

# Categorical sequence analysis with optimal matching: an application to the relationships history of women over 40

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# Motivation



Understand the similarities and differences in the romantic relationships history of a group of women over 40 years old and use this information as a predictor of personality traits.

# Women 40+ Healthy Aging Study (i)



**Universität  
Zürich**<sup>UZH</sup>

Dynamics of Healthy Aging



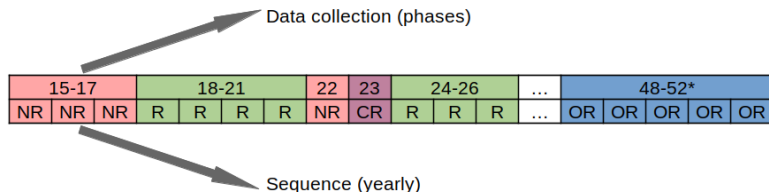
- ▶ Data from 250 individuals collected between June 2017 and February 2018.
- ▶ Psychometric instrument to obtain information about the history of romantic relationships of women aged between 40 and 75 years.

## Women 40+ Healthy Aging Study (ii)

- ▶ Information about relationship phases starting from the age of 15 years until the current age at the time of the data collection.
- ▶ The phases are defined by the start and end age and for each phase and information about civil status, relationship status, living situation, children and quality of the relationship was collected.
- ▶ Additional information about the individuals was collected, in particular, scores for personality scores.

# Data example

Consider the relationship status:



\*Current age

- ▶ No relationship (NR)
- ▶ In a relationship (R)
- ▶ Open relationship (OR)
- ▶ Changing relationships (CR)

# What is personality?



Personality refers to the enduring characteristics and behavior that comprise a person's unique adjustment to life, including major traits, interests, drives, values, self-concept, abilities, and emotional patterns.

# The “Big Five” personality traits

<b>O</b>	Openness to Experience	Appreciation for art, new ideas, variety of experiences imagination and curiosity	„I have many different interests“
<b>C</b>	Conscientiousness	Tendency towards self-discipline and striving for achievement against measures or outside expectations.	„I always follow my plans“
<b>E</b>	Extraversion	Gain energy from external situations and means, enjoy a breadth of activities and assert their viewpoints	„I am more the quite type“ (reverse coded)
<b>A</b>	Agreeableness	Value social harmony and getting along with others, optimistic, kind and generous towards others	„I am cooperative and prefer working in teams over competition“
<b>N</b>	Neuroticism	Tendency to experience negative emotions, such as anger, anxiety, or depression. Low tolerance of stress	„I worry a lot“

## Research question

- ▶ Can we get a good prediction of personality scores based on the relationship history sequences?



# Optimal Matching (OM)

- ▶ Technique used in social sciences for the comparison of sequences of categorical states indexed by time.
- ▶ Applications on life course and career path analysis.
- ▶ Uses the Needleman-Wunsch algorithm, that was developed to compare biological sequences.
- ▶ The Needleman-Wunsch algorithm is an application of dynamic programming, an iterative method that simplifies an optimization problem by breaking it into a recursion of smaller problems.

### Example (i)

# Analyzing Sequence Data: Optimal Matching in Management Research (T. Biemann and D. K. Datta)

- ▶ Goal: study career paths of deans at US business schools.
- ▶ Data source: 149 CVs of deans including public and private business schools.
- ▶ Coded into yearly data with the states: administration (A), corporation (C), faculty (F), government (G).

**Table 2.** Examples of Career Paths of U.S. Business School Deans.

[illegible]

## Example (ii)

Cost matrix:

**Table 3.** Absolute Frequency, Relative Frequency, and Substitution Costs Between States.

	Absolute Frequency	Relative Frequency (%)	Substitution Costs				
			F	A	C	G	NA
F	2,454	54.5	0.000	1.891	1.893	1.870	2.000
A	1,144	25.4	1.891	0.000	1.977	1.971	2.000
C	693	15.4	1.893	1.977	0.000	1.939	2.000
G	200	4.4	1.870	1.971	1.939	0.000	2.000
NA	14	0.3	2.000	2.000	2.000	2.000	0.000
Sum	4,505	100.00	(indel costs = 1)				

## Example (iii)

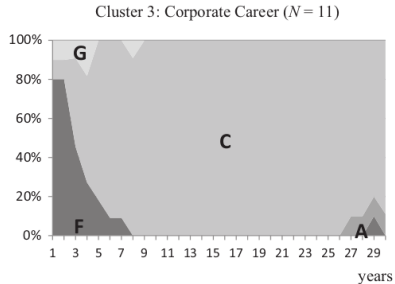
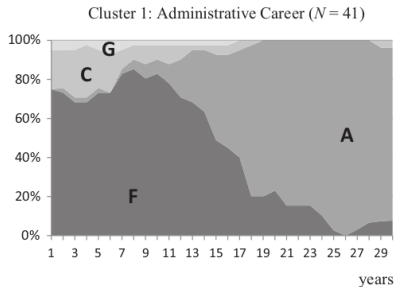
Distance/dissimilarities matrix for five deans:

**Table 4.** Distance Matrix for Five Deans.

	Dean 1	Dean 2	Dean 3	Dean 4	Dean 5
Dean 1	—				
Dean 2	23.13	—			
Dean 3	69.60	61.07	—		
Dean 4	28.52	18.22	64.14	—	
Dean 5	35.38	33.27	47.55	40.67	—

## Example (iv)

Two of the five resulting clusters:



## The OM algorithm (i)

- ▶ Set of  $n$  states:  $S = \{s_1, \dots, s_n\}$
- ▶ Sequence of size  $t > 0$ :  $X = (x_1, \dots, x_t)$ , with  $x_i \in S$  for  $i = 1, \dots, t$ .
- ▶  $\mathbf{S}$  is the set of all possible sequences with states belonging to  $S$ .

*Objective:* Find the optimal way to align these two sequences

- ▶ Let  $X, Y \in \mathbf{S}$  be two sequences of size  $t_1$  and  $t_2$ , respectively.
- ▶ Define an empty array  $F$  of size  $(t_1 + 1) \times (t_2 + 1)$

## The OM algorithm (ii)

```
1:  $F(1, 1) \leftarrow 0$ 
2: for  $j \leftarrow 2, t_2 + 1$  do
3:    $F(1, j) \leftarrow F(1, j - 1) + d$ 
4: end for
5: for  $i \leftarrow 2, t_1 + 1$  do
6:    $F(i, 1) \leftarrow F(i - 1, 1) + d$ 
7: end for
8: for  $i \leftarrow 2, t_1 + 1$  do
9:   for  $j \leftarrow 2, t_2 + 1$  do
10:     $F(i, j) \leftarrow$ 
         $\min\{F(i - 1, j) + d, F(i, j - 1) + d, F(i - 1, j - 1) + k(y_{i-1}, x_{j-1})\}$ 
11:   end for
12: end for
```

## The OM algorithm (iii)

- ▶  $d$  is the cost of inserting a gap (indel cost).
- ▶  $k(y_{i-1}, x_{j-1})$  is the cost associated to change from the state  $y_{i-1}$  to  $x_{j-1}$ .
- ▶ These costs are defined in a matrix  $K$  of size  $n \times n \rightarrow$  cost matrix.
- ▶ Lines 1-7 of the algorithm correspond to initialization.
- ▶ The remaining lines of the algorithm correspond to the row-wise recursion to fill the array  $F$ .
- ▶ When  $F$  is completely filled, the value  $F(t_1 + 1, t_2 + 1)$  corresponds to the optimal cost of aligning the sequences  $X$  and  $Y$ .



## Cost matrix (i)

The R package TraMineR provides several functions to work with sets of sequences. The package implements OM and offers several methods for computing the cost matrix  $K$ .

## Cost matrix (ii)

► Transition rates (TRATE):

The substitution cost between states  $s_i$  and  $s_j$ ,  $1 \leq i, j \leq n$ , is calculated as:

$$K(s_i, s_j) = c - P(s_i|s_j) - P(s_j|s_i), \quad (1)$$

where  $P(s_i|s_j)$  is the probability of transition from state  $s_j$  in time  $t$  to  $s_i$  in time  $t + 1$  and  $c$  is a constant, set to a value such that  $0 \leq K(s_i, s_j) \leq 2$ .

## Cost matrix (iii)

- ▶ Chi-squared distance (FUTURE):

$$K(s_i, s_j) = d_{\chi^2}(\mathbf{P}_i, \mathbf{P}_j), \quad (2)$$

where  $\mathbf{P}_. = (P(s_1|s_.), \dots, P(s_n|s_.))'$

## Cost matrix (iv)

- ▶ Relative frequencies (INDELS and INDELSLOG):

$$K(s_i, s_j) = \textit{indel}_i + \textit{indel}_j, \quad (3)$$

where  $\textit{indel}_i = 1/f_i$  for method INDEL,  $\textit{indel}_i = \log[2/(1 + f_i)]$  and  $f_i$  is the relative frequency of the state  $s_i$  for  $i = 1, \dots, n$ .

## Example (i)

Setup:

- ▶  $S$  = the alphabet
- ▶  $X = \{S, E, N, D\}, Y = \{A, N, D\} \in \mathbf{S}$
- ▶  $d = 2$

$$K(i, j) = \begin{cases} 0 & \text{if } i = j, \\ 3 & \text{otherwise} \end{cases}$$

## Example (ii)

Initialization of  $F$ :

		S	E	N	D
	0	2	4	6	8
A	2				
N	4				
D	6				

- ▶  $F(2, 2) = \min\{F(1, 2) + d, F(2, 1) + d, F(1, 1) + k(y_1, x_1)\} = \min\{2 + 2, 2 + 2, 0 + 3\} = 3$
- ▶  $F(2, 3) = \min\{F(1, 3) + d, F(2, 2) + d, F(1, 2) + k(y_1, x_2)\} = \min\{4 + 2, 3 + 2, 2 + 3\} = 5$
- ▶  $F(2, 4) = \min\{F(1, 4) + d, F(2, 3) + d, F(1, 3) + k(y_1, x_3)\} = \min\{6 + 2, 5 + 2, 4 + 3\} = 7$
- ▶  $F(2, 5) = \min\{F(1, 5) + d, F(2, 4) + d, F(1, 4) + k(y_1, x_4)\} = \min\{8 + 2, 7 + 2, 6 + 3\} = 9$

### Example (iii)

		S	E	N	D
	0	2	4	6	8
A	2	3	5	7	9
N	4				
D	6				

- ▶  $F(3, 2) = \min\{F(2, 2) + d, F(3, 1) + d, F(2, 1) + k(y_2, x_1)\} = \min\{3 + 2, 4 + 2, 2 + 3\} = 5$
- ▶  $F(3, 3) = \min\{F(2, 3) + d, F(3, 2) + d, F(2, 2) + k(y_2, x_2)\} = \min\{5 + 2, 5 + 2, 3 + 3\} = 6$
- ▶ ...

## Example (iv)

		S	E	N	D
	0	2	4	6	8
A	2	3	5	7	9
N	4	5	6	5	7
D	6	7	8	7	5

S E N D with

A - N D or —>

Optimal (equivalent) alignments

- A N D

- ▶ 2 matches: 0
- ▶ 1 mismatch: 3
- ▶ 1 gap: 2
- ▶ Total: 5



# Normalization

Given  $X, Y \in \mathbf{S}$  of length  $t_1$  and  $t_2$ , respectively. Let  $d(X, Y)$  be the distance between the sequences  $X$  and  $Y$ ,  $t_{max}$  the length of the longest sequence in  $\mathbf{S}$  and  $d_{max}$  the maximum distance between any pair of sequences in  $\mathbf{S}$ .

TraMineR provides the following options for normalization:

- ▶ maxlength:  $d(X, Y)/t_{max}$
- ▶ gmean:  $1 - \frac{d_{max} - d(X, Y)}{\sqrt{t_1 * t_2}}$
- ▶ maxdist:  $d(X, Y)/d_{max}$

## Application: data pre-processing

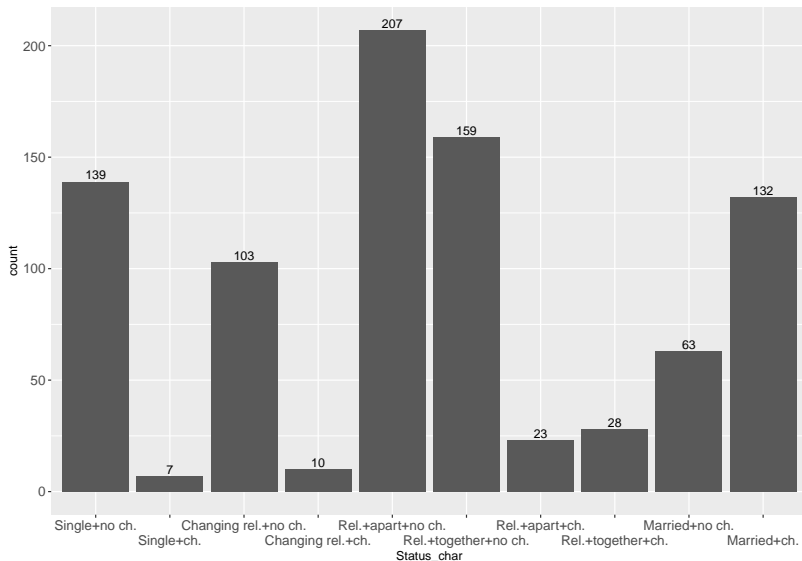
- ▶ Manual corrections of several inconsistent and incomplete records.
- ▶ Several additional automatic checks to identify sequences with inconsistent data.
- ▶ Corrections based on secondary data source.
- ▶ Identification and selection of the variables and patterns that provide a wider perspective of the individuals' situations.
- ▶ In total, 239 individuals are considered for the analysis.

## Application: considered states

- ▶ 1 = Single + no children
- ▶ 2 = Single + children
- ▶ 3 = Changing relationships + no children
- ▶ 4 = Changing rel. + children
- ▶ 5 = Relationship + living apart + no children
- ▶ 6 = Relationship + living together + no children
- ▶ 7 = Relationship + living apart + children
- ▶ 8 = Relationship + living together + children
- ▶ 9 = Married + no children
- ▶ 10 = Married + children

15-17			18-19		20	21-22		23-25			26	27-29			30-*		...
1	1	1	5	5	3	1	1	5	5	5	6	9	9	9	10	10	...

## Application: distribution of states

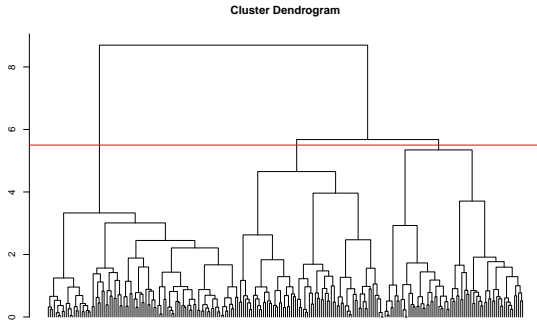


## Application: cost matrix

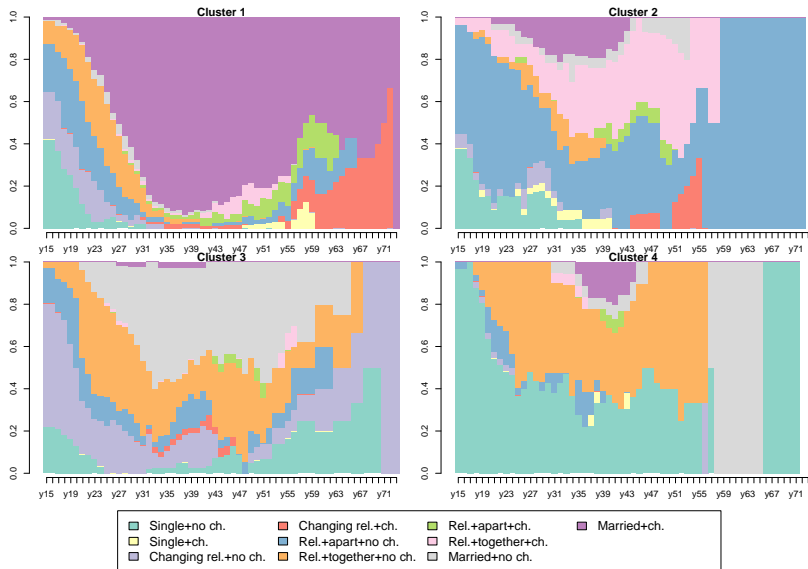
Base setup: TRATE and maxlength

[illegible]

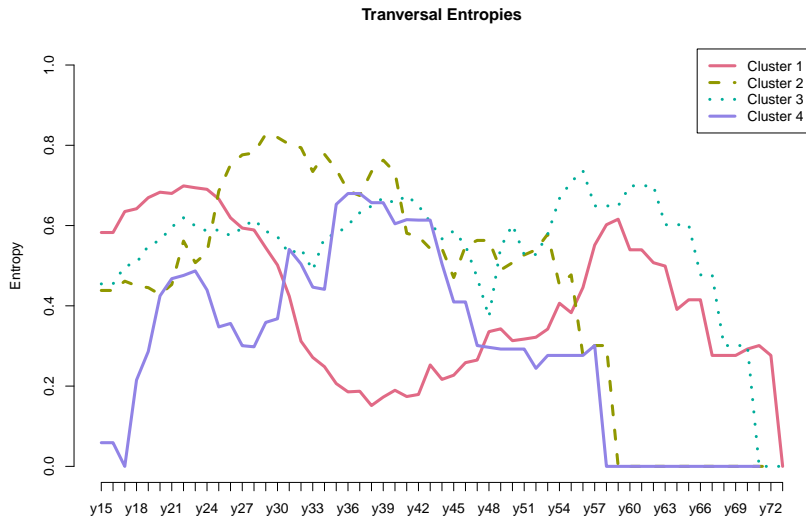
# Application: clustering



# Application: clustering



# Application: clustering



## [1] 4

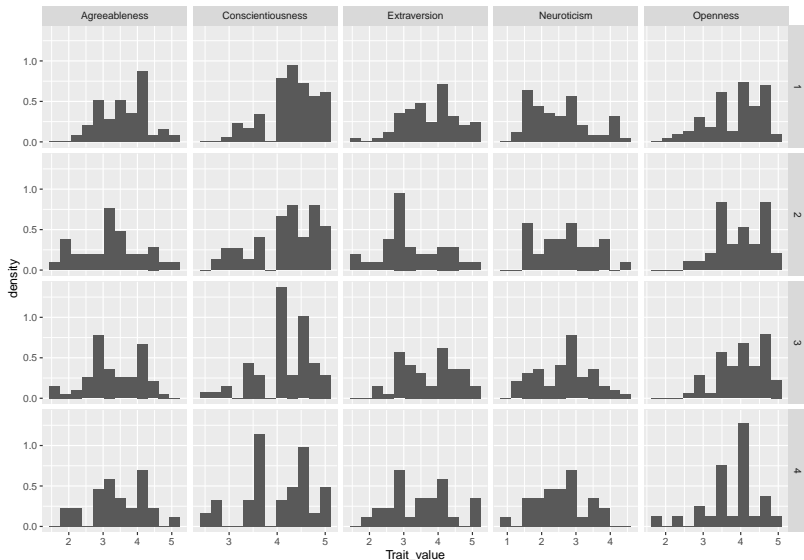


## Application: clustering

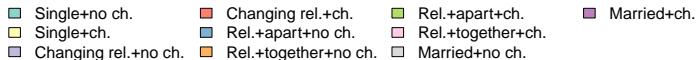
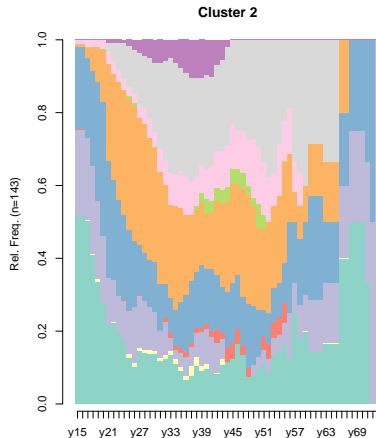
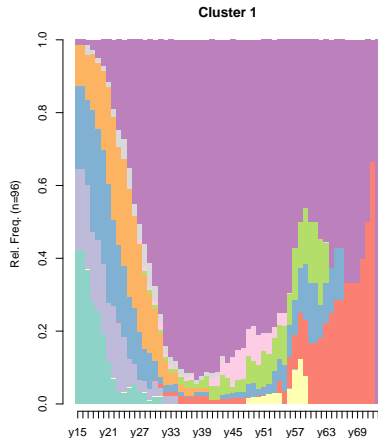
Cluster	n
1	96
2	37
3	73
4	33

- ▶ Cluster 1: Married young and had children
- ▶ Cluster 2: Often in relationships but not married
- ▶ Cluster 3: Older, mostly married or in long relationship without children
- ▶ Cluster 4: Younger, single or in a relationship without children

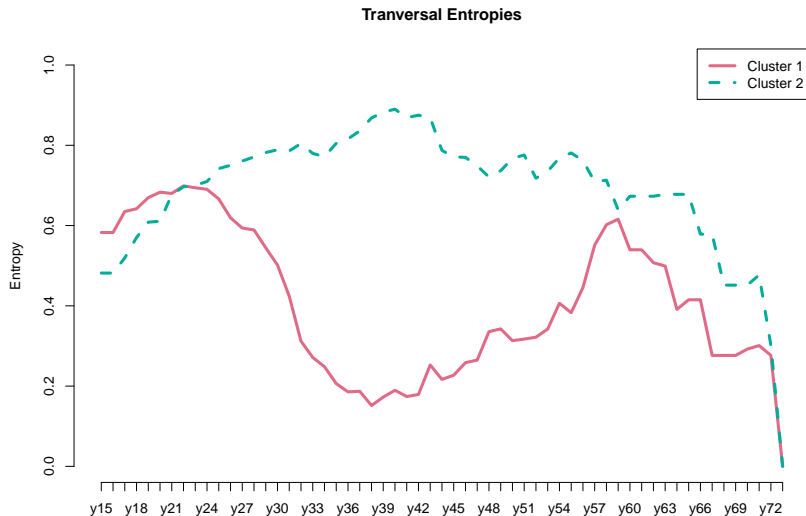
# Application: clustering



# Application: clustering

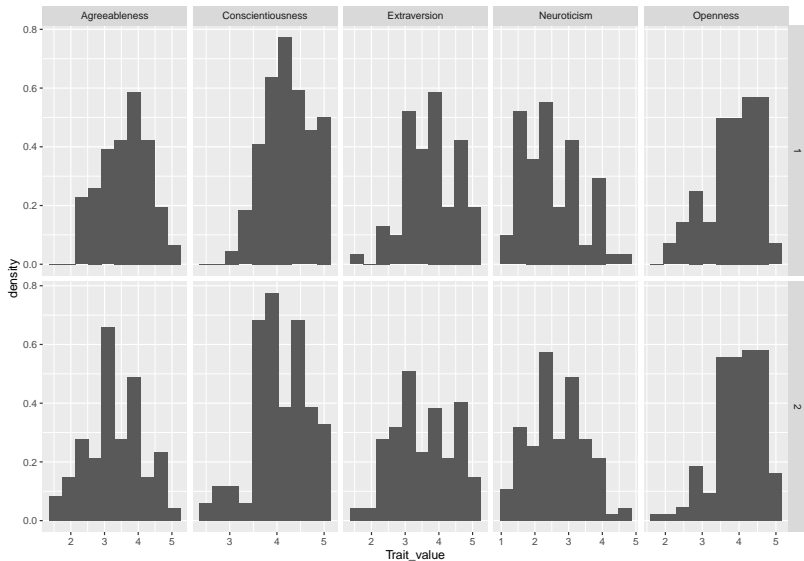


# Application: clustering



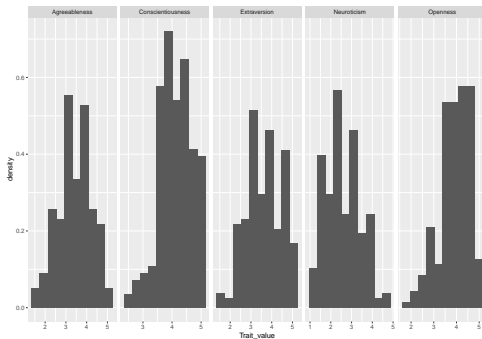
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# Application: clustering



## Application: Personality scores

Personality trait	Min	Max	Average	Std. deviation
Agreeableness	1.50	5.0	3.43	0.76
Conscientiousness	1.75	5.0	4.15	0.57
Extraversion	1.50	5.0	3.63	0.82
Neuroticism	1.00	4.5	2.62	0.77
Openness	1.80	5.0	3.89	0.70



## $k$ -nearest neighbors ( $k$ NN)

- ▶ Non-parametric method for prediction.
- ▶ Training set  $\mathcal{D} = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  of  $n$  labeled data points, where  $x_i \in \mathbb{R}^d$  and  $y_i \in \mathcal{Y}$  (a finite set of for classification or a continuous range of values for regression).
- ▶ Predict the label or value ( $y_{n+1}$  unknown) for  $x_{n+1}$  by finding the  $k$  training data points closest to  $x_{n+1}$  and taking a majority vote of their labels (for classification) or averaging the values of  $Y$  (for regression).
- ▶ To compare the performance of different values of  $k$ , we calculate the mean squared error (MSE) in a test set of size  $m$ .

$$\text{MSE} = \frac{1}{m} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

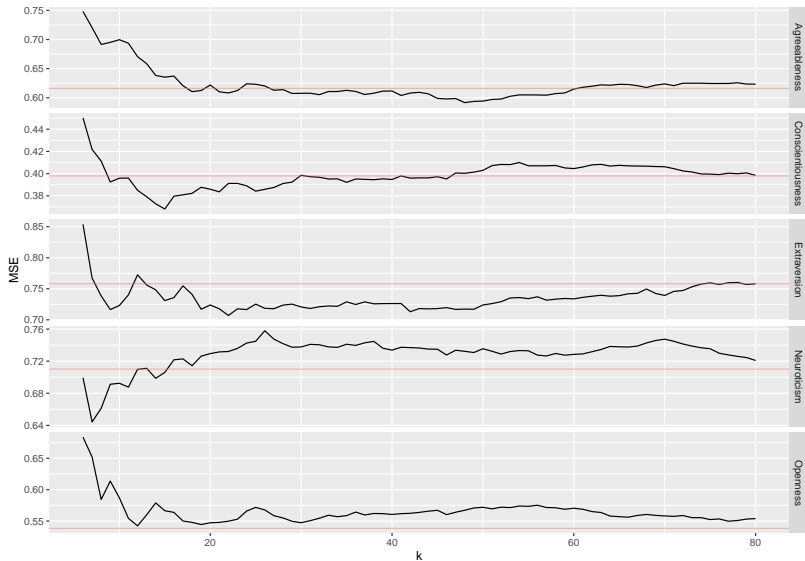
where  $y_i$  is the observed value and  $\hat{y}_i$  is the predicted value via  $k$ NN.

## Prediction of personality scores

- ▶ Only individuals who have available personality scores
- ▶ New sample size: 200 individuals.
- ▶ Data split into two subsets: train (70%) and test (30%).
- ▶ Evaluate the MSE of the predictions in the test set only using the data from the nearest neighbors available in the train set.



# Prediction of personality scores



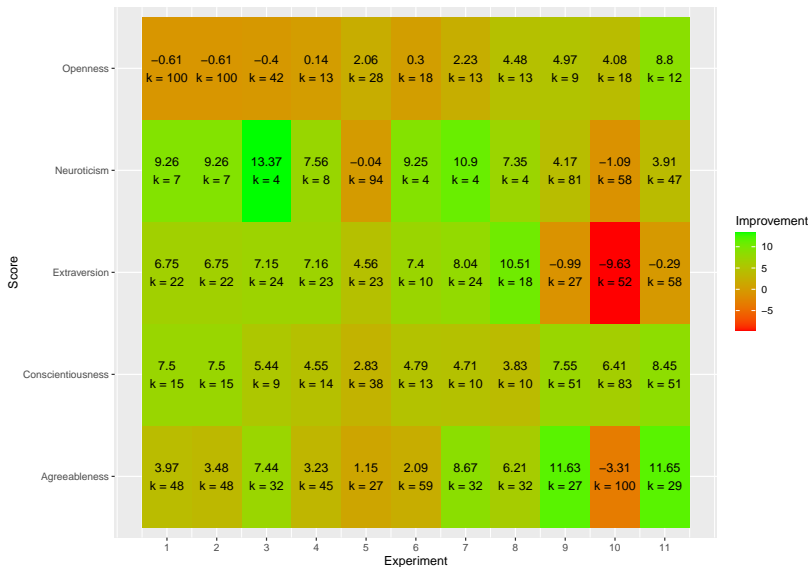
# Prediction of personality scores

- ▶ Method for calculation of cost matrix
- ▶ Constant  $c$  for cost matrix obtained from transition rates
- ▶ Normalization of distances
- ▶ Cost of transition from/to NA
- ▶ Limit range of the sequence

## Prediction of personality scores

cm_method	dm_norm	constant	missing_cost	min_age	max_age
TRATE	maxlength	NULL	NULL	NULL	NULL
TRATE	maxlength	1.998497	NULL	NULL	NULL
TRATE	gmean	NULL	NULL	NULL	NULL
FUTURE	maxlength	NULL	NULL	NULL	NULL
INDELS	maxlength	NULL	NULL	NULL	NULL
INDELSLOG	maxlength	NULL	NULL	NULL	NULL
FUTURE	gmean	NULL	NULL	NULL	NULL
FUTURE	gmean	NULL	1	NULL	NULL
FUTURE	gmean	NULL	NULL	20	55
FUTURE	gmean	NULL	NULL	20	40
FUTURE	gmean	NULL	1	20	55

# Prediction of personality scores



## Final comments



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# References

- ▶ Sequence Analysis: New Methods for Old Ideas - A. Abbott (1995)
- ▶ Optimal Matching Analysis: A Methodological Note on Studying Career Mobility - T. W. Chan (1995)
- ▶ Analyzing Sequence Data: Optimal Matching in Management Research - T. Biemann & D. K. Datta (2013)
- ▶ Analyzing and Visualizing State Sequences in R with TraMineR - A. Gabadinho, G. Ritschard, N. S. Müller, M. Studer (2011)