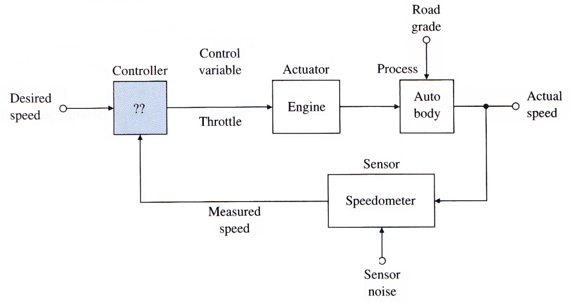
  - Control system: Finds a proper level of engine throttle such that the automobile speed is set to a given reference value.

- Objective




- Open-loop Control
  - Based on a mathematical/experimental model of the system, estimate the required control input for a given reference output value
  - A table for throttle level vs. speed can be obtained for nominal driving conditions
  - For a given reference speed, find the corresponding throttle level from the table.
  - Simple controller structure and lower cost
  - Cannot compensate any modeling error and disturbance

- Closed-loop Control (Feedback Control)
  - The system output becomes an input of the control system
  - Compare the system output with the reference value, and adjust control input accordingly
  - Example: Automobile cruise control



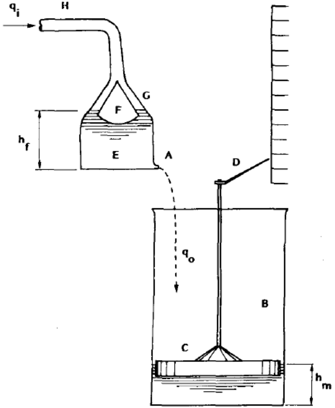

  - Compensates some modeling errors and disturbance
  - Can change dynamic characteristics completely, possibly breaks stability property

# Origin of Feedback Control Systems

- Water clock
  - Timepiece developed back in 500 BC
  - Time is measured by the flow of water into a vessel
  - The amount of water is transformed into time

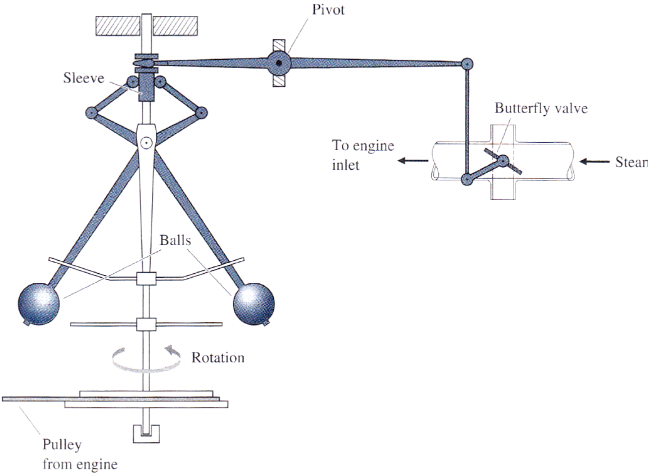
  - Water flow rate becomes slower

# Origin of Feedback Control Systems




- Water clock by Ktesibios
  - Greek inventor and mathematician (285-222 BC)
  - Constant water flow rate
  - Water is supplied by *H*
  - A conical float *F* obstructs a hollow cone *G* as the level reaches a reference value

- Rotation Speed Control of a Steam Engine




Fly-ball governor

Wright Brothers (Wilbur and Orville) at Kitty Hawk
December 17, 1903

# X-31 Enhanced Fighter

- X-31
  - NASA Experimental Aircraft for *Enhanced Maneuverability*
  - Demonstrated the value of thrust vectoring
  - Controlled motion at very high angle of attack
  - Significant advantage in maneuverability

# Contemporary Control Techniques...

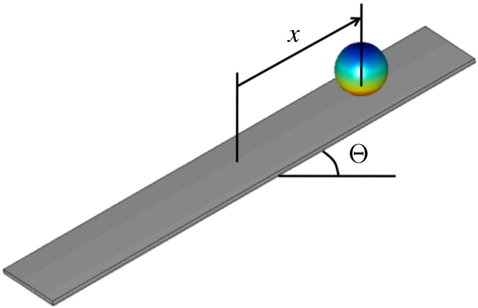
- Intelligent Control
  - Apply Artificial Intelligence techniques to control system engineering to develop a smart robotic system that can make a strategic decison.
  - Learning: adjust system behavior according to experience, control system is rewarded for good responses and punished for bad ones (reinforcement learning)
  - Perception: use sensor measurements (such as camera, microphones) to deduce certain aspects
  - Planning: control system should be able to get goals and achieve them by making proper decisions
  - Cooperation control systems of many agents cooperate and compete to achieve a given goal

- Applications
  - Humanoid Bipedel Robot
  - NASA Mars Helicopter
  - Autonomous Racing
  - Autonomous Helicopter
  - Alphago
  - Robot Soccer
  - Monocular Pose Capture

## Part II: Control System Design

Proportional and Derivative (PD) Control of a Ball and Beam System

# Ball and Beam System

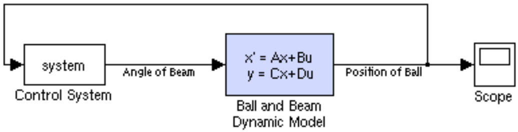



- Ball and Beam System
  - A ball rolling on a rotating beam (1D)
  - Input: Angle of beam $\Theta$
  - Output: Location of ball $x$
- Objective: control $\Theta$ such that $x \Rightarrow 0$
- This system is not stable without a control system

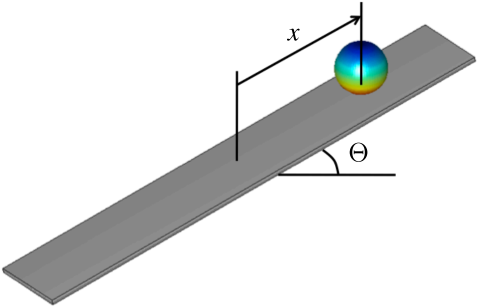
# Matlab/Simulink

# Control System Design

- Control System

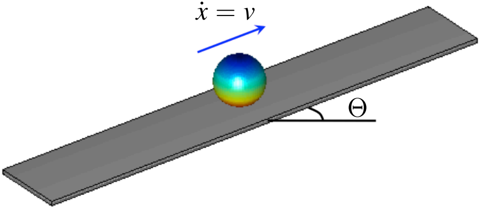



  - For a given current position $x$, control the angle of beam $\Theta$ such that the ball is centered, i.e. $x \Rightarrow 0$.
  - The angle $\Theta$ should be written as a function of position $x$.




- Control Strategy
  - Ball is at right ($x > 0$): Rotate beam counterclockwise ($\Theta > 0$)
  - Ball is at left ($x < 0$): Rotate beam clockwise ($\Theta < 0$)
- Proportional Controller
  - The angle of beam is proportional to the position: $\Theta = Kx$
  - Proportional controller gain: $K > 0$

# Matlab/Simulink




- Control Strategy
  - Ball is *moving* right ($\dot{x} > 0$): Rotate beam counterclockwise ($\Theta > 0$)
  - Ball is *moving* left ($\dot{x} < 0$): Rotate beam clockwise ($\Theta < 0$)
- Proportional and Derivative Controller
  - The angle of beam is proportional to the position and the velocity: $\Theta = Kx + K_p\dot{x}$
  - Proportional controller gain: $K > 0$, Derivative gain: $K_p > 0$.

Matlab/Simulink

- PID control
  - Proportional term: $Kx$ (**current** error)
  - Derivative term: $K_d\dot{x}$ (**prediction**)
  - Integral term
    - Control input is proportional to the integration of the **past** error:

$$K_i \int_0^t x(\tau)d\tau$$

    - Compensate the effects of modeling error and disturbance to improve robustness

# State Estimation

- Motivation
  - Control input requires the knowledge of the current state of the dynamic system (e.g., position and velocity)
  - It is challenging to measure all of the states directly
  - Sensor measurements are *always* corrupted by noise, bias, and delay
- Estimation
  - Estimate the state based on the noisy measurements and the best knowledge of the dynamics
  - Should be distinguished from *filtering* in signal processing to suppress undesirable features and components from a signal
  - Kalman Filter: estimator designed for linear systems with Gaussian distributions
  - Composed of
    - propagation or prediction based on the dynamics
    - correction from the measurements

# Feedback Control