Problem 2

Tracking data for an Earth orbiting satellite indicates the altitude = 600 km, $r\dot{f} = 7$ km/s, and $\dot{r} = 3.5$ km/s. Determine the eccentricity e and the true anomaly f at the data point. (Earth radius = 6378 km).

$r = 6978$ km

$r\dot{f} = 7\ \text{kms}^{-1}$

$\dot{r} = 3.5\ \text{kms}^{-1}$

$V = \dot{r} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$|V| = (\dot{r}^2 + r\dot{f}^2)^{1/2}$

$= (3.5^2 + 7^2)^{1/2} = 7.826\ \text{kms}^{-1}$

$$\boxed{V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}}$$

$$a = \left(\frac{2}{r} - \frac{V^2}{\mu}\right)^{1/2} = \left(\frac{2}{6978} - \frac{7.826^2}{398600}\right)^{1/2} = 7521\ \text{km}$$

$$h = r^2\dot{f} = r \times r\dot{f} = 6978 \times 7 = 48846$$

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos(f)} = \frac{a(1-e^2)}{1+e\cos(f)} \qquad \frac{h^2}{\mu} = a(1-e^2) \qquad \therefore\ e = \left(1 - \frac{h^2}{\mu a}\right)^{1/2}$$

$$e = \left(1 - \frac{48846^2}{398600 \times 7521}\right)^{1/2} = 0.452$$

$$\frac{h^2}{\mu}\cancel{\frac{1}{1+e\cos f}} = a(1-e^2)\cdot\cancel{\frac{1}{1+e\cos(f)}}$$

$$\cos f = \left(\frac{a(1-e^2)}{r} - 1\right)\frac{1}{e}$$

- For f...

$E_0 = 80.806$

$$\tan(f/2) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$\sin(E_0) = \frac{r\dot{r}}{e\sqrt{a\mu}}$$

$$f = 2\operatorname{atan}\left(\sqrt{\frac{1+0.4518}{1-0.4518}}\cdot\tan\left(\frac{1.498}{2}\right)\right)$$

$$= 108.344$$

$$\tan(E_0) = \frac{r\cdot\dot{r}}{\sqrt{a\mu}}\cdot\left(\frac{1}{1-\frac{r_0}{a}}\right)$$

$$= \frac{6978 \times 3.5}{\sqrt{7521 \times 398600}}\cdot\left(\frac{1}{1-\frac{6978}{7521}}\right)$$