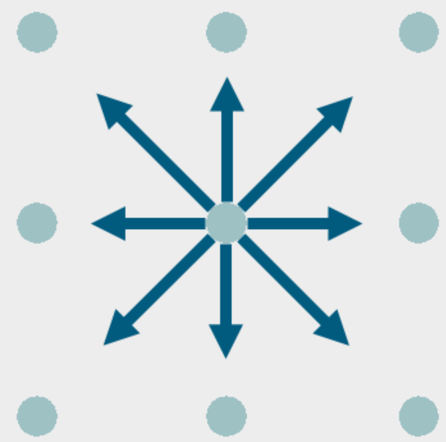


Lattice Boltzmann Method

Rycroft Group Journal Club

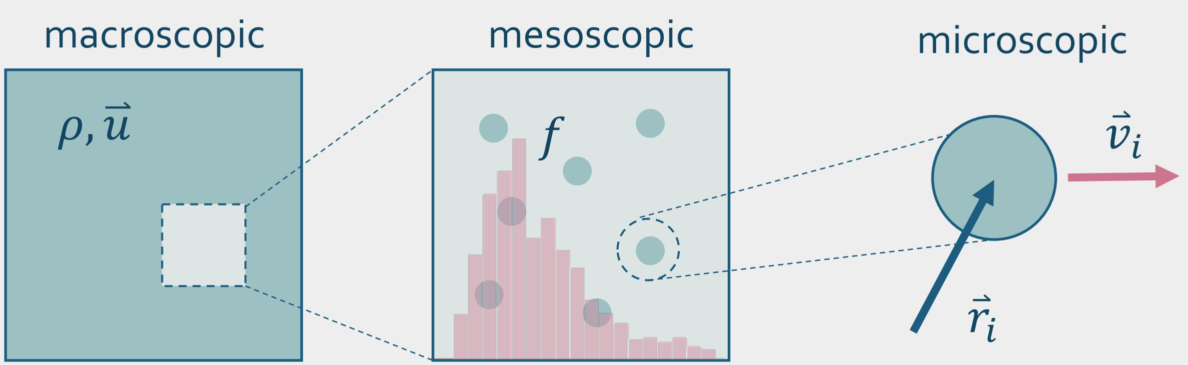
Oct. 25, 2018

Overview of lattice Boltzmann methods for fluids

- Replace a discretization of *macroscopic* continuum equations with a *mesoscopic* particle description which borrows from kinetic theory of gases

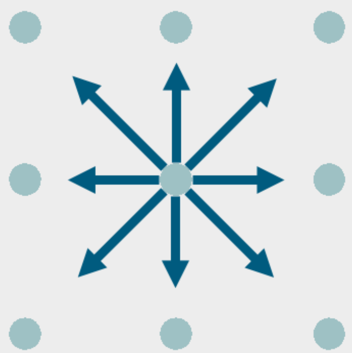
- Moments of particle distribution functions give back the macroscopic quantities such as density, velocity, etc.

- Particle transport governed by a discretized Boltzmann equation



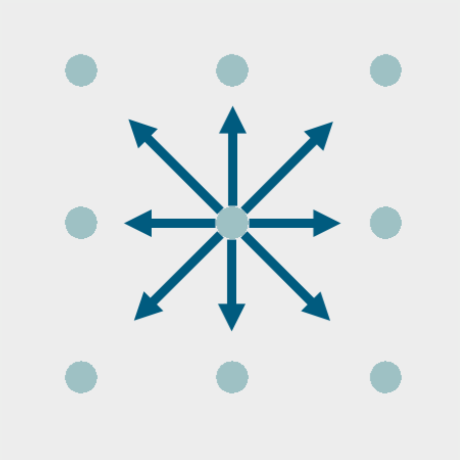
Advantages

- Simpler and more efficient for multiphase/multicomponent flows, complex obstacles and boundaries, and inhomogeneous media than other approaches
- Highly parallelizable
- Interactions are local, which helps in managing memory constraints
- Not limited to fluids



Challenges

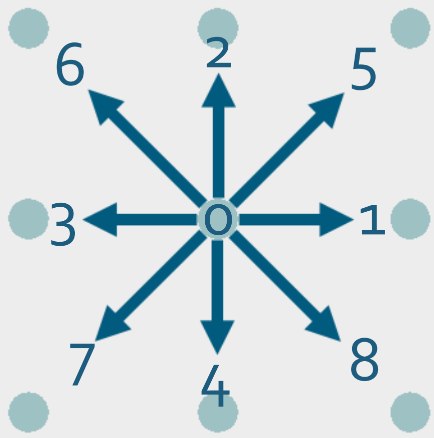
- By design, limited to low Mach number flows
- Larger number of variables to keep track of
- Higher complexity required for methods beyond 2nd order accuracy



From Boltzmann to Lattice Boltzmann

*From Dr. Federico Toschi's lectures, courtesy of Vamsi

9-Bit Model



$\vec{e}_\alpha$	$e_x = -c$	$e_x = 0$	$e_x = c$
$e_y = c$	$(-c, c)$	$(0, c)$	(c, c)
$e_y = 0$	$(-c, 0)$	$(0,0)$	$(c, 0)$
$e_y = -c$	$(-c, -c)$	$(0, -c)$	$(c, -c)$

w_α			
	$\frac{\pi}{36}$	$\frac{\pi}{9}$	$\frac{\pi}{36}$
	$\frac{\pi}{9}$	$\frac{4\pi}{9}$	$\frac{\pi}{9}$
	$\frac{\pi}{36}$	$\frac{\pi}{9}$	$\frac{\pi}{36}$

Simulation steps

1. If $t = 0$, initialize $\rho, \vec{u}$ at grid points; otherwise compute from current f_α's

2. Apply boundary conditions to populate missing f_α's at boundaries

3. Compute equilibrium particle distributions $f_\alpha^{eq}\left(\vec{r}_{ij}, t_n\right)$

Simulation steps

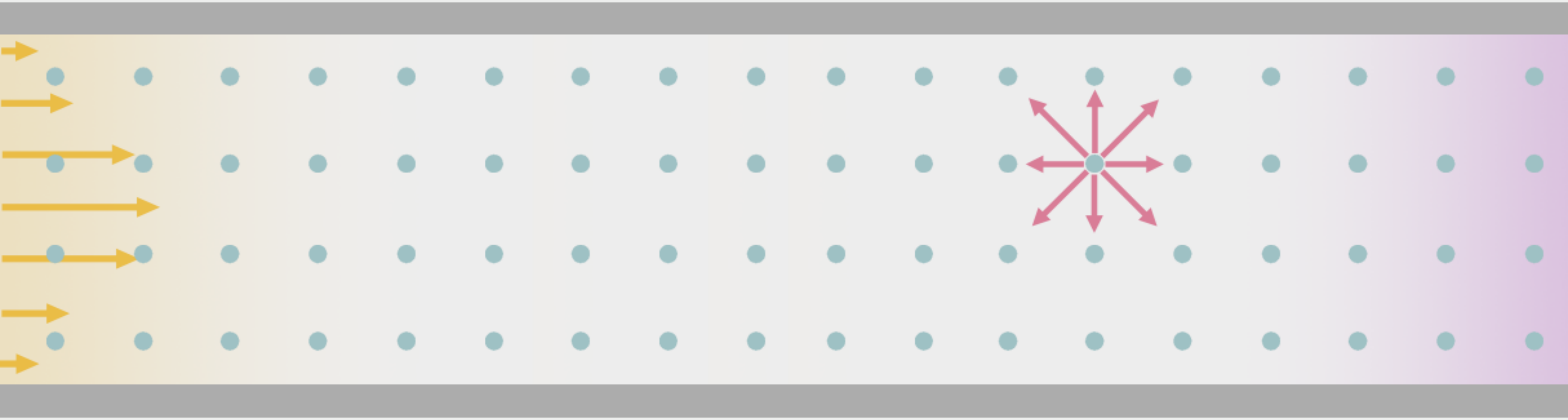
4. Collision step

$$f_\alpha^+(\vec{r}_{ij}, t_n) = f_\alpha(\vec{r}_{ij}, t_n) - \frac{1}{\tau}\Big(f_\alpha(\vec{r}_{ij}, t_n) - f_\alpha^{eq}(\vec{r}_{ij}, t_n)\Big)$$

5. Streaming step

$$f_\alpha(\vec{r}_{ij} + \vec{e}_\alpha \delta_t, t_n + \delta_t) = f_\alpha^+(\vec{r}_{ij}, t_n)$$

Boundary conditions



Constant velocity

$\vec{u} = \mathbf{0}$ — Wall

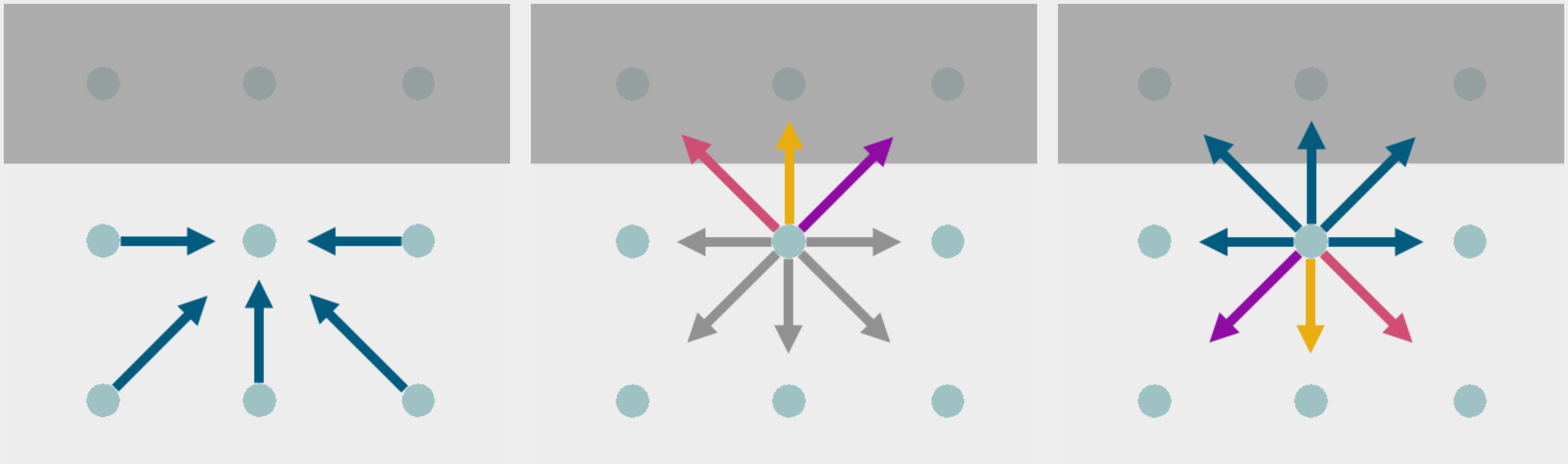
$\vec{u} = (u_x(y), 0)$ — Inlet with fixed velocity profile

Constant density

$\rho = \rho_o$ — Outlet with fixed pressure

$$\rho(\vec{r}_{ij})u_y(\vec{r}_{ij}) = \rho(\vec{r}_{ij} \pm c\delta_t\hat{x})u_y(\vec{r}_{ij} \pm c\delta_t\hat{x})$$

Bounce-back



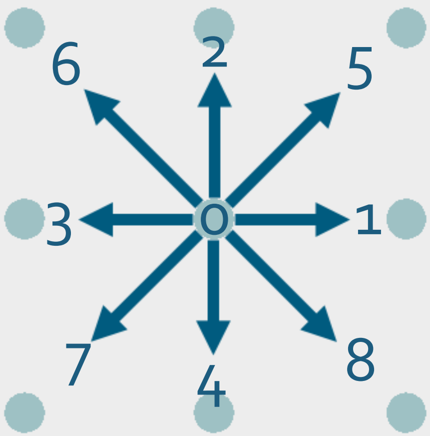
Particle populations streaming in

Particle populations streaming out, some directed at the wall

Reverse the flow of particle streams incident on the wall

$$f_\alpha(\vec{r}_{ij}, t_n) = f^+_{-\alpha}(\vec{r}_{ij}, t_n - \delta_t)$$

Corrector vector



$$f_\alpha(\vec{r}_{ij}, t_n) = f^* + \frac{w_\alpha}{c} \vec{e}_\alpha \cdot \vec{Q}$$

Conserved quantities

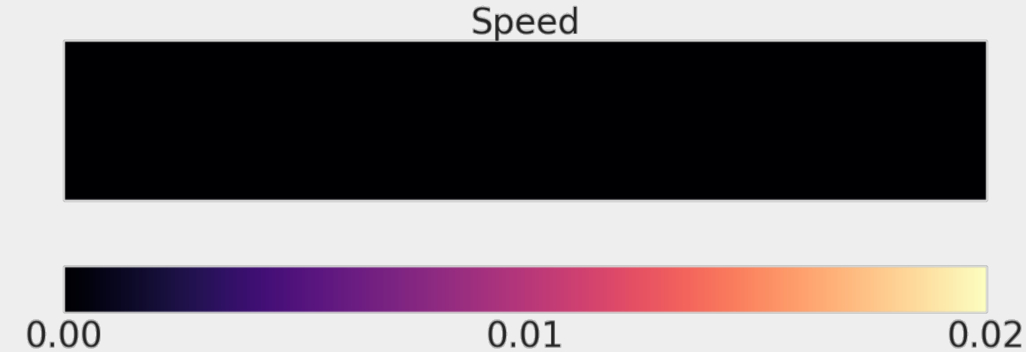
$$\rho = f_0 + f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 + f_8$$

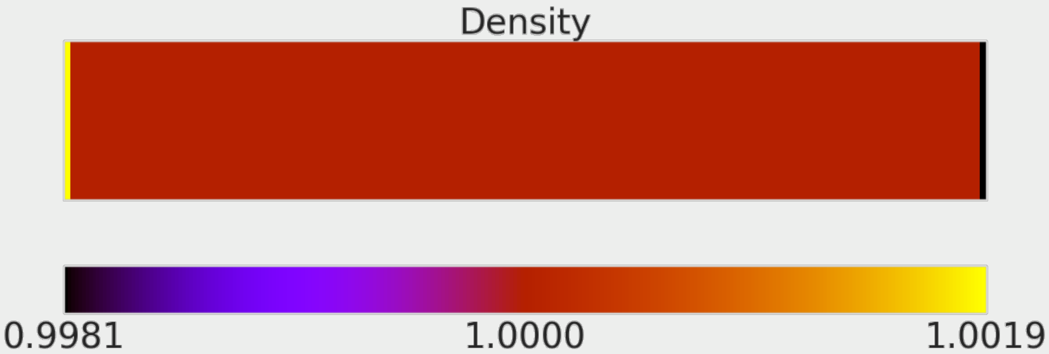
$$u_x = \frac{c}{\rho}(f_1 - f_3 + f_5 - f_6 - f_7 + f_8)$$

$$u_y = \frac{c}{\rho}(f_2 - f_4 + f_5 + f_6 - f_7 - f_8)$$

Zou-He: $f^* = f_{-\alpha}(\vec{r}_{ij}, t_n)$

Poiseuille Flow

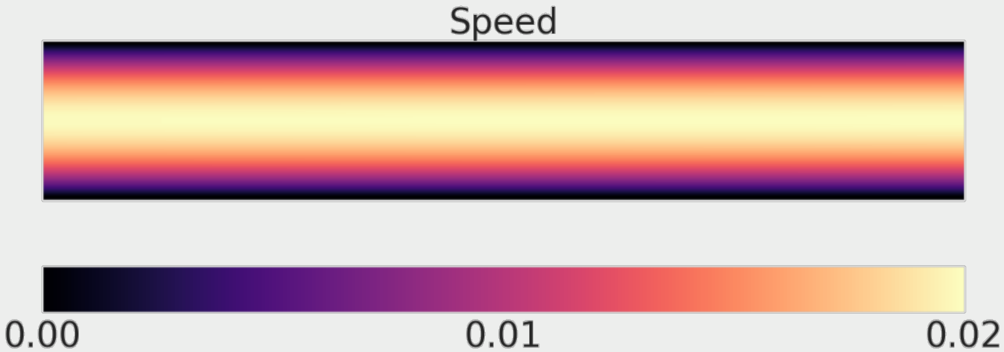


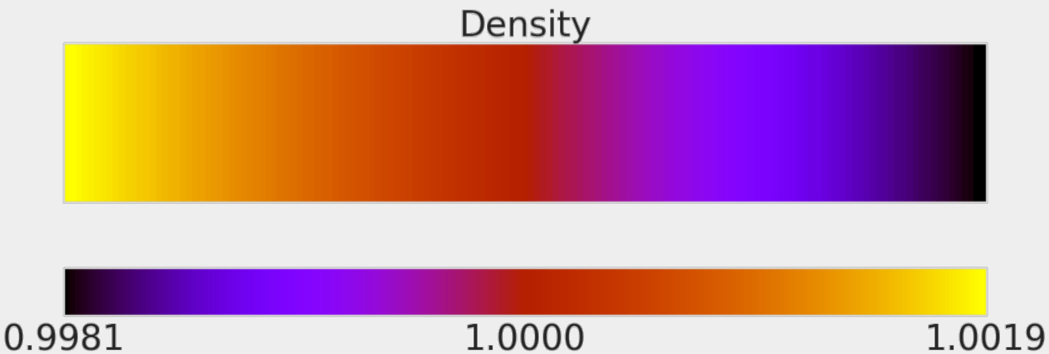


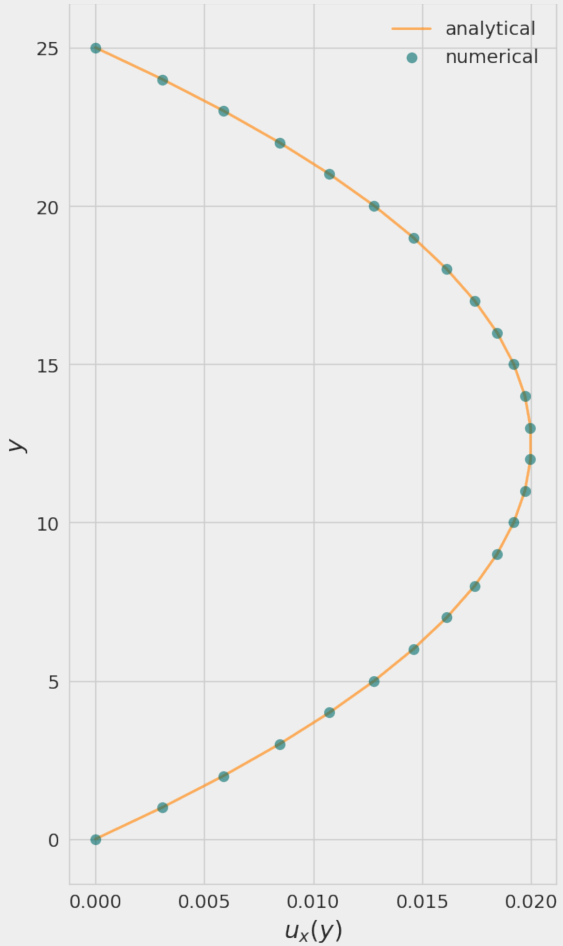
$n_x = 151$ $n_y = 26$ $\mathrm{Re} = 10$

$L_x = 150$ $L_y = 25$ $\tau = 0.6$

Poiseuille Flow







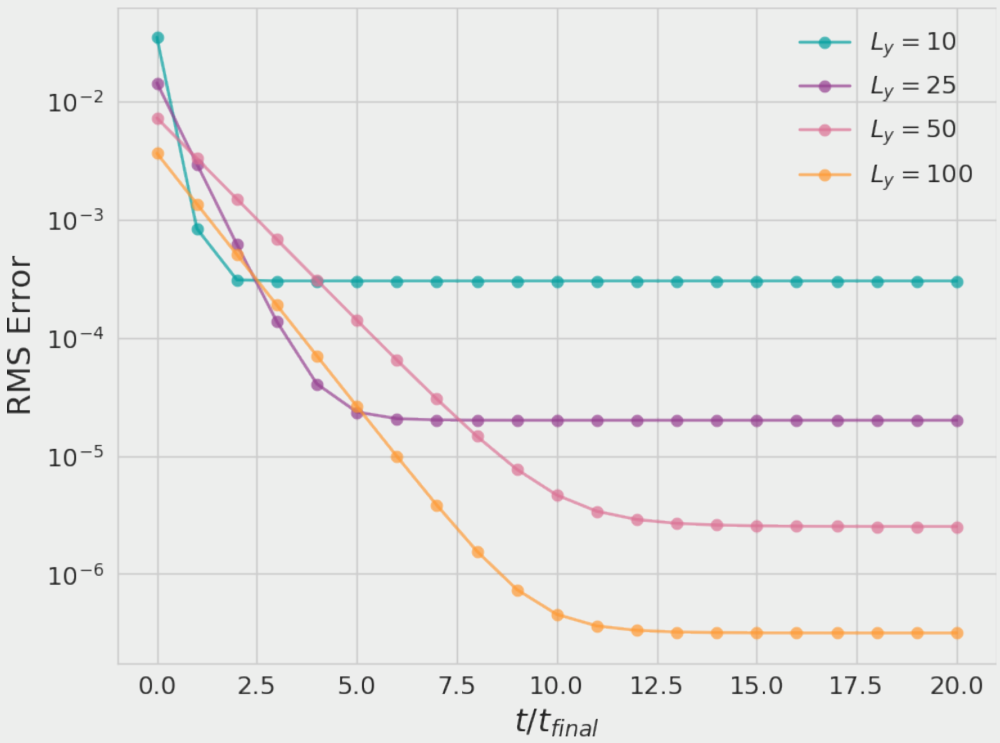
$$u_x(y) = \frac{6\,\mathrm{Re}\,\nu}{L_y} y(1-y)$$

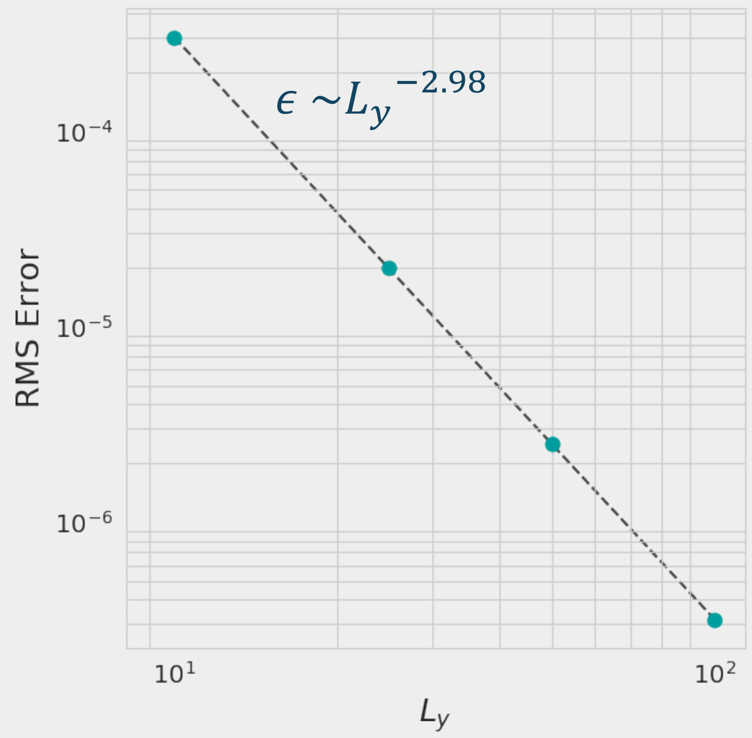
$$n_x = 151 \qquad n_y = 26 \qquad \mathrm{Re} = 10$$

$$L_x = 150 \qquad L_y = 25 \qquad \tau = 0.6$$

Poiseuille Flow

Convergence with varying grid resolution





$$\mathrm{Re} = 10$$
$$\tau = 0.6$$

RMS Error: $$\epsilon = \sqrt{\frac{1}{n_x n_y} \sum_{i=1}^{n_x n_y} \|\vec{u}_{num} - \vec{u}_{ana}\|^2}$$

Flow past a cylinder

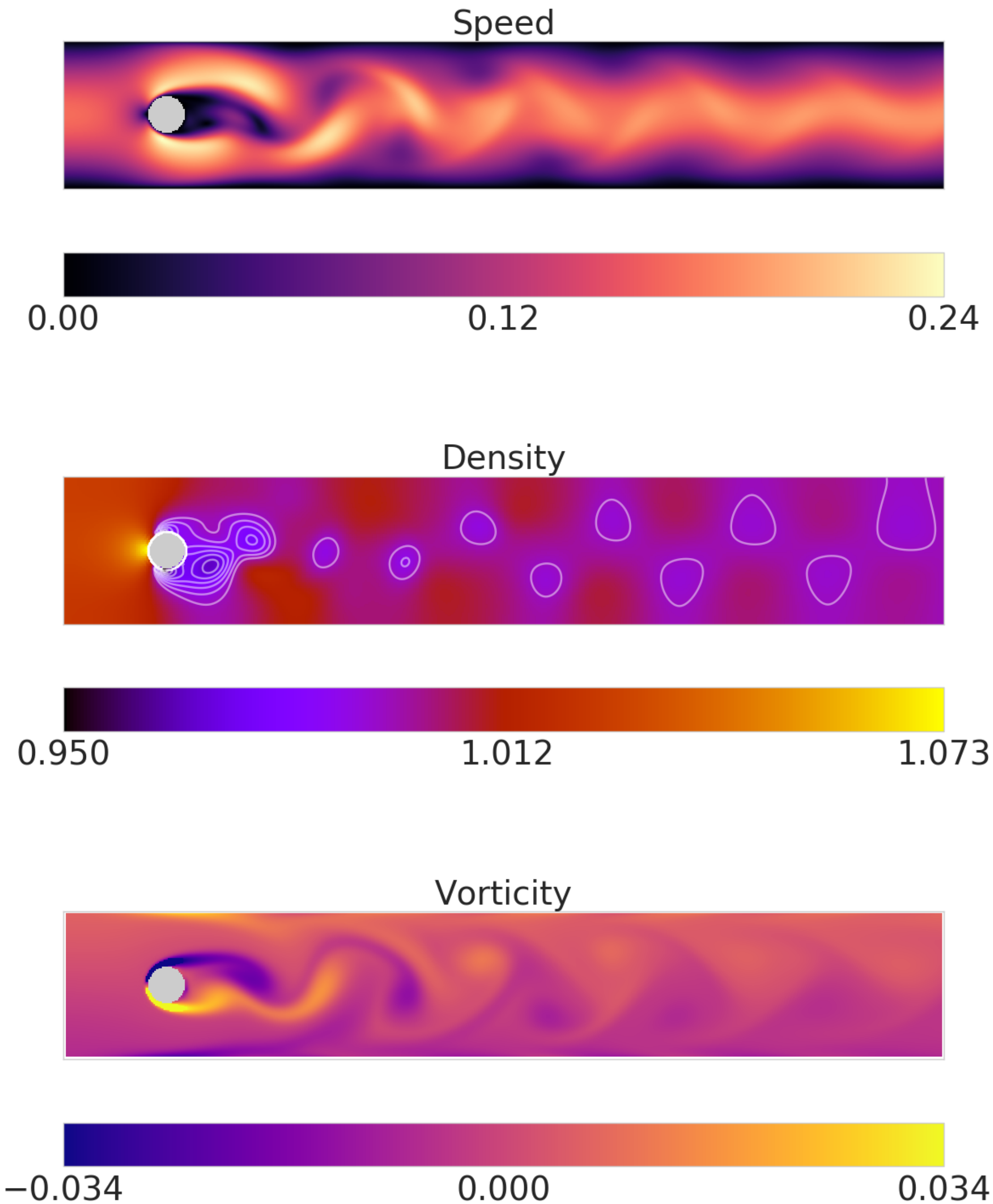
$$n_x = 601 \qquad n_y = 101$$
$$L_x = 600 \qquad L_y = 100$$

$$\mathrm{Re} = 80$$
$$\tau = 0.6$$

Boundary conditions
- Inlet: steady state Poiseuille velocity profile
- Outlet: constant density
- Walls and cylinder: zero velocity

Completing f_α's
- Inlet/Outlet/Walls: Zou-He
- Cylinder: Bounce-back

(see movies folder)



Flow past airfoil

$n_x = 1201 \qquad n_y = 301$

$L_x = 1200 \qquad L_y = 300$

$\mathrm{Re} = 600$

$\tau = 0.6$

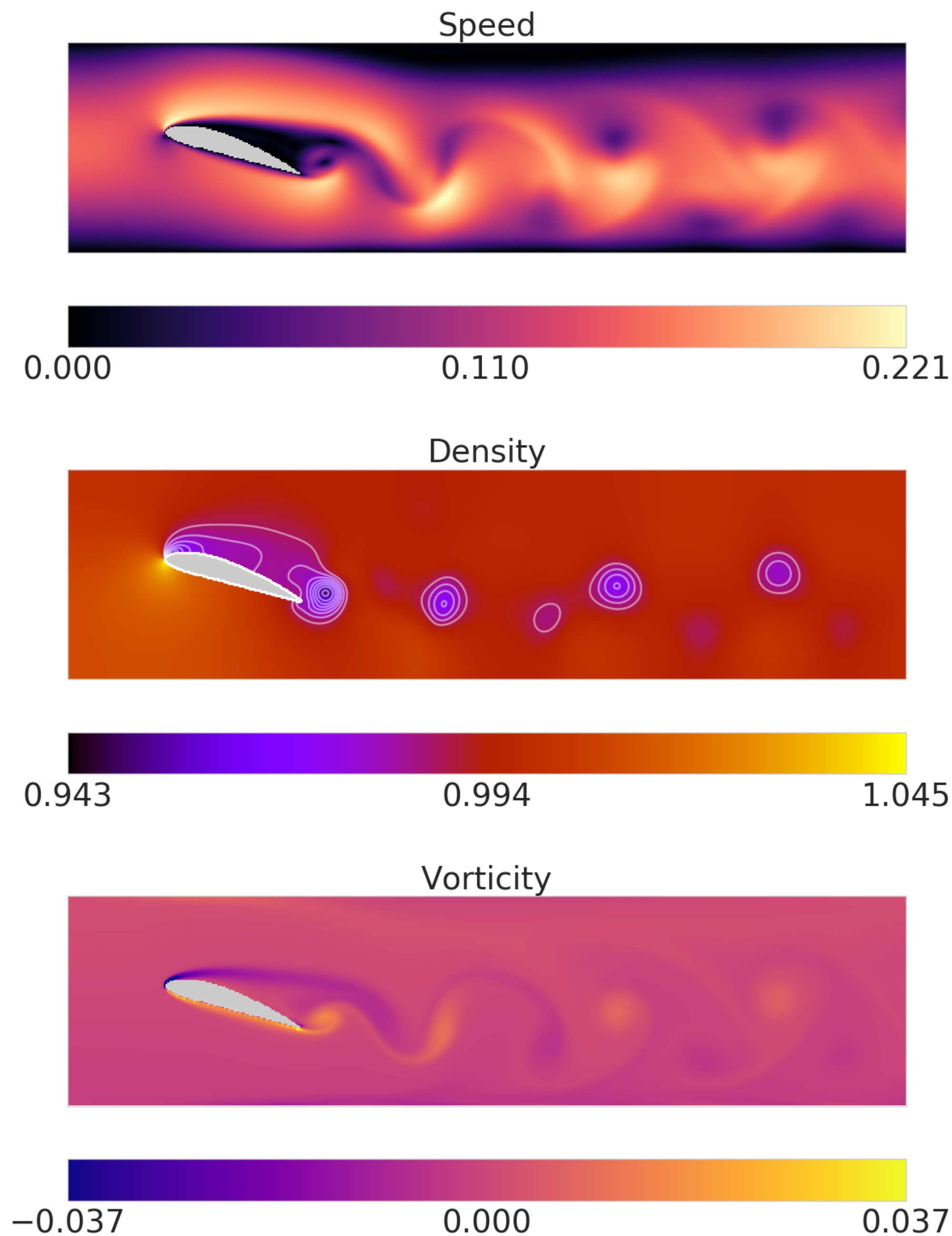
Boundary conditions

- Inlet: steady state Poiseuille velocity profile
- Outlet: constant density
- Walls and cylinder: zero velocity

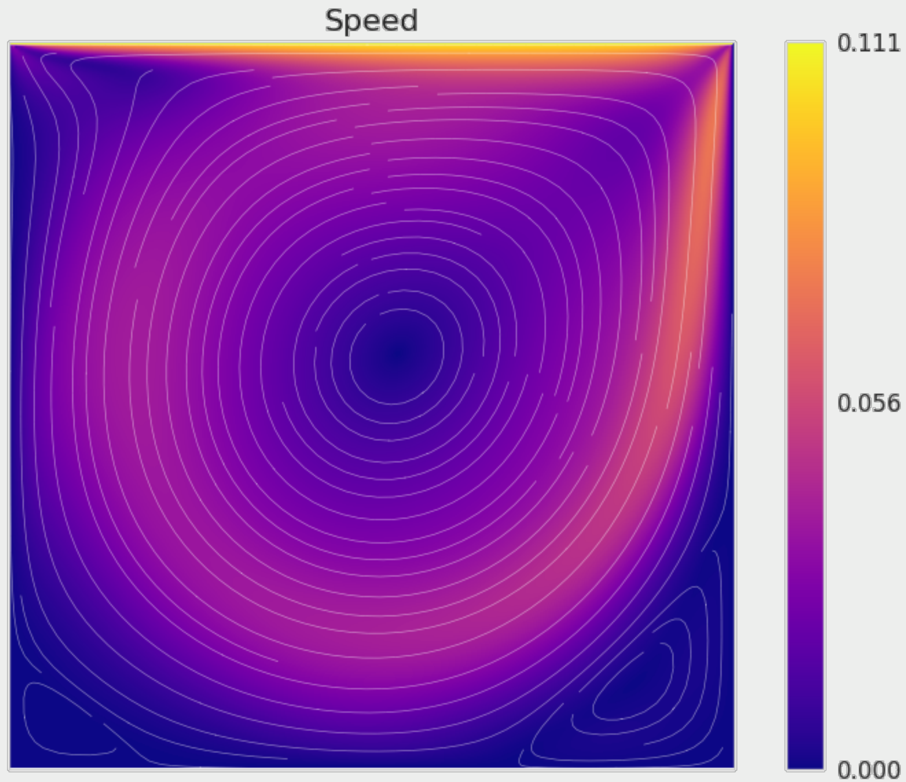
Completing f_α's

- Inlet/Outlet/Walls: Zou-He
- Airfoil: Bounce-back

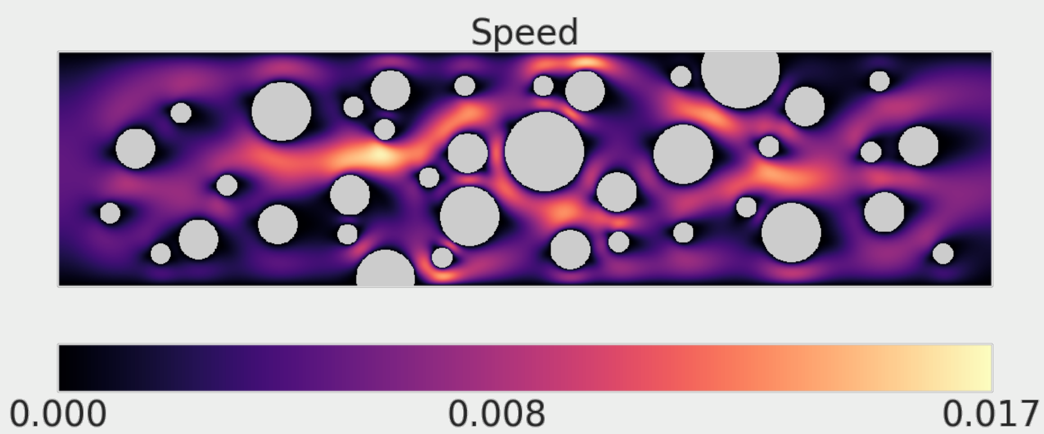
(see movies folder)



Lid-driven cavity flow



Flow through a porous medium



References

1. Chen, Shiyi, and Gary D. Doolen. "Lattice Boltzmann method for fluid flows." *Annual review of fluid mechanics* 30.1 (1998): 329-364.

2. He, Xiaoyi, and Li-Shi Luo. "A priori derivation of the lattice Boltzmann equation." *Physical Review E* 55.6 (1997): R6333.

3. Nourgaliev, R. Robert, et al. "The lattice Boltzmann equation method: theoretical interpretation, numerics and implications." *International Journal of Multiphase Flow* 29.1 (2003): 117-169.

4. Bao, Yuanxun Bill, and Justin Meskas. "Lattice Boltzmann method for fluid simulations." *Department of Mathematics, Courant Institute of Mathematical Sciences, New York University* (2011).

5. Liu, H., J. G. Zhou, and R. Burrows. "Inlet and outlet boundary conditions for the lattice-Boltzmann modelling of shallow water flows." *Progress in Computational Fluid Dynamics, an International Journal* 12.1 (2012): 11-18.

6. Chang, Cheng, Chih-Hao Liu, and Chao-An Lin. "Boundary conditions for lattice Boltzmann simulations with complex geometry flows." *Computers & Mathematics with Applications*58.5 (2009): 940-949.