

Categorical sequence analysis with optimal matching: an application to the relationships history of women over 40

Adriana Clavijo Daza

Statistics and Data Science Master's, Universität Bern

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Motivation



Understand the similarities and differences in the romantic relationships history of a group of women over 40 years old and use this information as a predictor of personality traits.

Women 40+ Healthy Aging Study (i)



**Universität
Zürich**^{UZH}

Dynamics of Healthy Aging



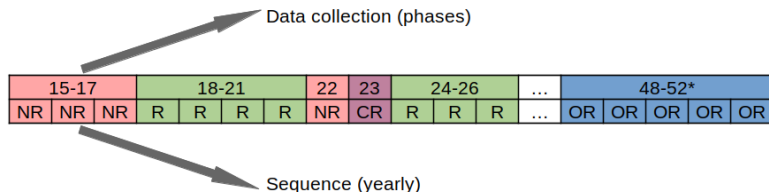
- ▶ Data from 250 individuals collected between June 2017 and February 2018.
- ▶ Psychometric instrument to obtain information about the history of romantic relationships of women aged between 40 and 75 years.

Women 40+ Healthy Aging Study (ii)

- ▶ Information about relationship phases starting from the age of 15 years until the current age at the time of the data collection.
- ▶ The phases are defined by the start and end age and for each phase and information about civil status, relationship status, living situation, children and quality of the relationship was collected.
- ▶ Additional information about the individuals was collected, in particular, scores for personality scores.

Data example

Consider the relationship status:



*Current age

- ▶ No relationship (NR)
- ▶ In a relationship (R)
- ▶ Open relationship (OR)
- ▶ Changing relationships (CR)

What is personality?



Personality refers to the enduring characteristics and behavior that comprise a person's unique adjustment to life, including major traits, interests, drives, values, self-concept, abilities, and emotional patterns.

The “Big Five” personality traits

O	Openness to Experience	Appreciation for art, new ideas, variety of experiences imagination and curiosity	„I have many different interests“
C	Conscientiousness	Tendency towards self-discipline and striving for achievement against measures or outside expectations.	„I always follow my plans“
E	Extraversion	Gain energy from external situations and means, enjoy a breadth of activities and assert their viewpoints	„I am more the quite type“ (reverse coded)
A	Agreeableness	Value social harmony and getting along with others, optimistic, kind and generous towards others	„I am cooperative and prefer working in teams over competition“
N	Neuroticism	Tendency to experience negative emotions, such as anger, anxiety, or depression. Low tolerance of stress	„I worry a lot“

Research question

- ▶ Can we get a good prediction of personality scores based on the relationship history sequences?

Optimal Matching (OM)

- ▶ Technique used in social sciences for the comparison of sequences of categorical states indexed by time.
- ▶ Applications on life course and career path analysis.
- ▶ Uses the Needleman-Wunsch algorithm, that was developed to compare biological sequences.
- ▶ The Needleman-Wunsch algorithm is an application of dynamic programming, an iterative method that simplifies an optimization problem by breaking it into a recursion of smaller problems.

Example (i)

Analyzing Sequence Data: Optimal Matching in Management Research (T. Biemann and D. K. Datta)

- ▶ Goal: study career paths of deans at US business schools.
- ▶ Data source: 149 CVs of deans including public and private business schools.
- ▶ Coded into yearly data with the states: administration (A), corporation (C), faculty (F), government (G).

Table 2. Examples of Career Paths of U.S. Business School Deans.

	Career Path
Dean 1	F-F-F-F-F-F-F-F-F-F-F-F-F-F-F-A-A-A-A-A-A-A-A-A-A-A-A-A
Dean 2	F-A-A
Dean 3	C-C
Dean 4	F-F-F-F-F-F-F-F-F-F-F-F-F-F-F-A-F-F-C-C-F-F-F-F-G-G-G-G-G-G-G-G-A-A
Dean 5	C-C-C-C-C-C-C-C-C-F-F-F-F-A-A-A-A-A-A-A-A-F-F-F-F-F-F-A

Example (ii)

Cost matrix:

Table 3. Absolute Frequency, Relative Frequency, and Substitution Costs Between States.

	Absolute Frequency	Relative Frequency (%)	Substitution Costs				
			F	A	C	G	NA
F	2,454	54.5	0.000	1.891	1.893	1.870	2.000
A	1,144	25.4	1.891	0.000	1.977	1.971	2.000
C	693	15.4	1.893	1.977	0.000	1.939	2.000
G	200	4.4	1.870	1.971	1.939	0.000	2.000
NA	14	0.3	2.000	2.000	2.000	2.000	0.000
Sum	4,505	100.00	(indel costs = 1)				

Example (iii)

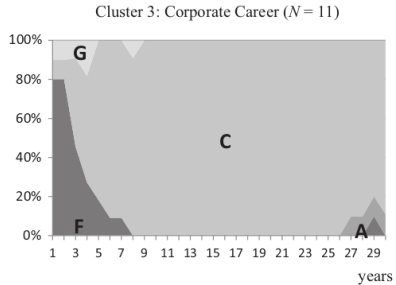
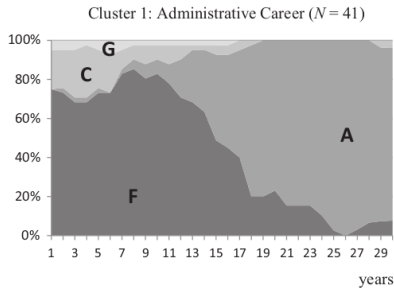
Distance/dissimilarities matrix for five deans:

Table 4. Distance Matrix for Five Deans.

	Dean 1	Dean 2	Dean 3	Dean 4	Dean 5
Dean 1	—				
Dean 2	23.13	—			
Dean 3	69.60	61.07	—		
Dean 4	28.52	18.22	64.14	—	
Dean 5	35.38	33.27	47.55	40.67	—

Example (iv)

Two of the five resulting clusters:



The OM algorithm (i)

- ▶ Set of n states: $S = \{s_1, \dots, s_n\}$
- ▶ Sequence of size $t > 0$: $X = (x_1, \dots, x_t)$, with $x_i \in S$ for $i = 1, \dots, t$.
- ▶ \mathbf{S} is the set of all possible sequences with states belonging to S .

Objective: Find the optimal way to align these two sequences

- ▶ Let $X, Y \in \mathbf{S}$ be two sequences of size t_1 and t_2 , respectively.
- ▶ Define an empty array F of size $(t_1 + 1) \times (t_2 + 1)$

The OM algorithm (ii)

```
1:  $F(1, 1) \leftarrow 0$ 
2: for  $j \leftarrow 2, t_2 + 1$  do
3:    $F(1, j) \leftarrow F(1, j - 1) + d$ 
4: end for
5: for  $i \leftarrow 2, t_1 + 1$  do
6:    $F(i, 1) \leftarrow F(i - 1, 1) + d$ 
7: end for
8: for  $i \leftarrow 2, t_1 + 1$  do
9:   for  $j \leftarrow 2, t_2 + 1$  do
10:     $F(i, j) \leftarrow$ 
         $\min\{F(i - 1, j) + d, F(i, j - 1) + d, F(i - 1, j - 1) + k(y_{i-1}, x_{j-1})\}$ 
11:   end for
12: end for
```

The OM algorithm (iii)

- ▶ d is the cost of inserting a gap (indel cost).
- ▶ $k(y_{i-1}, x_{j-1})$ is the cost associated to change from the state y_{i-1} to x_{j-1} .
- ▶ These costs are defined in a matrix K of size $n \times n \rightarrow$ cost matrix.
- ▶ Lines 1-7 of the algorithm correspond to initialization.
- ▶ The remaining lines of the algorithm correspond to the row-wise recursion to fill the array F .
- ▶ When F is completely filled, the value $F(t_1 + 1, t_2 + 1)$ corresponds to the optimal cost of aligning the sequences X and Y .

Cost matrix (i)

The R package TraMineR provides several functions to work with sets of sequences. The package implements OM and offers several methods for computing the cost matrix K .

Cost matrix (ii)

► Transition rates (TRATE):

The substitution cost between states s_i and s_j , $1 \leq i, j \leq n$, is calculated as:

$$K(s_i, s_j) = c - P(s_i|s_j) - P(s_j|s_i), \quad (1)$$

where $P(s_i|s_j)$ is the probability of transition from state s_j in time t to s_i in time $t + 1$ and c is a constant, set to a value such that $0 \leq K(s_i, s_j) \leq 2$.

Cost matrix (iii)

- ▶ Chi-squared distance (FUTURE):

$$K(s_i, s_j) = d_{\chi^2}(\mathbf{P}_i, \mathbf{P}_j), \quad (2)$$

where $\mathbf{P}_. = (P(s_1|s.), \dots, P(s_n|s.))'$

Cost matrix (iv)

- ▶ Relative frequencies (INDELS and INDELSLOG):

$$K(s_i, s_j) = \textit{indel}_i + \textit{indel}_j, \quad (3)$$

where $\textit{indel}_i = 1/f_i$ for method INDEL, $\textit{indel}_i = \log[2/(1 + f_i)]$ and f_i is the relative frequency of the state s_i for $i = 1, \dots, n$.

Example (i)

Setup:

- ▶ S = the alphabet
- ▶ $X = \{S, E, N, D\}, Y = \{A, N, D\} \in \mathbf{S}$
- ▶ $d = 2$

$$K(i, j) = \begin{cases} 0 & \text{if } i = j, \\ 3 & \text{otherwise} \end{cases}$$

Example (ii)

Initialization of F :

		S	E	N	D
	0	2	4	6	8
A	2				
N	4				
D	6				

- ▶ $F(2, 2) = \min\{F(1, 2) + d, F(2, 1) + d, F(1, 1) + k(y_1, x_1)\} = \min\{2 + 2, 2 + 2, 0 + 3\} = 3$
- ▶ $F(2, 3) = \min\{F(1, 3) + d, F(2, 2) + d, F(1, 2) + k(y_1, x_2)\} = \min\{4 + 2, 3 + 2, 2 + 3\} = 5$
- ▶ $F(2, 4) = \min\{F(1, 4) + d, F(2, 3) + d, F(1, 3) + k(y_1, x_3)\} = \min\{6 + 2, 5 + 2, 4 + 3\} = 7$
- ▶ $F(2, 5) = \min\{F(1, 5) + d, F(2, 4) + d, F(1, 4) + k(y_1, x_4)\} = \min\{8 + 2, 7 + 2, 6 + 3\} = 9$

Example (iii)

		S	E	N	D
	0	2	4	6	8
A	2	3	5	7	9
N	4				
D	6				

- ▶ $F(3, 2) = \min\{F(2, 2) + d, F(3, 1) + d, F(2, 1) + k(y_2, x_1)\} = \min\{3 + 2, 4 + 2, 2 + 3\} = 5$
- ▶ $F(3, 3) = \min\{F(2, 3) + d, F(3, 2) + d, F(2, 2) + k(y_2, x_2)\} = \min\{5 + 2, 5 + 2, 3 + 3\} = 6$
- ▶ ...

Example (iv)

		S	E	N	D
	0	2	4	6	8
A	2	3	5	7	9
N	4	5	6	5	7
D	6	7	8	7	5

S E N D with

A - N D or —>

Optimal (equivalent) alignments

- A N D

- ▶ 2 matches: 0
- ▶ 1 mismatch: 3
- ▶ 1 gap: 2
- ▶ Total: 5

Normalization

Given $X, Y \in \mathbf{S}$ of length t_1 and t_2 , respectively. Let $d(X, Y)$ be the distance between the sequences X and Y , t_{max} the length of the longest sequence in \mathbf{S} and d_{max} the maximum distance between any pair of sequences in \mathbf{S} .

TraMineR provides the following options for normalization:

- ▶ maxlength: $d(X, Y)/t_{max}$
- ▶ gmean: $1 - \frac{d_{max} - d(X, Y)}{\sqrt{t_1 * t_2}}$
- ▶ maxdist: $d(X, Y)/d_{max}$

Application: data pre-processing

- ▶ Manual corrections of several inconsistent and incomplete records.
- ▶ Several additional automatic checks to identify sequences with inconsistent data.
- ▶ Corrections based on secondary data source.
- ▶ Identification and selection of the variables and patterns that provide a wider perspective of the individuals' situations.
- ▶ In total, 239 individuals are considered for the analysis.

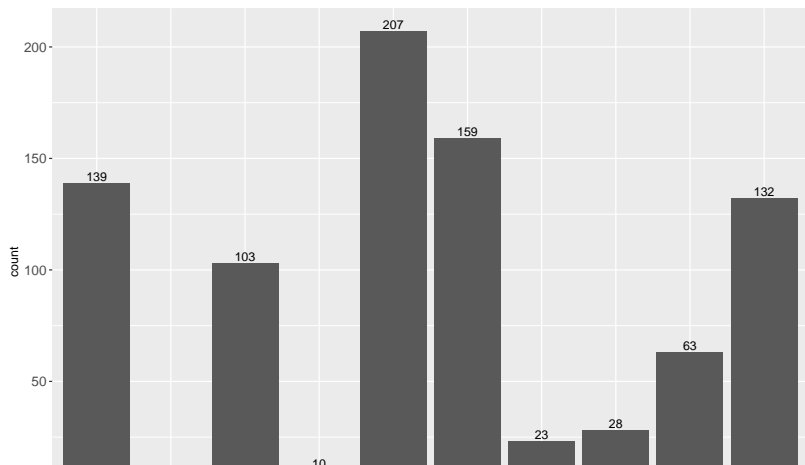
Application: considered states

- ▶ 1 = Single + no children
- ▶ 2 = Single + children
- ▶ 3 = Changing relationships + no children
- ▶ 4 = Changing rel. + children
- ▶ 5 = Relationship + living apart + no children
- ▶ 6 = Relationship + living together + no children
- ▶ 7 = Relationship + living apart + children
- ▶ 8 = Relationship + living together + children
- ▶ 9 = Married + no children
- ▶ 10 = Married + children

15-17			18-19		20	21-22		23-25			26	27-29			30-*		...
1	1	1	5	5	3	1	1	5	5	5	6	9	9	9	10	10	...

Application: distribution of states

```
## Warning: The dot-dot notation (`..count..`) was deprecated  
## i Please use `after_stat(count)` instead.  
## This warning is displayed once every 8 hours.  
## Call `lifecycle::last_lifecycle_warnings()` to see where  
## generated.
```

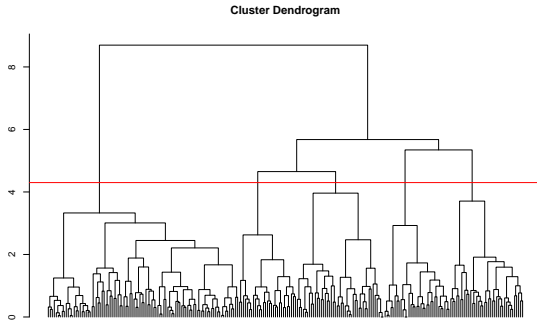


Application: cost matrix

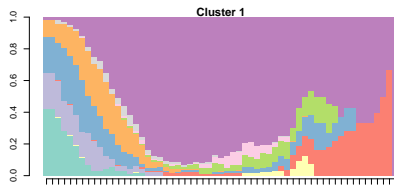
Base setup: TRATE and maxlength

[illegible]

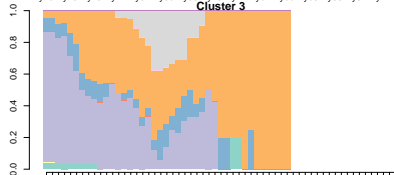
Application: clustering



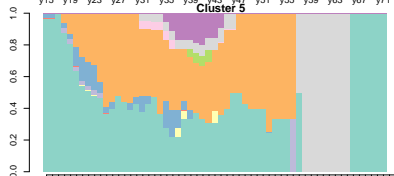
Application: clustering



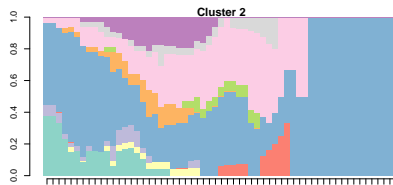
y15 y19 y23 y27 y31 y35 y39 y43 y47 y51 y55 y59 y63 y67 y71



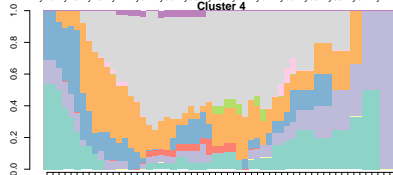
y15 y19 y23 y27 y31 y35 y39 y43 y47 y51 y55 y59 y63 y67 y71



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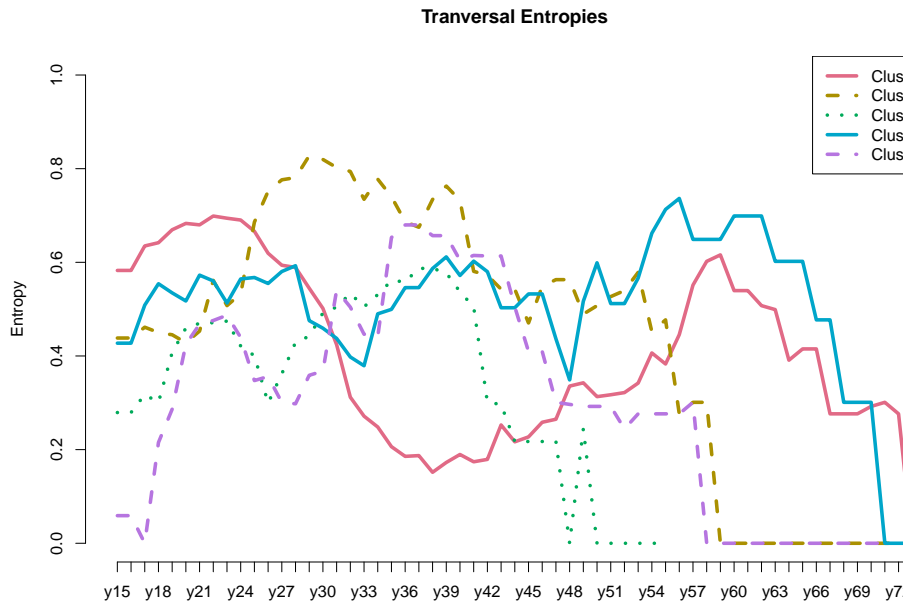


Application: clustering

Cluster	n
1	96
2	37
3	29
4	44
5	33

- ▶ Cluster 1: Married young and had children
- ▶ Cluster 2: Constantly in relationship but not married
- ▶ Cluster 3: Older, mostly married or in long relationship without children
- ▶ Cluster 4: Younger, single or in a relationship without children

Application: clustering



Distance matrix

- ▶ Given $x, y \in X$ two sequences of interest. There different mappings from $T : X \rightarrow X$ such that $T(x) = y$.
- ▶ T is composed of elements (operations) that can be insertion, deletion or substitution.
- ▶ There is a cost associated with each operation: The substitution cost are given by the cost matrix and insertion/deletion costs are set in a way that reduces/increases the importance of time shifts (low/high).
- ▶ The distance between x and y is given by the lower cost mapping.