Homework 1

AE 402 – Fall 2021

Due: Tuesday, September 21, 2021 @ 11pm CT

Starred problem (*) to be complete by 4-credit students only.

Round all answers to 3 decimal places and type into Gradescope. Upload all figures and code into Gradecope.

Problem 1

At time t_0 in units for which $\mu = 1$ (so-called canonical units), the following data is given for a two-body problem:

$$\boldsymbol{r} = [0 \quad 2 \quad 0] \text{ and } \boldsymbol{v} = [-1 \quad 1 \quad 0]/\sqrt{2}$$

(a) Calculate the vectors $\boldsymbol{h}$ and $\boldsymbol{e}$ and verify that $\boldsymbol{h} \cdot \boldsymbol{e} = 0$.
(b) Write the polar equation $r(f)$ for this conic orbit.
(c) What is the value of f at t_0?
(d) Determine the speed of the spacecraft when $r = 32$.
(e) Determine the value of f when $r = 32$.
(f) For the Earth $\mu = 398600$ km^3/s^2. Determine the value in kilometers of the length unit (LU) for which $\mu = 1$ LU^3/s^2.

Problem 2

Tracking data for an Earth orbiting satellite indicates the altitude = 600 km, $r\dot{f} = 7$ km/s, and $\dot{r} = 3.5$ km/s. Determine the eccentricity e and the true anomaly f at the data point. (Earth radius = 6378 km).

Problem 3

Write a computer program to compute the position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ for the two-body problem at arbitrary time, given $\boldsymbol{r}_0(t)$ and $\boldsymbol{v}_0(t)$. Run the program for the following initial conditions and determine position $\boldsymbol{r}(t)$ and velocity $\boldsymbol{v}(t)$ after $t = 3$ hours. Compute $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$ for 100 equally spaced times between $t_0 = 0$ and $t = 3$ hours. Plot the resulting orbit.

$\boldsymbol{r}_0(t) = [-8903.833 \quad 1208.356 \quad 213.066]$ km
$\boldsymbol{v}_0(t) = [-0.971 \quad -6.065 \quad -1.069]$ km/s

Problem 4

Using Kepler's Equation, regenerate the figure for E vs M shown at the beginning of lecture 5. Is Mean anomaly (M) swept out faster or slower than Eccentric anomaly (E) at perigee? Is Mean anomaly (M) swept our faster or slower than Eccentric anomaly (E) at apogee?

Problem 5*

Use a numerical integrator (e.g. MATLAB's ode45) with the tolerance set to 1×10^{-13} propagate the two-body equations of motion for the initial conditions given in Problem 3. Plot the orbit. Compute and plot the variation in specific Energy $(|E(t) - E(t_0)|/|E(t_0)|)$ and specific angular momentum $(\left|h(t) - h(t_0)/|h(t_0)|\right|)$ as a function of time. Plot the result using a log scale on the *y*-axis. To what extend are these quantities conserved? Repeat using a tolerance of 1×10^{-7} in your integrator. To what extend are these quantities conserved?