2019/05/19

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第 I 部
真空中のマクスウェル方程式

$$F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2} \qquad E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

1. ガウスの法則

S S'

$$\begin{aligned}\int E\cdot n\ dS &= \frac{q}{4\pi\epsilon_0}\frac{1}{|r|^3}\int |r||n|\cos\theta\ dS' \\ &= \frac{q}{4\pi\epsilon_0}\frac{1}{|r|^2}\int dS' \\ &= \frac{q}{\epsilon_0}\end{aligned}$$

$q=\int\rho dV$

$$\begin{aligned}\int E\cdot n\ dS = \int \mathrm{div}E dV = \frac{1}{\epsilon_0}\int \rho dV \\ \mathrm{div}E = \frac{\rho}{\epsilon_0}\end{aligned}$$

■帯電球の静電場 a

$r\leq a$

$$\begin{aligned}4\pi r^2\cdot E &= \frac{r^3}{a^3}\cdot\frac{q}{\epsilon_0} \\ E &= \frac{1}{4\pi\epsilon_0}\frac{qr}{a^3}\end{aligned}$$

$r>a$

$$\begin{aligned}4\pi r^2\cdot E &= \frac{q}{\epsilon_0} \\ E &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\end{aligned}$$

$$E=\begin{cases}\frac{1}{4\pi\epsilon_0}\frac{qr}{a^3} & (r\leq a)\\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} & (r>a)\end{cases}$$

2. ファラデーの法則

F 0

E 0

$$\begin{aligned}\oint E\cdot ds = \int \mathrm{rot}\ E\cdot n dS = 0 \\ \mathrm{rot}E = 0\end{aligned}$$

■ポアソン方程式 E ϕ

$$
\begin{aligned}
E &= -\nabla\phi (= -\mathrm{grad}\phi) \\
&= \left(-\frac{\partial\phi}{\partial x}, -\frac{\partial\phi}{\partial y}, -\frac{\partial\phi}{\partial z}\right)
\end{aligned}
$$

$$
\begin{aligned}
-\mathrm{div}\ \mathrm{grad}\phi &= -\frac{\rho}{\epsilon_0} \\
\Delta\phi &= -\frac{\rho}{\epsilon_0}
\end{aligned}
$$

Δ

$$
\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}
$$

$$
\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = -\frac{\rho}{\epsilon_0}
$$

ϕ Φ

$$
\phi = -k\frac{d\Phi}{dt}
$$

$$
\begin{aligned}
\phi &= \int E\cdot ds \\
\Phi &= \int B\cdot ndS
\end{aligned}
$$

$$
\begin{aligned}
\int \mathrm{rot}E\cdot ndS &= -k\frac{d}{dt}\int B\cdot ndS \\
\int\left(\mathrm{rot}E + k\frac{\partial B}{\partial t}\right)\cdot ndS &= 0
\end{aligned}
$$

k 1

$$
\mathrm{rot}E + \frac{\partial B}{\partial t} = 0
$$

3. 磁束保存の法則

$$dB = \frac{\mu_0}{4\pi}\frac{Ids \times r}{|r|^3}$$

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{i(r') \times (r - r')}{|r - r'|^3} dr'$$

$$A = \frac{\mu_0}{4\pi}\int \frac{i(r')}{|r - r'|} dr'$$

$B = \mathrm{rot}A$

$$\mathrm{div}B = \mathrm{div}\ \mathrm{rot}A = 0$$

4. アンペールの法則

$$\begin{aligned}\mathrm{rot}B &= \mathrm{rot}\ \mathrm{rot}A \\ &= \mathrm{grad}\ \mathrm{div}A - \Delta A \\ &= \mu_0 i\end{aligned}$$

$\mathrm{div}\ i = -\frac{\partial \rho}{\partial t}$

$$\mathrm{rot}B - \epsilon_0\mu_0\frac{\partial E}{\partial t} = \mu_0 i$$

5. ローレンツ力

$$|F| = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

$$dF = I\ ds \times B$$

$1\ m$ n

$I = qnv$

$$dF = qnv\ ds \times B$$

$n\ ds$

$$F = qv \times B$$

$$F = q(E + v \times B)$$

第 II 部
物質中の電磁場

E B

$$\mathrm{div}E = \frac{\rho}{\epsilon_0}$$
$$\mathrm{rot}B - \epsilon_0\mu_0\frac{\partial E}{\partial t} = \mu_0 i$$

1. 電束密度

D

$$\int D \cdot dS = q$$
$$\mathrm{div}D = \rho$$

P

$$\int \epsilon_0 E \cdot dS = q - \int P \cdot dS$$
$$\int (\epsilon_0 E + P) \cdot dS = q$$

$$D = \epsilon_0 E + P$$

$P = \chi E$ χ

$\epsilon = \epsilon_0 + \chi$ $D = \epsilon E$

2. 磁場

H

$$\int H \cdot dl = I$$
$$\mathrm{rot}H = i$$

i_m

$$\mathrm{rot}B = \mu_0(i + i_m)$$

$\mathrm{rot}J = \mu_0 i_m$ J 0

$$\mathrm{rot}(B - J) = \mu_0 i$$

$$H = \frac{1}{\mu_0}(B - J)$$

$J = \chi_m H$ χ_m

$\mu = \mu_0 + \chi_m$ $B = \mu H$

$$\mathrm{rot}H - \frac{\partial D}{\partial t} = i$$

$$\begin{aligned}\mathrm{div}\ i &= -\frac{\partial \rho}{\partial t} \\ &= -\mathrm{div}\left(\frac{\partial D}{\partial t}\right)\end{aligned}$$

$$\mathrm{rot}H - \frac{\partial D}{\partial t} = i$$

$$\begin{aligned}\mathrm{div}D &= \rho \\ \mathrm{div}B &= 0\end{aligned}$$

$$\begin{aligned}\mathrm{rot}E + \frac{\partial B}{\partial t} &= 0 \\ \mathrm{rot}H - \frac{\partial D}{\partial t} &= i\end{aligned}$$

第 III 部

電磁ポテンシャル

1. 電磁ポテンシャル

ϕ A

$$
\begin{aligned}
E &= -\mathrm{rot}\phi - \frac{\partial A}{\partial t} \\
B &= \mathrm{rot} A
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{rot}E + \frac{\partial B}{\partial t} &= \mathrm{rot}E + \frac{\partial(\mathrm{rot}A)}{\partial t} \\
&= \mathrm{rot}\left(E + \frac{\partial A}{\partial t}\right) \\
&= -\mathrm{rot}\ \mathrm{grad}\phi \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{div}B &= \mathrm{div}\ \mathrm{rot}A \\
&= 0
\end{aligned}
$$

2. ゲージ変換

χ

$$
\begin{aligned}
\phi &\to \phi - \frac{\partial \chi}{\partial t} \\
A &\to A + \mathrm{grad}\chi
\end{aligned}
$$

3. ローレンツゲージにおけるマクスウェル方程式

$$
\begin{aligned}
\mathrm{rot}E &= \frac{\rho}{\epsilon_0} \\
\mathrm{rot}B - \mu_0\epsilon_0\frac{\partial E}{\partial t} &= \mu_0 i
\end{aligned}
$$

$$\Delta\phi + \mathrm{div}\frac{\partial A}{\partial t} = -\frac{\rho}{\epsilon_0}$$
$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)A - \mathrm{grad}\left(\mathrm{div}A + \epsilon_0\mu_0\frac{\partial\phi}{\partial t}\right) = -\mu_0 i$$

$$\mathrm{div}A + \epsilon_0\mu_0\frac{\partial\phi}{\partial t} = 0$$

$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)\phi = \frac{\rho}{\epsilon_0}$$
$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)A = -\mu_0 i$$

ϕ, A

χ χ

$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)\chi = -\mathrm{div}A + \epsilon_0\mu_0\frac{\partial\phi}{\partial t}$$

$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)\phi = \frac{\rho}{\epsilon_0}$$
$$\left(\Delta - \epsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)A = -\mu_0 i$$
$$\mathrm{div}A + \epsilon_0\mu_0\frac{\partial\phi}{\partial t} = 0$$

第 IV 部
マクスウェル方程式の解

1. 電磁波

波動方程式

0

$$\mathrm{rot}E + \frac{\partial B}{\partial t} = 0$$
$$\mathrm{rot}B - \mu_0\epsilon_0\frac{\partial E}{\partial t} = 0$$
$$\mathrm{div}E = 0$$
$$\mathrm{div}B = 0$$

rot

$$\mathrm{rot}\ \mathrm{rot}E + \frac{\partial}{\partial t}(\mathrm{rot}B) = 0$$

$\mathrm{rot}\ \mathrm{rot}X = \mathrm{grad}\ \mathrm{div}X - \Delta X$

$$\mathrm{grad}\ \mathrm{div}E - \Delta E + \frac{\partial}{\partial t}(\mathrm{rot}B) = 0$$
$$\left(\Delta - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)E = 0$$

$$\left(\Delta - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)B = 0$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

3

$$E = F(e\cdot r - ct) + G(e\cdot r + ct)$$

e

電磁波の性質

$$\mathrm{div}E = 0$$

$$\begin{aligned}
&\mathrm{div}F(e\cdot r - ct) + G(e\cdot r + ct)\\
&\qquad = \frac{\partial}{\partial x}(F_x + G_x) + \frac{\partial}{\partial y}(F_y + G_y) + \frac{\partial}{\partial z}(F_z + G_z)\\
&\qquad = e_x(F'_x + G'_x) + e_y(F'_y + G'_y) + e_z(F'_z + G'_z)\\
&\qquad = e\cdot(F' + G') = 0
\end{aligned}$$

z

$$(E_x, E_y, E_z) = (F(z-ct) + G(z+ct), 0, 0)$$

$$\left(\frac{\partial B_x}{\partial t}, \frac{\partial B_y}{\partial t}, \frac{\partial B_z}{\partial t}\right) = (0, -F'(z-ct) - G'(z+ct), 0)$$

$$(B_x, B_y, B_z) = (0, -\frac{1}{c}F(z-ct) - \frac{1}{c}G(z+ct), 0)$$

2. 遅延ポテンシャル

- ρ i r,t
-
- ρ, i 0
- ρ, i 0
-
-

$t_r = t - \frac{|r-r'|}{c}$

$$\phi(r,t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r',t_r)}{|r-r'|}dr'$$
$$A(r,t) = \frac{\mu_0}{4\pi}\int \frac{i(r',t_r)}{|r-r'|}dr'$$

$t_r = t - \frac{|r-r'|}{c}$ $t_r = t + \frac{|r-r'|}{c}$

$R = r - r'$

$$E(r,t) = \frac{1}{4\pi\epsilon_0}\int \left[\frac{\rho(r',t_r)R}{|R|^3} + \frac{\rho(r',t_r)R}{c|R|^2} - \frac{i(r',t_r)R}{c^2|R|^2}\right]dr'$$
$$B(r,t) = \frac{\mu_0}{4\pi}\int \left[\frac{i(r',t_r)R}{|R|^3} + \frac{i(r',t_r)R}{c|R|^2}\right]dr'$$

3. リエナール・ヴィーヘルト・ポテンシャル

$s(t)$

$$\rho(r,t) = q\delta(r - s(t))$$
$$i(r,t) = q\dot{s}(t)\delta(r - s(t))$$

$$\phi(r,t) = \frac{q}{4\pi\epsilon_0}\frac{1}{|r-s(t_r)| - \beta(t_r)\cdot(r-s(t_r))}$$
$$A(r,t) = \frac{\mu_0 q}{4\pi}\frac{\dot{s}(t_r)}{|r-s(t_r)| - \beta(t_r)\cdot(r-s(t_r))}$$

$$t_r = t - \frac{|r-s(t)|}{c}$$
$$\beta(t_r) = \frac{\dot{s}(t_r)}{c}$$

4. 等速運動する点電荷

x v $t=0$

$$\phi = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(x-vt)^2 + (1-\frac{v^2}{c^2})(y^2+z^2)}}$$
$$A = \frac{\mu_0 q}{4\pi}\frac{v}{\sqrt{(x-vt)^2 + (1-\frac{v^2}{c^2})(y^2+z^2)}}$$

$(x-vt, y, z)$ x

yz

5. 加速運動する点電荷

$s(t)$ r

$R = |r-s(r)|, n = (r-s(r))/|r-s(r)|$

$$S(r,t) = \frac{q^2}{16\pi^2\epsilon_0 c}\frac{n}{(1-n\cdot v)^6 R^2}[n\times\{(n-v)\times\dot{v}\}]^2$$

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第 V 部

電磁場の力と保存量

1. 電磁場のエネルギー

静電場のエネルギー

r

q

q'

ϕ $E = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}$ 0

$$\phi = \frac{q'}{4\pi\epsilon_0 r}$$

dq'

$$\begin{aligned}
U(r) &= \int_0^q \frac{q'}{4\pi\epsilon_0 r} \\
&= \frac{q^2}{8\pi\epsilon_0 r}
\end{aligned}$$

$r, r + dr$

dr $u(E)$

$$\begin{aligned}
U(r + dr) - U(r) &= -u(E) \cdot 4\pi r^2 dr \\
\frac{U(r + dr) - U(r)}{dr} &= -u(E) \cdot 4\pi r^2
\end{aligned}$$

$$\begin{aligned}
u(E) &= -\frac{1}{4\pi r^2}\frac{dU}{dr} \\
&= -\frac{1}{4\pi r^2} \cdot -\frac{q^2}{8\pi\epsilon_0 r^2} = \frac{q^2}{32\pi^2\epsilon_0 r^4} \\
&= \frac{1}{2}\epsilon_0 \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)^2 \\
&= \frac{1}{2}\epsilon_0 E^2
\end{aligned}$$

静磁場のエネルギー

電磁場のエネルギー

$$m\frac{dv}{dt} = q(E(r) + v \times B(r))$$

v

$$mv \cdot \frac{dv}{dt} = E(r) \cdot (qv) + qv \cdot (v \times B(r))$$
$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = E(r) \cdot i(r)$$

$qv = i(r)$ v $v \times B(r)$ $v \cdot (v \times B(r)) = 0$

$$\frac{d}{dt}\left(\sum_j \frac{1}{2}m_j v_j^2\right) = E \cdot i$$

E i

$$i = \mathrm{rot}H - \frac{\partial D}{\partial t}$$
$$\begin{aligned} E \cdot (\nabla \times H) &= H \cdot (\nabla \times E) - \nabla \cdot (E \times H) \\ &= -H \cdot \frac{\partial B}{\partial t} - \nabla \cdot (E \times H) \end{aligned}$$

$$\begin{aligned} E \cdot i &= E \cdot \left(\mathrm{rot}H - \frac{\partial D}{\partial t}\right) \\ &= -E \cdot \frac{\partial D}{\partial t} - H \cdot \frac{\partial B}{\partial t} - \nabla \cdot (E \times H) \\ &= -\frac{\partial}{\partial t}\left(\frac{1}{2}E \cdot D + \frac{1}{2}H \cdot B\right) - \nabla \cdot (E \times H) \end{aligned}$$

$u = \frac{1}{2}(E \cdot D + H \cdot B)$ $S = E \times H$

$$-\frac{\partial u}{\partial t} = E \cdot i + \mathrm{div}S$$

(Poynting)

2. マクスウェルの応力

V

$$dF = \rho(r)E(r) + i(r) \times B(r)$$

$$F = \int_V \rho(r)E(r) + i(r) \times B(r)dV$$

$$\mathbf{div}\ D = \rho$$
$$\mathbf{rot}\ H - \frac{\partial D}{\partial t} = i$$

$$\begin{aligned} F &= \int \left[E\mathbf{div}\ D + \left(\mathbf{rot}\ H - \frac{\partial D}{\partial t} \right) \times B \right] dV \\ &= \int \left[E\mathbf{div}\ D - B \times \mathbf{rot}\ H - \frac{\partial D}{\partial t} \times B \right] dV \end{aligned}$$

$$-\frac{\partial D}{\partial t} \times B = -D \times \mathbf{rot}\ E - \frac{d}{dt}(D \times B)$$

$$= \int \left[\epsilon_0 E\mathbf{div}\ E - \frac{1}{\mu_0} B \times \mathbf{rot}\ B - \epsilon_0 E \times \mathbf{rot}\ E - \epsilon_0 \mu_0 \frac{d}{dt}(E \times H) \right] dV$$

$\mathbf{div}\ B = 0$ $B\mathbf{div}\ B$

$$\begin{aligned} &= \int \left[\epsilon_0 (E\mathbf{div}\ E - E \times \mathbf{rot}\ E) + \frac{1}{\mu_0}(B\mathbf{div}\ B - B \times \mathbf{rot}\ B) \right] dV \\ &\quad - \frac{1}{c^2} \frac{d}{dt} \left[\int (E \times H) dV \right] \end{aligned}$$

$E \times H$

$E\mathbf{div}\ E - E \times \mathbf{rot}\ E$

x

$$
\begin{aligned}
(E\mathbf{div}\ E - E\times\mathbf{rot}\ E)_x &= E_x\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)\\
&\quad -E_y\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)\\
&\quad +E_z\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)\\
&= E_x\frac{\partial E_x}{\partial x}-E_y\frac{\partial E_y}{\partial x}-E_z\frac{\partial E_z}{\partial x}\\
&\quad +E_x\frac{\partial E_y}{\partial y}+E_y\frac{\partial E_x}{\partial y}\\
&\quad +E_x\frac{\partial E_z}{\partial z}+E_z\frac{\partial E_x}{\partial z}\\
&= \frac{1}{2}\frac{\partial E_x^2}{\partial x}-\frac{1}{2}\frac{\partial E_y^2}{\partial x}-\frac{1}{2}\frac{\partial E_z^2}{\partial x}\\
&\quad +\frac{\partial(E_xE_y)}{\partial y}+\frac{\partial(E_xE_z)}{\partial z}\\
&= \frac{\partial(E_x^2-\frac{1}{2}E^2)}{\partial x}+\frac{\partial(E_xE_y)}{\partial y}+\frac{\partial(E_xE_z)}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
T &= T_e + T_m\\
T_e &= \epsilon_0\begin{bmatrix} E_x^2-\frac{1}{2}E^2 & E_xE_y & E_xE_z\\ E_yE_x & E_y^2-\frac{1}{2}E^2 & E_yE_z\\ E_zE_x & E_zE_y & E_z^2-\frac{1}{2}E^2\end{bmatrix}\\
T_m &= \frac{1}{\mu_0}\begin{bmatrix} B_x^2-\frac{1}{2}B^2 & B_xB_y & B_xB_z\\ B_yB_x & B_y^2-\frac{1}{2}B^2 & B_yB_z\\ B_zB_x & B_zB_y & B_z^2-\frac{1}{2}B^2\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
F &= \int \mathbf{div}\ T\ dV - \frac{1}{c^2}\frac{d}{dt}\int SdV\\
&= \int T\cdot dS - \frac{1}{c^2}\frac{d}{dt}\int SdV
\end{aligned}
$$

T

3. 電磁場の運動量

m q

$$
m\frac{d^2r}{dt^2} = \int\left[q\delta(r-r')E + q\delta(r-r')\frac{dr}{dt}\times B\right]dr'
$$

$$
\sum_i m_i\frac{d^2r_i}{dt^2} = \int\left[q_i\delta(r_i-r')E + q_i\delta(r_i-r')\frac{dr_i}{dt}\times B\right]dr'
$$

$$\mathbf{div}\ D = \sum_i q_i \delta(r_i - r')$$

$$\mathbf{rot}\ H - \frac{\partial D}{\partial t} = \sum_i q_i \delta(r_i - r') \frac{dr_i}{dt}$$

V

$$\sum_i m_i \frac{d^2 r_i}{dt^2} = \int T \cdot dS - \frac{1}{c^2} \frac{d}{dt} \int (E \times H) dV$$

$$\frac{d}{dt} \left[\sum_i m_i \frac{dr_i}{dt} + \frac{1}{c^2} \int (E \times H) dV \right] = \int T \cdot dS$$

第 VI 部

諸現象

1. 電気双極子モーメント

$+q, -q$ s p

$$p = qs$$

$$\begin{aligned} U &= -\left[q\frac{s}{2}|E|\cos\theta + (-q)\left(-\frac{s}{2}\right)|E|\cos\theta \right] \\ &= -p \cdot E \end{aligned}$$

$(-\frac{s}{2}, 0)(+\frac{s}{2}, 0)$

$r = (x, y, z)$

$$U(r) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$r_1 = \sqrt{r^2 + (s/2)^2 - rs\cos\theta}$$

$$r_2 = \sqrt{r^2 + (s/2)^2 + rs\cos\theta}$$

$s \ll r$

$$
\begin{aligned}
\frac{1}{r_1} &= \frac{1}{\sqrt{r^2 + (s/2)^2 - rs\cos\theta}} \\
&= \frac{1}{r}\left(1 + \left(\frac{s}{2r}\right)^2 - \frac{s}{r}\cos\theta\right)^{-\frac{1}{2}} \\
&\fallingdotseq \frac{1}{r}\left(1 - \frac{1}{2}\left[\left(\frac{s}{2r}\right)^2 - \frac{s}{r}\cos\theta\right]\right) \\
&= \frac{1}{r}\left(1 - \frac{s^2}{8r^2} + \frac{s}{2r}\cos\theta\right)
\end{aligned}
$$

$$
\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{s^2}{8r^2} - \frac{s}{2r}\cos\theta\right)
$$

$$
\begin{aligned}
U(r) &= \frac{q}{4\pi\epsilon_0}\frac{s}{r^2}\cos\theta \\
&= \frac{p \cdot r}{4\pi\epsilon_0 r^3}
\end{aligned}
$$

$E(r) = -\frac{\partial U}{\partial r}$

$$
\begin{aligned}
-\frac{\partial U}{\partial x} &= -\frac{\partial}{\partial x}\left(\frac{1}{4\pi\epsilon_0}\frac{p_x x + p_y y + p_z z}{(x^2+y^2+z^2)^{3/2}}\right) \\
&= -\frac{1}{4\pi\epsilon_0}\left[\frac{p_x}{(x^2+y^2+z^2)^{3/2}} - (p_x x + p_y y + p_z z)(\frac{3}{2})(x^2+y^2+z^2)^{5/2}(2x)\right] \\
&= -\frac{1}{4\pi\epsilon_0}\left[\frac{p_x}{r^3} - \frac{3x(p\cdot r)}{r^5}\right]
\end{aligned}
$$

y, z

$$
E(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{p}{r^3} - \frac{3r(p\cdot r)}{r^5}\right]
$$

2. 磁気モーメント

N S

$+q_m(N), -q_m(S)$ Wb $F = q_m H$ S N

s

$$
m = q_m s
$$

$$
\begin{aligned}
U &= -m \cdot H \\
H(r) &= -\frac{1}{4\pi\mu_0}\left[\frac{m}{r^3} - \frac{3r(m\cdot r)}{r^5}\right]
\end{aligned}
$$

m'

$$U = -m' \cdot B$$

$m' = m/\mu_0$

$$B(r) = -\frac{\mu_0}{4\pi}\left[\frac{m'}{r^3} - \frac{3r(m' \cdot r)}{r^5}\right]$$