(a)

$r = [0 \quad 2 \quad 0]$ and $v = [-1 \quad 1 \quad 0]/\sqrt{2}$

$$r = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$a = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad a \cdot b = x \times Q + y \times R + z \times S$$

$$b = \begin{bmatrix} Q \\ R \\ S \end{bmatrix}$$

* Cross product vector orthogonal to both other vectors

$$v = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$e = \frac{\dot{r} \times h}{\mu} - \frac{\mathbf{r}}{r}$$

$$h = r \times \dot{r} = r \times v$$

$$e = \frac{\dot{r} \times h}{\mu} - \frac{\mathbf{r}}{r} = \dot{r} \times h - \frac{\mathbf{r}}{r}$$

$$\boxed{e \cdot h = 0}$$

$$\boxed{\vec{a} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \vec{b} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}}$$

$$h = r \times v = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \times \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \times 0 - 1/\sqrt{2} \times 0 \\ 0 \times 0 - 0 \times -1/\sqrt{2} \\ 0 - 1/2 - 2/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

$$e = \frac{\dot{r} \times h}{\mu} - \frac{\mathbf{r}}{r} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \times \sqrt{2} - 0 \cdot 0 \\ 0 \times 0 - (-1/\sqrt{2} \times \sqrt{2}) \\ -1/\sqrt{2} \times 0 - \frac{1}{\sqrt{2}} \times 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$h \cdot e = \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0 \times 1 + 0 \times 0 + 0 \times \sqrt{2} = \underline{0}$$

(b) $r(f) = \boxed{r = \frac{h^2}{\mu} \cdot \frac{1}{1 + e\cos(f)}}$ $\mu = 1$ $r = \frac{2}{1} \cdot \frac{1}{1 + e\cos(f)}$

$|\vec{h}| = \left|\begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix}\right| = \sqrt{2}$, $e = \left|\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right| = 1$

$\therefore \boxed{r = \frac{2}{1 + \cos(f)}}$

c) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \frac{2}{1 + \cos(f)}$

$2 = \frac{2}{1 + \cos(f)}$ $\cos(f) = 0$

$f = \pi/2 \,, \; 3\pi/2$

d) $$\boxed{V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}}$$

Speed when $r = 32$

$r = \frac{h^2}{\mu} \frac{1}{1 + e\cos f}$ or $r = \frac{p}{1 + e\cos f}$

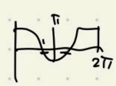
$p = a(1 - e^2)$

$h^2 = 2$ $2 = a(1 - e^2)$

$e = 1$

$a = 0 \ldots ?$

$V = \sqrt{\mu\left(\frac{2}{r}\right)} = \sqrt{1 \times \frac{2}{32}} = 1/4 = 0.25$



e) Value of f when $r = 32$

$32 = \frac{2}{1 + \cos(f)}$ $\cos(f) = \frac{2}{32} - 1$ $f = 159.64$ $200.36°$

f) $\mu = 398600 \text{ km}^3/\text{s}^2$

$\mu = 1$, LU^2/s^2

$1\,LU^2 = 398600 \text{ km}^3$

$1\,LU^2 = 1\,LU$

$\therefore 1\,LU = (398600)^{1/3} = 73.595$

Problem 2

Tracking data for an Earth orbiting satellite indicates the altitude = 600 km, $r\dot{f} = 7$ km/s, and $\dot{r} = 3.5$ km/s. Determine the eccentricity e and the true anomaly f at the data point. (Earth radius = 6378 km).

$r = 6978$ km

$r\dot{f} = 7$ kms^{-1}

$\dot{r} = 3.5$ kms^{-1}

$V = \dot{r} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$|V| = (\dot{r}^2 + r\dot{f}^2)^{1/2}$

$= (3.5^2 + 7^2)^{1/2} = 7.826$ kms^{-1}

$$\boxed{V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}}$$

$$a = \left(\frac{2}{r} - \frac{V^2}{\mu}\right)^{1/2} = \left(\frac{2}{6978} - \frac{7.826^2}{398600}\right)^{1/2} = 7521 \text{ km}$$

$$h = r^2\dot{f} = r \times r\dot{f} = 6978 \times 7 = 48846$$

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos(f)} = \frac{a(1-e^2)}{1+e\cos(f)} \qquad \frac{h^2}{\mu} = a(1-e^2) \qquad \therefore\ e = \left(1 - \frac{h^2}{\mu a}\right)^{1/2}$$

$$e = \left(1 - \frac{48846^2}{398600 \times 7521}\right)^{1/2} = 0.452$$

$$\frac{h^2}{\mu}\cancel{\frac{1}{1+e\cos f}} = a(1-e^2)\cdot\cancel{\frac{1}{1+e\cos(f)}}$$

$$\cos f = \left(\frac{a(1-e^2)}{r} - 1\right)\frac{1}{e}$$

- For f...

$E_0 = 80.806$

$$\tan(f/2) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$\sin(E_0) = \frac{r\dot{r}}{e\sqrt{a\mu}}$$

$$f = 2\,\text{atan}\left(\sqrt{\frac{1+0.4518}{1-0.4518}}\cdot\tan\left(\frac{1.498}{2}\right)\right)$$

$$= 108.344$$

$$\tan(E_0) = \frac{r\cdot\dot{r}}{\sqrt{a\mu}}\cdot\left(\frac{1}{1-\frac{r_0}{a}}\right)$$

$$= \frac{6978 \times 3.5}{\sqrt{7521 \times 398600}}\cdot\left(\frac{1}{1-\frac{6978}{7521}}\right)$$