

## Task 1

a) Adding padding to the image :

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 3 & 1 & 0 \\ 0 & 3 & 9 & 1 & 1 & 4 & 0 \\ 0 & 4 & 5 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We then apply hadamard product between the kernel matrix and each of the matrix below, obtained by sliding the kernel with stride = 1 :

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 3 & 9 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 9 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 9 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 9 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 3 & 9 & 1 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 9 & 1 & 1 \\ 5 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 4 \\ 0 & 7 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ 1 & 4 & 0 \\ 7 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 9 \\ 0 & 4 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 9 & 1 \\ 4 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 & 1 & 1 \\ 5 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 7 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We obtain :

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -4 & 0 & 4 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -2 & 0 & 6 \\ -9 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -4 & 0 & 2 \\ -1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -6 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 18 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 \\ -6 & 0 & 2 \\ -4 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ -18 & 0 & 2 \\ -5 & 0 & 7 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ -2 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ -2 & 0 & 0 \\ -7 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 9 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \\ -8 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -9 & 0 & 1 \\ -10 & 0 & 14 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -14 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

the result is finally the  $3 \times 5$  matrix where the  $ij^{th}$  component is the sum of the elements of the  $ij^{th}$  above :

$$\begin{bmatrix} 11 & -2 & -4 & 1 & -7 \\ 24 & -8 & -12 & 5 & -12 \\ 19 & -10 & -4 & 3 & -15 \end{bmatrix}$$

b) The Convolutional layer (i) is the one reducing translational sensibility since a kernel for the convolution will detect a feature regardless of its position in the image

c) The output height  $H_O$  and width  $W_O$  are computed as follow, where  $H_I$  and  $W_I$  are the image height and width,  $S$  the stride,  $F$  the kernel (receptive **F**ield) side size and  $P$  the padding size :

$$H_O = \frac{H_I - F + 2P}{S} + 1 = H_I - 6 + 2P$$

$$W_O = \frac{W_I - F + 2P}{S} + 1 = W_I - 6 + 2P$$

Because we want  $H_O = H_I$  and  $W_O = W_I$ , we need  $2P = 6$ , thus,

$$P = 3$$

Note that the number of filter does not impact the output image shape.

d) Using same equation as above, we have

$$F = H_I + 2P - S(H_O - 1) = 512 - 508 - 1 = 3$$

Kernel has dimensions  $3 \times 3$

e) Again using same equation,

$$H_O = \frac{H_I - F + 2P}{S} + 1 = \frac{508 - 2}{2} + 1 = 254$$

$$W_O = \frac{W_I - F + 2P}{S} + 1 = \frac{508 - 2}{2} + 1 = 254$$

Pooled feature maps have dimensions  $254 \times 254$

f) Similarly, feature maps of the second layer have dimension  $252 \times 252$

g)

## Task 2