

Cauchy Formula

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{it}}{e^{it} - z} f(e^{it}) dt$$

+

Cauchy-Goursat Formula

$$0 = \frac{1}{2\pi} \int_0^{2\pi} \frac{\bar{z}}{1 - \bar{z} e^{it}} f(e^{it}) dt$$

Poisson Formula Analytic Scaled

$$f(rz) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - |z|^2}{|e^{it} - z|^2} f(re^{it}) dt$$

$r \nearrow 1$

Poisson Formula Analytic

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} \operatorname{Re} \left(\frac{e^{it} + z}{e^{it} - z} \right) f(e^{it}) dt$$

\Downarrow Harmonic is real of Holomorphic

Poisson Formula Harmonic Scaled

$$u(rz) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - |z|^2}{|e^{it} - z|^2} u(re^{it}) dt$$

\Downarrow $r \nearrow 1$, Dominated Conv.

Poisson Formula Harmonic

$$u(z) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - |z|^2}{|e^{it} - z|^2} u(e^{it}) dt$$