HW2

Sebastian Kirkpatrick

2024-09-18

Question 1

a)

```
f_1 <- function(x){x^3 - 2*x + 5}
fp_1 <- function(x){3*x^2 - 2}
xvec <- c()
xvec[1] <- 1000
for (i in 2:41){
xvec[i] <- xvec[i-1] - f_1(xvec[i-1])/fp_1(xvec[i-1])
}
xvec
```

```
##  [1] 1000.0000000  666.6671094  444.4454025  296.2979266  197.5334321
##  [6]  131.6911620   87.7973869   58.5364377   39.0313994   26.0312302
## [11]   17.3687818   11.5992963    7.7589223    5.2025429    3.4928157
## [16]    2.3186328    1.4106713    0.1547749    2.5893346    1.6407888
## [21]    0.6310513    5.5845907    3.7497757    2.4998430    1.5670330
## [26]    0.5023478    3.8187376    2.5480204    1.6069815    0.5741443
## [31]    4.5708547    3.0652738    2.0086534    1.1093140   -1.3417048
## [36]   -2.8909141   -2.3110527   -2.1170025   -2.0948302   -2.0945515
## [41]   -2.0945515
```

Root: -2.0945836

Starting at 1000, it took 40 iterations to find the negative root

b)

```
secant <- function(f, x0 = c(1000,1000.1), max.iter = 100, tol = 0.00001){

  x <- x0
  n <- 2
  eps <- 1000
  while (n < max.iter & eps > tol) {

    x[n + 1] <- x[n] - f(x[n]) * ((x[n] - x[n - 1]) / (f(x[n]) - f(x[n - 1])))

    eps <- abs(f(x[n+1]))
    n <- n + 1
```

```
  }

  return(x)

}

secant(f_1)
```

```
##  [1] 1000.00000000 1000.10000000  666.70044053  526.35169598  390.38050658
##  [6]  296.68026028  223.37707184  168.79217712  127.37022759   96.16548079
## [11]   72.59208480   54.80332568   41.37464739   31.23962260   23.59070797
## [16]   17.81909604   13.46498358   10.18121400    7.70535162    5.83849017
## [21]    4.42883053    3.35854162    2.53223066    1.86384375    1.24927332
## [26]    0.41930231  -15.84948824    0.40185492    0.38431908    3.17539385
## [31]   -0.06939367   -0.72251027    3.54154101   -1.43589431   -2.08907361
## [36]   -2.09728192   -2.09454305   -2.09455147
```

38 iterations

Question 2

```
dumb <- function(lol, n=100){
  i <- 0:n
  y<- sum((-1)^i * lol^i / (factorial(i))^2)
  return(y)
}

dumb(2)
```

```
## [1] -0.1965481
```

```
secant(dumb, c(0,1))
```

```
## [1] 0.000000 1.000000 1.288478 1.424367 1.444675 1.445788
```

Root: 1.445788

Question 3

a)

```
rrb <- (26/52) * (25/51) * (26/50)
rbr <- (26/52) * (26/51) * (25/50)
brr <- (26/52) * (26/51) * (25/50)

choose(26,2) * choose(26,1) / choose(52,3)
```

```
## [1] 0.3823529
```

```
prob_win <- rrb + rbr + brr
prob_win
```

```
## [1] 0.3823529
```

.3824 probability to win

b)

```
(1-prob_win) * -1
```

```
## [1] -0.6176471
```

```
win <- function(greg_is_lame) {
  ((-1) * (1-prob_win)) + ((greg_is_lame) * (prob_win))
}
```

Expected Value: 0.3824x - 0.6176

c)

```
secant(win, c(0,1))
```

```
## [1] 0.000000 1.000000 1.615385
```

To be a fair game, x = $1.62

Question 4

```
silly <- function(x) {(x^3) - 1957*(x^2) + 1187296 * x - 210305472}
uniroot(silly, c(0,1000))
```

```
## $root
## [1] 312
##
## $f.root
## [1] 0.1569403
##
## $iter
## [1] 11
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 6.103516e-05
```

```r
uniroot(silly, c(0,350))
```

```
## $root
## [1] 312
##
## $f.root
## [1] 0.01055354
##
## $iter
## [1] 9
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 6.103516e-05
```

```r
uniroot(silly, c(350, 800))
```

```
## $root
## [1] 773
##
## $f.root
## [1] 0.2810857
##
## $iter
## [1] 6
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 6.103516e-05
```

```r
uniroot(silly, c(800, 1000))
```

```
## $root
## [1] 872
##
## $f.root
## [1] -0.110182
##
## $iter
## [1] 8
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 6.103516e-05
```

Roots: 312, 773, 872

Question 5

a)

```
exp_CDF <- function(x,lam=2) {
  1-(exp(-(x*lam)))
}

exp_CDF(1,lam=1)
```

```
## [1] 0.6321206
```

CDF $= 1 - e^{-\lambda x}$

b)

```
find_med <- function(x, lam=2) {
  exp_CDF(x,lam) - 0.5
}

uniroot(find_med, c(0,1))$root
```

```
## [1] 0.3465728
```

Median: x $= 0.3466$

c)

```
a <- function(x) {
  exp_CDF(x,lam) - (perc*0.01)
}

mp <- function(lam,perc){
med_p <- function(x,lam,perc) {
  pexp(x,lam) - (perc * 0.01)
}
uniroot(med_p,c(-3,3),lam,perc)$root
}

mp(2,50)
```

```
## [1] 0.3465772
```

d)

```
dat <- data.frame(i = numeric(), p = numeric(), root = numeric())

percs <- c(5,25,50,75,95)

for (i in 1:100) {
```

```
  for (p in percs) {
  root <- mp(i,p)
  dat <- rbind(dat, data.frame(i=i,p=p,root=root))
  }
}
```

```
dat |>
  ggplot(
    aes(x=i,y=root,color=factor(p))
  ) +
  geom_point() +
  labs(
    x = "Lambda",
    y = "Median"
  ) +
  scale_color_discrete(name="Percentile")
```