Homework 2

AE 402 – Fall 2021

Due: Tuesday, October 19, 2021 @ 11pm CT

Starred problem (*) to be complete by 4-credit students only.

Round all answers to 3 decimal places and type into Gradescope. Upload all figures and code into Gradecope.

Problem 1

Write a computer program to convert position and velocity to orbital elements. Run the program for the following cases and upload your numbers and code into Gradescope. For each case determine if the orbit is a low-Earth-orbit (LEO), medium-Earth-orbit (MEO), Geostationary orbit (GEO) or a Geostationary-transfer-orbit (GTO).

$\mu = 398600 \text{ km}^3/\text{s}^2$

Case 1

$\boldsymbol{r}_0 = [-6115.75, -6586.18, -58.65]$ km
$\boldsymbol{v}_0 = [4.42, -4.26, -1.08]$ km/s

Case 2

$\boldsymbol{r}_0 = [6590, 0, 0]$ km
$\boldsymbol{v}_0 = [0, 10.153, 1.247]$ km/s

Problem 2

Write a computer program to convert orbital elements to position and velocity. Run the program for the following cases and upload your numbers and code into Gradescope.

(a) Case 1: $a = 8000 \text{ km}, e = 0.125, i = 10°, \Omega = 45°, \omega = 10°, M = 170°$
(b) Case 2: $r_p = 6611 \text{ km}, e = 0.01, i = 90°, \Omega = 0°, \omega = 0°, M = 355°$

Problem 3

Considering the following information, answer parts (a), (b) and (c) below.

$M_\odot = 1.989 \times 10^{30}$
$M_\oplus = 5.972 \times 10^{24}$
$M_{\text{Moon}} = 7.347 \times 10^{22}$
$M_{\text{Jupiter}} = 1.898 \times 10^{27}$
$M_{\text{Europa}} = 4.799 \times 10^{22}$
$M_{\text{Apophis}} = 2.699 \times 10^{10}$
$d_{\text{SunApophis}} = 149 \times 10^6 \text{ km}$
$d_{\text{SunJupiter}} = 750 \times 10^6 \text{ km}$
$d_{\text{EarthMoon}} = 384 \times 10^3 \text{ km}$
$d_{\text{JupiterEuropa}} = 670 \times 10^3 \text{ km}$

(a) Compute and plot the position (non-dimensional units) of the five Lagrange points for the Sun-Jupiter system. Your plot must also include the positions of the primary and secondary bodies.
(b) For the Jupiter-Europa system, compute the distance of L_1 and L_2 in km, with respect to Europa.
(c) For the Sun-Apophis system, compute the distance of L_1 in km, with respect to Apophis.

Problem 4*

Use a numerical integrator (e.g. MATLAB's ode45) with the tolerance set to 1×10^{-13} to propagate the circular restricted three-body problem equations of motion for the sets of initial conditions and corresponding time duration given below. Plot the results. Which set of initial conditions produces a Halo orbit?

Case 1

$\mu = 0.012150585609262$

$\boldsymbol{r}_0 = [1.118824382902157, 0.0, 0.014654873101278]$ Distance Units

$\boldsymbol{v}_0 = [0.0, 0.180568501159703, 0.0]$ Velocity Units

$t = 3.41213481127321$ Time Units

Case 2

$\mu = 0.012150585609262$

$\boldsymbol{r}_0 = [0.7, 0.0, 0.01]$ Distance Units

$\boldsymbol{v}_0 = [\,0.0, 0.8, 0.0]$ Velocity Units

$t = 30$ Time Units

Case 3

$\mu = 0.012150585609262$

$\boldsymbol{r}_0 = [1 - \mu, 0.5, 0.0]$ Distance Units

$\boldsymbol{v}_0 = [-1.0 \times 10^{-15}, 0.0, 0.0]$ Velocity Units

$t = 30$ Time Units

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}$$

$$\ddot{z} = \frac{\partial U}{\partial z}$$

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

$$r_1 = \sqrt{(x - x_1)^2 + y^2 + z^2}, \;\; r_2 = \sqrt{(x - x_2)^2 + y^2 + z^2}$$