ME40064 System Modelling and Simulation - Coursework 2

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1. Introduction

Finite Element Method (FEM) is a powerful numerical technique for solving partial differential equations (PDEs) over a discrete domain. This coursework focuses on the implementation and verification of a FEM solver for the transient diffusion-reaction equation, given by:

$$\frac{\delta c}{\delta t} = D\frac{\delta^2 c}{\delta x^2} + \lambda c + f$$

2. Part 1: Software Verification

2.1. Overview

For this section, a static FEM solver was adapted to solve the transient diffusion equation (i.e $\lambda = 0$ and $f = 0$), to give an equation of the form:

$$\frac{\delta c}{\delta t} = D\frac{\delta^2 c}{\delta x^2}$$

Furthermore, the domain was specified with:

- An x range of 0 to 1
- Dirichlet boundary conditions of c(0, t) = 1 and c(1, t) = 0
- An initial condition of c(x, 0) = 0

2.2. Implementation

2.3. Results

The solver was implemented and the following plots generated:

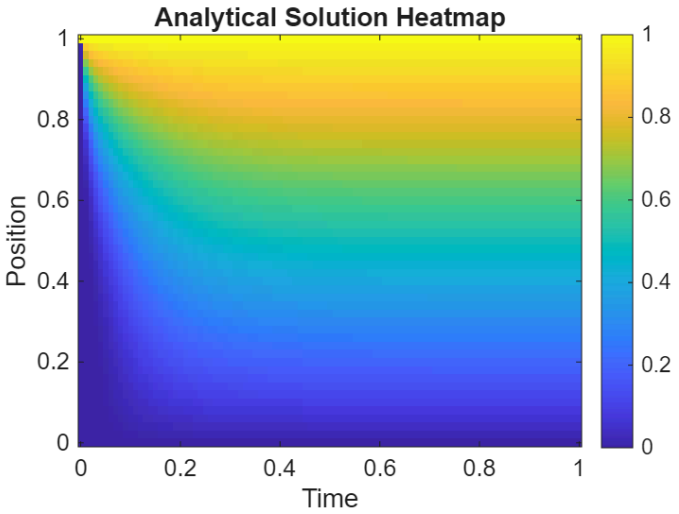

Figure 1: Linear Reaction Operator Unit Test Results

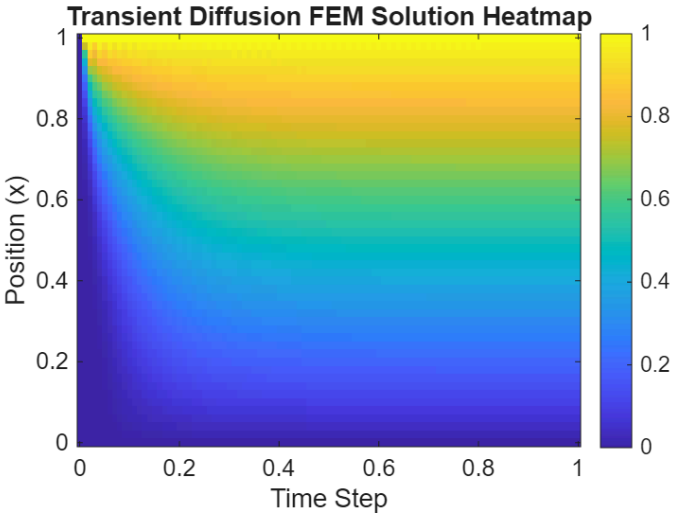

Figure 2: Linear Reaction Operator Unit Test Results

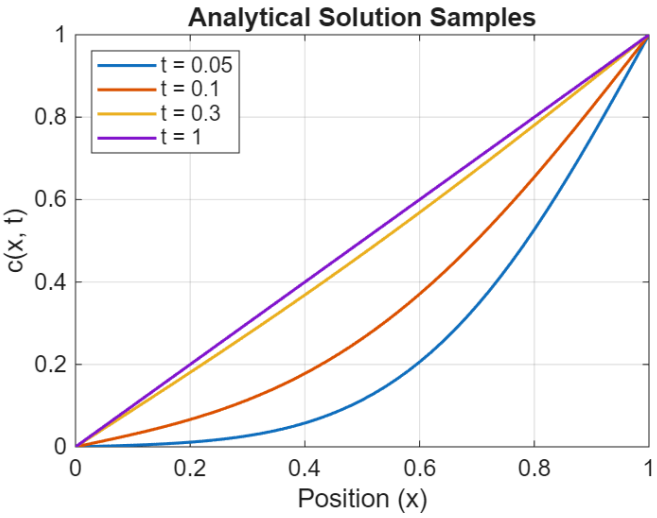

Figure 3: Linear Reaction Operator Unit Test Results

Figure 4: Linear Reaction Operator Unit Test Results

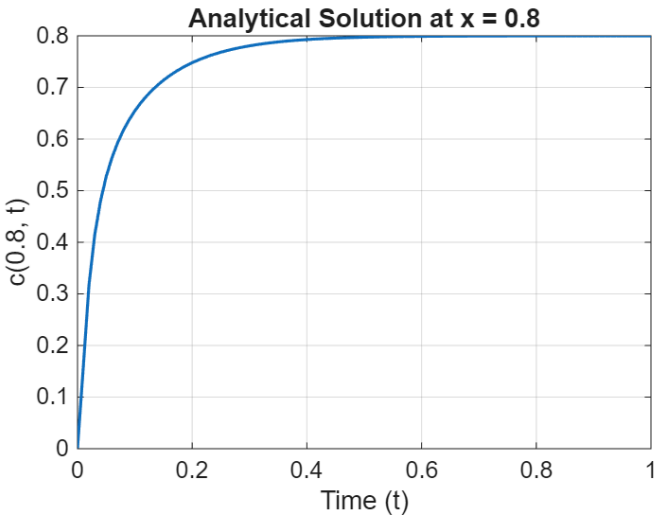

Figure 5: Linear Reaction Operator Unit Test Results

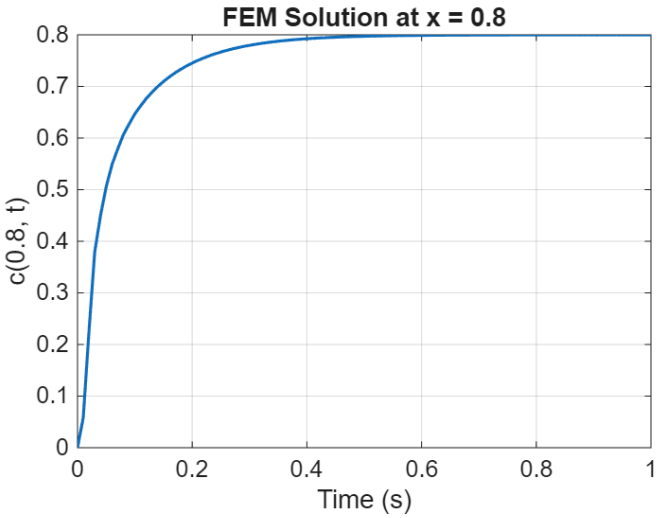

Figure 6: Linear Reaction Operator Unit Test Results

2.4. Validation

3. Part 2: Software features

Next, the FEM solver was extended to account for the following advanced features:

- Different solver methods (Explicit Euler, Implicit Euler, Crank-Nicolson)
- Gaussian quadrature for numerical integration
- Quadratic basis functions
- Using L2 norm to evaluate solution accuracy

- The **Mesh** and **MeshElement** classes were modified to support higher-order elements.
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4. Part 3: Modelling & Simulation Results

5. Conclusion

6. Use of Generative AI

This coursework was completed in Visual Studio Code (with the MATLAB Extension), using Typst for report writing. The GitHub Copilot AI tool was enabled for this, and provided generative suggestions for code snippets and report phrasing.