ON THE KERNEL FUNCTION SELECTION OF NONLOCAL FILTERING FOR IMAGE DENOISING

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Abstract:

Nonlocal filtering has been proved to yield attractive performance for removing additive Gaussian noise from the image by replacing the intensity value of each pixel via a weighted average of that of the full image. The key challenge of the nonlocal filtering is to establish the kernel function for computing the above-mentioned weighting factors, which control the quality of the denoised image result. In contrast to that the exponential function is used in the conventional nonlocal filtering, several new kernel functions are proposed in this paper to be further incorporated into the conventional nonlocal filtering framework to develop new filters. Extensive experiments are conducted to demonstrate not only that the kernel function is essential to control the performance of the algorithm, but also that the new kernel functions proposed in this paper yield superior performance to that of the conventional nonlocal filtering.

Keywords:

Image Restoration; Image Reconstruction

1. Introduction

Image denoising aims to remove various noise that are incurred during image acquisition or image transmission from the observed noisy image, such as the additive noise or the multiplicative noise. Among of them, removing the additive Gaussian noise has remained as a fairly active research field in image processing. A noisy image that is corrupted with the additive white Gaussian noise can be mathematically modeled as

$$\mathbf{I} = \mathbf{S} + \mathbf{N}, \tag{1}$$

where **I** is the observed noisy image, **S** is the unknown original image, both of them are presented in the column vector. **N** represents the additive white Gaussian noise with a zero mean and a variance σ_n^2. The goal is to recover **S** from its noisy counterpart **I**.

The spatial domain smoothing has proved to be effective to remove the additive Gaussian noise from the noisy image. The basic idea of the spatial domain smoothing is to replace the intensity value of each pixel via a weighted average of those intensity values of its neighborhood. The above weights can be computed via the *box filter*, where each pixel value is replaced by the mean of its local neighbors, or via the *Gaussian filter*, where the values of the neighboring pixels are given different weighting that is defined by a spatial Gaussian distribution. Lee proposed a *sigma filter* [1] that takes an average of only those neighboring pixels whose values are larger than a user defined threshold. Tomasi and Manduchi proposed a *bilateral filter* [2] to compute the weighting factor as a function of the spatial distance measured from the center pixel, and the difference in the pixel intensity. This kind of filter has a fundamental relationship with robust estimation [3] and nonlinear diffusion [4].

In contrast to that only local neighborhood is considered in the conventional filtering methods [1], [2], Buades et al. proposed a nonlocal filtering technique in [5]–[7] to consider nonlocal neighborhoods of a given pixel; that is a full image. The aim of the nonlocal filtering is to search the full image to find the similar region, where the similarity is determined by a certain kernel function, to smooth each pixel. Furthermore, the spatial domain nonlocal filtering was further introduced by Souidene et al. into the wavelet domain [8]. The key challenge of the nonlocal filtering is the determination of the above-mentioned kernel function, since it controls the degree of the filtering and affects the quality of the denoised image result.

To tackle the above challenge, the establishment of the kernel function is discussed in this paper. Rather than exploiting the exponential function as in [5], several modified kernel functions are proposed in this paper to be incorporated into the conventional nonlocal filtering to arrive at new filtering methods.

The rest of the paper is organized as follows. A brief introduction to the nonlocal filtering will be first presented in Section II-A. Then the issue of establishing the kernel function is discussed in Section II-B, where several

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modified kernel functions will be proposed. Experimental results are presented in Section III to verify our proposed filters. Finally, Section IV concludes this paper.

2. Kernel Functions Selection For Nonlocal Filtering

In this section, a brief introduction to the conventional nonlocal filtering [5] will be first provided, followed by several modified kernel functions will be proposed and incorporated into the conventional nonlocal filtering framework to develop new filtering methods.

2.1. Nonlocal Filtering

Let p be the location of the pixel under the consideration in the noisy image I. The nonlocal filtering aims to replace the intensity value of the pixel p (denoted as I(p)) via a weighted average (denoted as $\hat{I}(\mathbf{p})$ of all the pixels in a search region $\Omega(\mathbf{p})$ which is either the full image [5], or a neighborhood centered at the pixel p with a size of $L \times L$. The above-mentioned filtering operation can be mathematically expressed as

$$\hat{I}(\mathbf{p}) = \sum_{\mathbf{q}\in\Omega(\mathbf{P})} w(\mathbf{p},\mathbf{q}) I(\mathbf{q}), \tag{2}$$

where I(q) represents the intensity value of the position q, w(p, q) is a weighting factor that depends on the similarity between the current pixel p and the pixel q, and it is determined by

$$w(\mathbf{p},\mathbf{q}) = \frac{1}{Z(\mathbf{p})} f\left(\left\|J(\mathbf{p}) - J(\mathbf{q})\right\|_2\right), \tag{3}$$

where J(p) and J(p) denote the neighborhoods of pixels centered at the position p and q with a size of $M \times M$, respectively. Z(p) is a normalizing factor which is defined as

$$Z(\mathbf{p}) = \sum_{\mathbf{q}\in\Omega(\mathbf{p})} f\left(\left\|J(\mathbf{p}) - J(\mathbf{q})\right\|_2\right), \tag{4}$$

in which the function $\|\cdot\|_2$ is the L-2 norm. The conventional nonlocal filter [5] selects the exponential function to formulate the function $f(\cdot)$; that is

$$f(x) = e^{\left(-\frac{x}{\lambda}\right)}, \tag{5}$$

in which the parameter λ is the factor to adjust the decay of the exponential function.

The key challenge of the nonlocal filtering (2) is the establishment of the function $f(\cdot)$ defined in (5), which controls the performance of the algorithm and the quality of the denoised image. The only criterion of designing the kernel function is that its output should decrease as its input increase.

2.2. Proposed Kernel Functions For Nonlocal Filtering

To build up the kernel function for the nonlocal filtering, in contrast to the exponential function used in (5), several modified functions are considered in this paper. These functions are mathematically expressed as the below and illustrated in Figure 2.

- *Cosine function* is expressed as

$$f(x) = \begin{cases} \cos\left(\frac{\pi x}{2\lambda}\right) & 0 < x \le \lambda \\ 0 & \text{else} \end{cases} \tag{6}$$

- *Flat function* is expressed as

$$f(x) = \begin{cases} \frac{1}{x} & 0 < x \le \lambda \\ 0 & \text{else} \end{cases} \tag{7}$$

- *Gaussian function* is expressed as

$$f(x) = e^{\left(-\frac{x^2}{2\lambda^2}\right)}, \quad \text{for} \quad x > 0 \tag{8}$$

in which the parameter λ is the factor to adjust the decay of the exponential function.

- *Turkey bi-weight function* is expressed as

$$f(x) = \begin{cases} \frac{1}{2}\left(1 - \left(\frac{x}{\lambda}\right)^2\right)^2 & 0 < x \le \lambda \\ 0 & \text{else} \end{cases} \tag{9}$$

- *Wave function* is expressed as

$$f(x) = \begin{cases} \frac{\sin\left(\frac{\pi x}{\lambda}\right)}{\pi x \lambda} & 0 < x \le \lambda \\ 0 & \text{else} \end{cases} \tag{10}$$

The above functions (i.e., (6)-(10)) can be respectively incorporated into the conventional nonlocal filtering framework (i.e., (2)) and replace the exponential function defined in (5) to arrive at new filtering method, whose performances will be evaluated in the next section.

3 Experimental Results

3.1 Experimental Setup and Implementation

To evaluate the performance of the proposed approach, five test images are used in the experiments, as shown in Figure 1. In our simulation, the original image is added with a zero mean white Gaussian noise with various standard derivation σ_n to obtain the noisy image.

The nonlocal filtering (i.e., (2)) with the incorporation of various kernel functions (i.e. (5)-(10)) are exploited to perform denoising, respectively. The parameters L in (2) and M in (3) are experimentally set to be 15 and 3, respectively. The adjusting parameter λ used for the above filtering methods is experimentally selected.

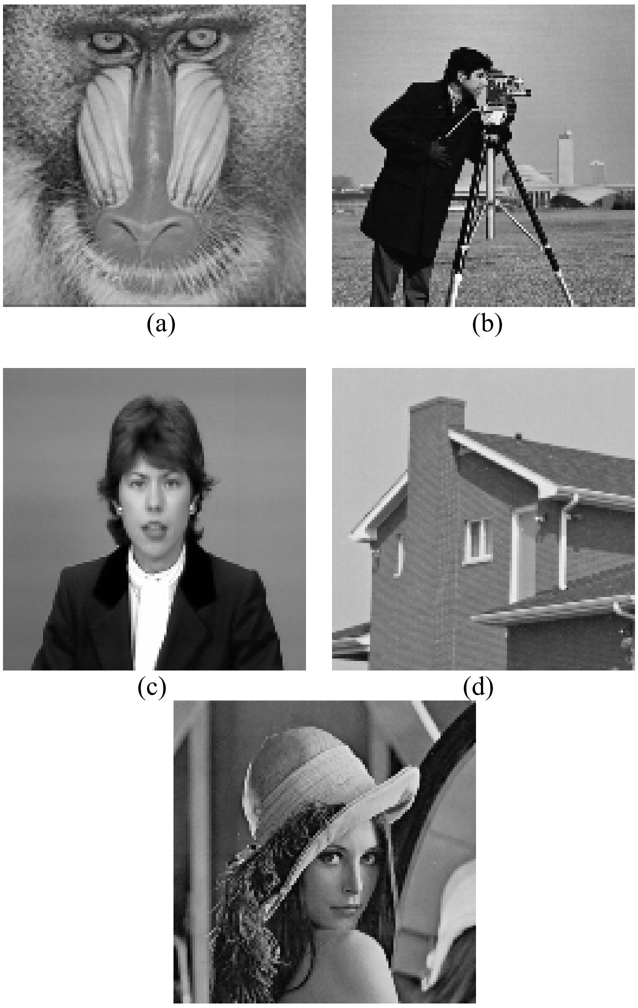

Figure. 1. Test images: (a) *Baboon*, (b) *Camera*, (c) *Claire*, (d) *House* and (e) *Lena*.

3.2 Experimental Results

Experiments are conducted to exploit the performance of the proposed modified nonlocal filterings with the incorporation of the kernel functions defined in (6)-(10), and compare them with that of the conventional nonlocal filtering [5], respectively. To provide the objective performance comparison, Table I compares their PSNR performances. On the other hand, for the subjective performance comparison, Figure 3 and Figure 4 present various image results obtained by the test images Lena and House, respectively. As seen from Table I and Figures 3, 4, one can clearly see that the kernel function used in the nonlocal filtering is essential to determine the quality of the denoised image result. Furthermore, the modified nonlocal filtering with the incorporation of the proposed kernel functions (6)-(10) outperform the conventional nonlocal filtering (5), in terms of the both the PSNR performance and visual image quality.

Table 1

The psnr performance comparison of the nonlocal filtering with the incorporation of various kernel functions.

Function	The additive white Gaussian noise ($\sigma_n = 10$)				
	Baboon	*Camera*	*Claire*	*House*	*Lena*
Exponential function (5)	28.73 dB	27.96 dB	32.59 dB	30.55 dB	28.83 dB
Cosine function (6)	29.21 dB	31.20 dB	34.11 dB	32.29 dB	30.56 dB
Flat function (7)	29.16 dB	31.13 dB	34.13 dB	32.23 dB	30.48 dB
Gaussian function (8)	28.77 dB	29.69 dB	32.91 dB	30.82 dB	28.98 dB
Turkey function (9)	29.20 dB	31.21 dB	34.20 dB	32.32 dB	30.57 dB
Wave function (10)	29.21 dB	31.21 dB	34.16 dB	32.31 dB	30.57 dB

The additive white Gaussian noise ($\sigma_n = 15$)				
Baboon	*Camera*	*Claire*	*House*	*Lena*
27.26 dB	26.81 dB	30.85 dB	29.03 dB	27.60 dB
27.80 dB	28.78 dB	31.57 dB	30.08 dB	28.08 dB
27.76 dB	28.70 dB	31.67 dB	30.10 dB	28.12 dB
27.35 dB	27.70 dB	31.60 dB	29.24 dB	27.71 dB
27.32 dB	28.82 dB	31.82 dB	30.23 dB	28.23 dB
27.38 dB	28.79 dB	30.72 dB	30.23 dB	28.25 dB

4. Conclusions

The kernel function serves as an essential role in the nonlocal filtering [5] for image denoising. For that, several modified kernel functions have been proposed in this paper to be incorporated into the conventional nonlocal filtering framework to develop new filtering methods, rather than exploiting the exponential function as used in the conventional nonlocal filtering [5]. The proposed new filtering methods outperform the conventional nonlocal filtering [5], as verified in the experimental results. There are several significant issues need to be investigated for the proposed new filtering methods for future research works. The first issue is the determination of various parameters used for the nonlocal filtering; that is, L in (2), M in (3) and λ in (5). The second issue is how to select the kernel function from the proposed ones (i.e., (5)-(10)) in the real-life simulations. The third issue is regarding the fairly large computational cost of the nonlocal filtering. Therefore, the development of fast version [9], [10] of the proposed algorithm is a promising research direction.

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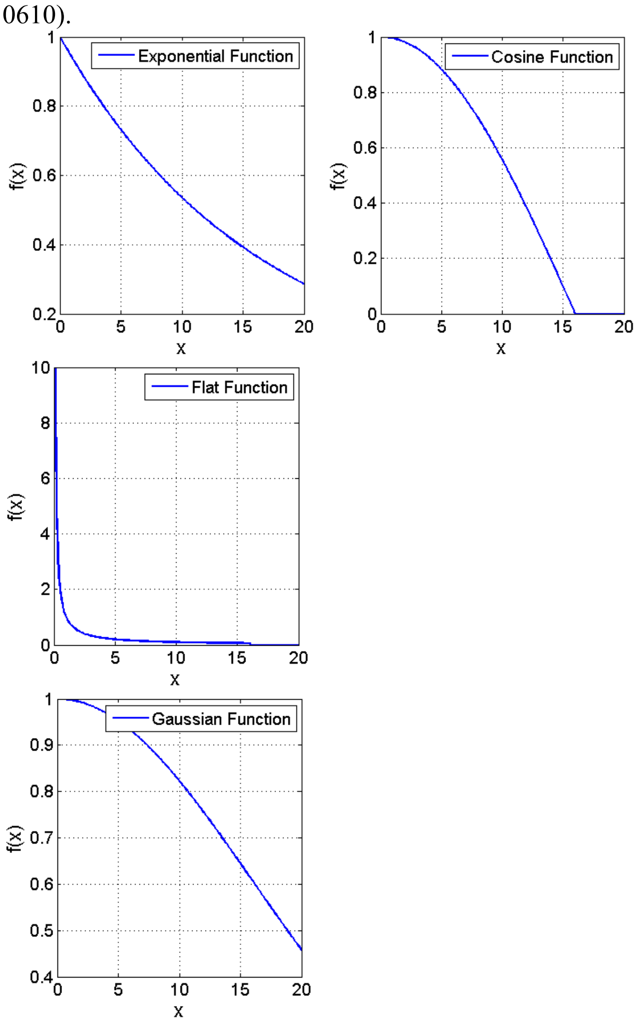


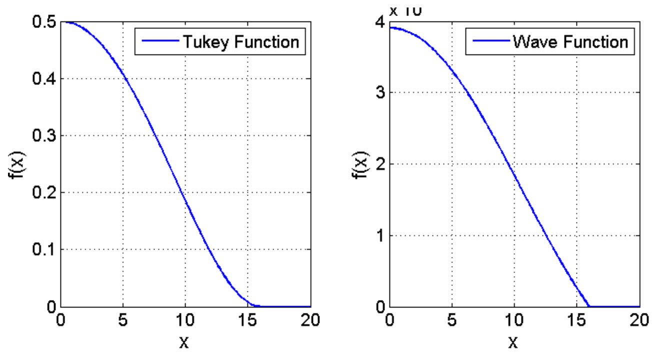
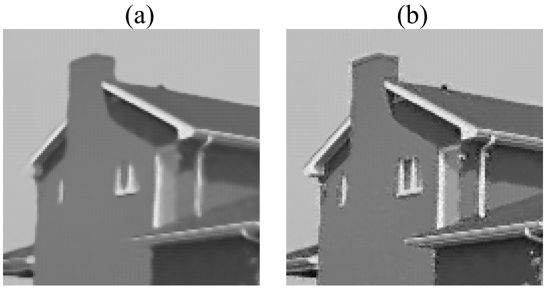


Figure. 2. Various kernel functions with the parameter λ = 16: the exponential function defined in (5), the cosine function defined in (6), the flat function defined in (7), the Gaussian function defined in (8), the Turkey function defined in (9), and the wave function defined in (10).

(g) (h)

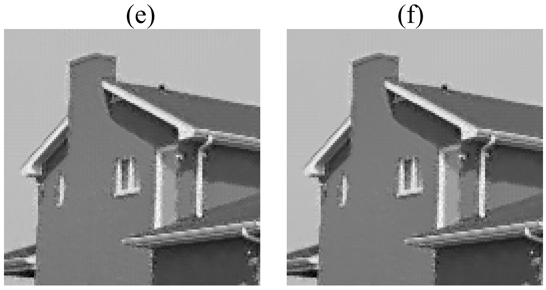
Figure. 3. Various denoised image result using the test image Lena: (a) original image; (b) noisy image with the additive white Gaussian noise (σ_n = 10); (c) denoised image using the conventional nonlocal filtering with the exponential function (5) (PSNR = 28.83 dB); (d) denoised image using the nonlocal filtering with the incorporation of the proposed cosine function (6) (PSNR = 30.56 dB); (e) denoised image using the nonlocal filtering with the incorporation of the proposed flat function (7) (PSNR = 30.48 dB); (f) denoised image using the nonlocal filtering with the incorporation of the proposed Gaussian function (8) (PSNR = 28.98 dB); (g) denoised image using the nonlocal filtering with the incorporation of the proposed Turkey function (9) (PSNR = 30.57 dB); (h) denoised image using the nonlocal filtering with the incorporation of the proposed wave function (10) (PSNR = 30.57 dB).

(a) (b)



(c) (d)

(e) (f)



(g) (h)

Figure 4. Various denoised image result using the test image House: (a) original image; (b) noisy image with the additive white Gaussian noise (σ_n = 10); (c) denoised image using the conventional nonlocal filtering with the exponential function (5) (PSNR = 30.55 dB); (d) denoised image using the nonlocal filtering with the incorporation of the proposed cosine function (6) (PSNR = 32.29 dB); (e) denoised image using the nonlocal filtering with the incorporation of the proposed flat function (7) (PSNR = 32.23 dB); (f) denoised image using the nonlocal filtering with the incorporation of the proposed Gaussian function (8) (PSNR = 30.82 dB); (g) denoised image using the nonlocal filtering with the incorporation of the proposed Turkey function (9) (PSNR = 32.32 dB); (h) denoised image using the nonlocal filtering with the incorporation of the proposed wave function (10) (PSNR = 32.31 dB).

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