Real-Time Attitude-Independent Three-Axis Magnetometer Calibration

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Abstract

In this paper new real-time approaches for three-axis magnetometer sensor calibration are derived. These approaches rely on a conversion of the magnetometer-body and geomagnetic-reference vectors into an attitude independent observation by using scalar checking. The goal of the full calibration problem involves the determination of the magnetometer bias vector, scale factors and non-orthogonality corrections. Although the actual solution to this full calibration problem involves the minimization of a quartic loss function, the problem can be converted into a quadratic loss function by a centering approximation. This leads to a simple batch linear least squares solution, which is easily converted into a sequential algorithm that can executed in real time. Alternative real-time algorithms are also developed in this paper, based on both the extended Kalman filter and Unscented filter. With these real-time algorithms, a full magnetometer calibration can now be performed on-orbit during typical spacecraft mission-mode operations. The algorithms are tested using both simulated data of an Earth-pointing spacecraft and actual data from the Transition Region and Coronal Explorer.

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Introduction

Three-axis magnetometers (TAMs) are widely used for onboard spacecraft operations. A paramount issue to the attitude accuracy obtained using magnetometer measurements is the precision of the onboard calibration. The accuracy obtained using a TAM depends on a number of factors, including: biases, scale factors and non-orthogonality corrections. Scale factors and non-orthogonality corrections occur because the individual magnetometer axes are not orthonormal, typically due to thermal gradients within the magnetometer or to mechanical stress from the spacecraft.[1] Magnetometer calibration is often accomplished using batch methods, where an entire set of data must be stored to determine the unknown parameters. This process is often repeated many times during the lifetime of a spacecraft in order to ensure the best possible precision obtained from magnetometer measurements.

If an attitude is known accurately, then the magnetometer calibration problem is easy to solve. However, this is generally not the case. Fortunately, an attitude-independent scalar observation can be obtained using the norms of the body-measurement and geomagnetic-reference vectors. For the noise-free case, these norms are identical because the attitude matrix preserves the length of a vector. This process is also known as “scalar checking”.[2] Unfortunately, even for the simpler magnetometer-bias determination problem, the loss function to be minimized is quartic in nature. The most common technique to overcome this difficulty has been proposed by Gambhir, who applies a “centering” approximation to yield a quadratic loss function that can be minimized using simple linear least squares.[3] Alonso and Shuster expand upon Gambhir’s approach by using a second step that employs the centered estimate as an initial value to an iterative Gauss-Newton method. Their algorithm, called “TWOSTEP”,[4] has been shown to perform well when other algorithms fail due to divergence problems. Furthermore, Alonso and Shuster have extended this approach to perform a complete calibration involving biases as well as scale factors and non-orthogonality corrections.[1]

One of the current goals for modern-day spacecraft is the ability to perform onboard and autonomous calibrations in real time without ground support. The TWOSTEP algorithm requires an iterative process on a batch of data, so it cannot be performed in real time. The main objective of this paper is to present and compare several sequential algorithms that are suitable for real-time applications. The centering approximation leads to a non-iterative least-squares solution, and has been shown to be nearly optimal for many realistic cases.[5] Since this approximation is linear, then it can be converted into a sequential process, which is the first real-time algorithm shown in this paper. The second algorithm uses an extended Kalman filter approach that is developed with commonly employed estimation techniques. The third algorithm uses an Unscented filter approach that offers very good results for robust calibration when the initial conditions are poorly known. Simulated test cases and results using real data obtained from the Transition Region and Coronal Explorer (TRACE) spacecraft show the validity of the new real-time algorithms to perform onboard and autonomous calibrations.

Measurement Model

In this section the TAM measurement model and attitude-independent observation are summarized. More details on these concepts can be found in Ref. 1. The magnetometer measurements can be modelled as

$$\mathbf{B}_k = (I_{3\times 3} + D)^{-1}(\mathcal{O}^T A_k \mathbf{H}_k + \mathbf{b} + \boldsymbol{\epsilon}_k), \quad k = 1, 2, \ldots, N \tag{1}$$

where $\mathbf{B}_k$ is the measurement of the magnetic field by the magnetometer at time t_k, $\mathbf{H}_k$ is the corresponding value of the geomagnetic field with respect to an Earth-fixed coordinate system, A_k is the unknown attitude matrix of the magnetometer with respect to the Earth-fixed coordinates, D is an unknown fully-populated matrix of scale factors (the diagonal elements) and non-orthogonality corrections (the off-diagonal elements), $\mathcal{O}$ is an orthogonal matrix (see Ref. 1 for a discussion on the physical connotations of this matrix), $\mathbf{b}$ is the bias vector, and $\boldsymbol{\epsilon}_k$ is the measurement noise vector that is assumed to be a zero-mean Gaussian process with covariance Σ_k. The matrix D can be assumed to be symmetric without loss of generality. Also, $I_{n\times n}$ is an $n \times n$ identity matrix. The goal of the full calibration problem is to estimate D and $\mathbf{b}$. We first define the following quantities:

$$\boldsymbol{\theta} \equiv \begin{bmatrix} \mathbf{b}^T & \mathbf{D}^T \end{bmatrix}^T \tag{2a}$$

$$\mathbf{D} \equiv \begin{bmatrix} D_{11} & D_{22} & D_{33} & D_{12} & D_{13} & D_{23} \end{bmatrix}^T \tag{2b}$$

$$E \equiv 2D + D^2 \tag{2c}$$

$$\mathbf{c} \equiv (I_{3\times 3} + D)\mathbf{b} \tag{2d}$$

$$S_k \equiv \begin{bmatrix} B_{1_k}^2 & B_{2_k}^2 & B_{3_k}^2 & 2B_{1_k}B_{2_k} & 2B_{1_k}B_{3_k} & 2B_{2_k}B_{3_k} \end{bmatrix} \tag{2e}$$

$$\mathbf{E} \equiv \begin{bmatrix} E_{11} & E_{22} & E_{33} & E_{12} & E_{13} & E_{23} \end{bmatrix}^T \tag{2f}$$

An attitude-independent observation can be computed from

$$z_k \equiv ||\mathbf{B}_k||^2 - ||\mathbf{H}_k||^2 = L_k\boldsymbol{\theta}' - ||\mathbf{b}(\boldsymbol{\theta}')||^2 + v_k \tag{3}$$

where

$$L_k \equiv \begin{bmatrix} 2\mathbf{B}_k^T & -S_k \end{bmatrix} \tag{4a}$$

$$\boldsymbol{\theta}' \equiv \begin{bmatrix} \mathbf{c}^T & \mathbf{E}^T \end{bmatrix}^T \tag{4b}$$

$$v_k \equiv 2[(I_{3\times 3} + D)\mathbf{B}_k - \mathbf{b}]^T\boldsymbol{\epsilon}_k - ||\boldsymbol{\epsilon}_k||^2 \tag{4c}$$

The effective measurement noise, v_k, is approximately Gaussian with mean denoted by μ_k and variance denoted by σ_k^2, each given by

$$\mu_k \equiv E\left\{v_k\right\} = -\mathrm{Tr}(\Sigma_k) \tag{5a}$$

$$\sigma_k^2 \equiv E\left\{v_k^2\right\} - \mu_k^2 = 4[(I_{3\times3}+D)\mathbf{B}_k - \mathbf{b}]^T \Sigma_k [(I_{3\times3}+D)\mathbf{B}_k - \mathbf{b}] + 2\left(\mathrm{Tr}\,\Sigma_k^2\right) \tag{5b}$$

$$\Sigma_k = E\left\{\boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_k^T\right\} \tag{5c}$$

where $E\{\ \}$ denotes expectation. Note that the measurement variance in Eq. (5b) is a function of the unknown parameters. A conversion from $\mathbf{c}$ and E to the sought variables $\mathbf{b}$ and D can be found in Ref. 1.

Sequential Centered Algorithm

The measurement model in Eq. (3) is clearly nonlinear in the unknown parameter vector $\boldsymbol{\theta}'$. Therefore, linear least squares cannot be applied directly. However, it is possible to determine an approximate linear solution by applying a centering approach. The complete batch algorithm is shown in Ref. 1. Since this solution is linear, then a sequential formulation can be derived that provides real-time estimates. A formal derivation of this process can be found in Ref. 6; we only present the final algorithm here. First, the sequential formulas for the averaged quantities are given by

$$\bar{L}_{k+1} = \frac{1}{\sigma_{k+1}^2 + \bar{\sigma}_k^2}\left(\sigma_{k+1}^2 \bar{L}_k + \bar{\sigma}_{k+1}^2 L_{k+1}\right) \tag{6a}$$

$$\bar{z}_{k+1} = \frac{1}{\sigma_{k+1}^2 + \bar{\sigma}_k^2}\left(\sigma_{k+1}^2 \bar{z}_k + \bar{\sigma}_{k+1}^2 z_{k+1}\right) \tag{6b}$$

$$\bar{\mu}_{k+1} = \frac{1}{\sigma_{k+1}^2 + \bar{\sigma}_k^2}\left(\sigma_{k+1}^2 \bar{\mu}_k + \bar{\sigma}_{k+1}^2 \mu_{k+1}\right) \tag{6c}$$

where

$$\frac{1}{\bar{\sigma}_{k+1}^2} = \frac{1}{\bar{\sigma}_k^2} + \frac{1}{\sigma_{k+1}^2} \tag{7}$$

Next, the following centered variables are defined:

$$\tilde{L}_{k+1} \equiv L_{k+1} - \bar{L}_{k+1} \tag{8a}$$

$$\tilde{z}_{k+1} \equiv z_{k+1} - \bar{z}_{k+1} \tag{8b}$$

$$\tilde{\mu}_{k+1} \equiv \mu_{k+1} - \bar{\mu}_{k+1} \tag{8c}$$

Finally, the sequential formulas for the optimal centered estimate of $\boldsymbol{\theta}'$, denoted by $\tilde{\boldsymbol{\theta}}'^*$, and covariance of $\tilde{\boldsymbol{\theta}}'^*$, denoted by $\tilde{P}_{\theta'\theta'}$, are given by

$$\tilde{\boldsymbol{\theta}}'^*_{k+1} = K_k \tilde{\boldsymbol{\theta}}'^*_k + \frac{1}{\sigma^2_{k+1}} \left(\tilde{z}_{k+1} - \tilde{\mu}_{k+1}\right) \tilde{P}_{\theta'\theta'_{k+1}} \tilde{L}^T_{k+1} \tag{9a}$$

$$\tilde{P}_{\theta'\theta'_{k+1}} = K_k \tilde{P}_{\theta'\theta'_k} \tag{9b}$$

$$K_k \equiv I_{9\times 9} - \tilde{P}_{\theta'\theta'_k} \tilde{L}^T_{k+1} \left(\tilde{L}_{k+1} \tilde{P}_{\theta'\theta'_k} \tilde{L}^T_{k+1} + \sigma^2_{k+1}\right)^{-1} \tilde{L}_{k+1} \tag{9c}$$

Note that only an inverse of a scalar quantity is required in the sequential process. The sequential process can be initialized using a small batch of data. A conversion from $\tilde{P}_{\theta'\theta'}$ to the covariance of the parameters $\mathbf{b}$ and $\mathbf{D}$ can be found in Ref. 1. Also, an approach for determining σ^2_{k+1} involves using the previous estimate in Eq. (5b).

Kalman Filter Formulation

In this section an extended Kalman filter (EKF) is derived to determine the calibration parameters in real time. An advantage of the EKF formulation over the sequential centered approach is that $\mathbf{b}$ and $\mathbf{D}$ can be computed directly without a conversion from $\mathbf{c}$ and $\mathbf{E}$. A summary of the EKF equations can be found in Ref. 7. Since the vector $\boldsymbol{\theta}$ in Eq. (2a) is constant, then the state model is given by $\dot{\hat{\mathbf{x}}}(t) = \mathbf{0}$, where $\hat{\mathbf{x}} \equiv \boldsymbol{\theta}^*$, which is used to denote the optimal estimate of $\boldsymbol{\theta}$. The measurement model is given by $z_k = h_k(\mathbf{x}_k) + v_k$, where

$$h_k(\mathbf{x}_k) \equiv -\mathbf{B}^T_k (2D_k + D^2_k)\mathbf{B}_k + 2\mathbf{B}^T_k (I_{3\times 3} + D_k)\mathbf{b}_k - ||\mathbf{b}_k||^2 \tag{10}$$

Since no process noise appears in the state model, then the updated quantities (state and covariance) are given by their respective propagated quantities. The EKF equations then reduce down to

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + K_k[z_{k+1} - h_{k+1}(\hat{\mathbf{x}}_k)] \tag{11a}$$

$$P_{k+1} = [I_{9\times 9} - K_k H_{k+1}(\hat{\mathbf{x}}_k)]P_k \tag{11b}$$

$$K_k = P_k H_{k+1}^T(\hat{\mathbf{x}}_k)\left[H_{k+1}(\hat{\mathbf{x}}_k)P_k H_{k+1}^T(\hat{\mathbf{x}}_k) + \sigma_{k+1}^2(\hat{\mathbf{x}}_k)\right]^{-1} \tag{11c}$$

where $P \equiv P_{\theta\theta}$, which is the covariance of the estimated parameters for $\mathbf{b}$ and $\mathbf{D}$. The state dependence of the measurement variance is shown through Eq. (5b). The 1×9 matrix $H(\mathbf{x})$ is the partial derivative of $h(\mathbf{x})$ with respect to $\mathbf{x}$. This quantity is given by

$$H(\mathbf{x}) = \begin{bmatrix} 2\mathbf{B}^T(I_{3\times 3} + D) - 2\mathbf{b}^T & -S\dfrac{\partial \mathbf{E}}{\partial \mathbf{D}} + 2J \end{bmatrix} \tag{12}$$

where S is defined in Eq. (2e) and

$$\frac{\partial \mathbf{E}}{\partial \mathbf{D}} = \begin{bmatrix} 2(1+D_{11}) & 0 & 0 & 2D_{12} & 2D_{13} & 0 \\ 0 & 2(1+D_{22}) & 0 & 2D_{12} & 0 & 2D_{23} \\ 0 & 0 & 2(1+D_{33}) & 0 & 2D_{13} & 2D_{23} \\ D_{12} & D_{12} & 0 & 2+D_{11}+D_{22} & D_{23} & D_{13} \\ D_{13} & 0 & D_{13} & D_{23} & 2+D_{11}+D_{33} & D_{12} \\ 0 & D_{23} & D_{23} & D_{13} & D_{12} & 2+D_{22}+D_{33} \end{bmatrix} \tag{13a}$$

$$J \equiv \begin{bmatrix} B_1 b_1 & B_2 b_2 & B_3 b_3 & B_1 b_2 + B_2 b_1 & B_1 b_3 + B_3 b_1 & B_2 b_3 + B_3 b_2 \end{bmatrix} \tag{13b}$$

The sensitivity matrix $H(\hat{\mathbf{x}})$ in the EKF evaluates $H(\mathbf{x})$ at its current estimate, and the notations $h_{k+1}(\hat{\mathbf{x}}_k)$, $H_{k+1}(\hat{\mathbf{x}}_k)$ and $\sigma_{k+1}^2(\hat{\mathbf{x}}_k)$ denote an evaluation at the $k+1$ time-step measurement using $\mathbf{B}_{k+1}$ and at the k time-step estimate using $\hat{\mathbf{x}}_k$.

Unscented Filter Formulation

In this section a new approach, developed by Julier, Uhlmann and Durrant-Whyte,[8] is discussed as an alternative to the EKF. This approach, which they called the *Unscented filter* (UF), works on the premise that with a fixed number of parameters it should be easier to approximate a Gaussian distribution than to approximate an arbitrary nonlinear function. The Unscented filter uses a different propagation than the form given by the standard extended Kalman filter. Given an $n \times n$ covariance matrix P, a set of order n points can be generated from the columns (or rows) of the matrices $\pm\sqrt{nP}$. The set of points is zero-mean, but if the distribution has mean $\boldsymbol{\mu}$, then simply adding $\boldsymbol{\mu}$ to each of the points yields a symmetric set of $2n$ points having the desired mean and covariance. Due to the symmetric nature of this set, its odd central moments are zero, so its first three moments are the same as the original Gaussian distribution (see Ref. 9 for more details).

The implementation of the UF for real-time magnetometer calibration is straightforward. First, the following set of *sigma points* are computed from $P \equiv P_{\theta\theta}$:

$$\boldsymbol{\sigma}_k \leftarrow 2n \text{ columns from } \pm\gamma\sqrt{P_k} \tag{14a}$$

$$\boldsymbol{\chi}_k(0) = \hat{\mathbf{x}}_k \tag{14b}$$

$$\boldsymbol{\chi}_k(i) = \boldsymbol{\sigma}_k(i) + \hat{\mathbf{x}}_k, \quad i = 1, 2, \ldots, 2n \tag{14c}$$

The parameter γ is given by $\gamma = \sqrt{n+\lambda}$, where the composite scaling parameter, λ, is given by $\lambda = \alpha^2(n+\kappa) - n$. The constant α determines the spread of the sigma points and is usually set to a small positive value (e.g. $1\times 10^{-4} \leq \alpha \leq 1$).[9] Also, the parameter κ is usually given by $\kappa = 3 - n$. Efficient methods to compute the matrix square root can be found by using the Cholesky decomposition.[10] The following weights are now defined:

$$W_0^{\text{mean}} = \frac{\lambda}{n+\lambda} \tag{15a}$$

$$W_0^{\text{cov}} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \tag{15b}$$

$$W_i^{\text{mean}} = W_i^{\text{cov}} = \frac{1}{2(n+\lambda)}, \quad i = 1, 2, \ldots, 2n \tag{15c}$$

where β is used to incorporate prior knowledge of the distribution (for Gaussian distributions $\beta = 2$ is optimal).

Since the state model estimate is given by $\dot{\hat{\mathbf{x}}}(t) = \mathbf{0}$, then the propagated values for the state and covariance are given by their respective updated values, which significantly reduces the computational requirements in the UF. Hence, the only essential difference between the EKF and UF formulations is in the computation of the innovations covariance, where the EKF uses a first-order expansion to compute this quantity, while the UF uses a nonlinear transformation to compute this quantity. For the TAM calibration algorithm using the UF, the state estimate is calculated by

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + K_k(z_{k+1} - \hat{z}_k) \tag{16}$$

where $\hat{z}_k$ is the mean observation, given by

$$\hat{z}_k = \sum_{i=0}^{2n} W_i^{\text{mean}}\, h_{k+1}\left[\boldsymbol{\chi}_k(i)\right] \tag{17}$$

where $h_{k+1}\left[\boldsymbol{\chi}_k(i)\right]$ is defined in Eq. (10). Note that $h_{k+1}\left[\boldsymbol{\chi}_k(i)\right]$ denotes an evaluation at the $k+1$ time-step measurement using $\mathbf{B}_{k+1}$ and at the k time-step sigma point using $\boldsymbol{\chi}_k(i)$. The gain K_k is computed by

$$K_k = P_k^{xz}[P_k^{zz} + \sigma_{k+1}^2(\hat{\mathbf{x}}_k)]^{-1} \tag{18}$$

where P_k^{xz} is the cross-correlation matrix between $\hat{\mathbf{x}}_k$ and $\hat{z}_k$, given by

$$P_k^{xz} = \sum_{i=0}^{2n} W_i^{\text{cov}} \left\{\boldsymbol{\chi}_k^x(i) - \hat{\mathbf{x}}_k\right\} \left\{h_{k+1}\left[\boldsymbol{\chi}_k(i)\right] - \hat{z}_k\right\}^T \tag{19}$$

and P_k^{zz} is the output covariance, given by

$$P_k^{zz} = \sum_{i=0}^{2n} W_i^{\text{cov}} \left\{h_{k+1}\left[\boldsymbol{\chi}_k(i)\right] - \hat{z}_k\right\} \left\{h_{k+1}\left[\boldsymbol{\chi}_k(i)\right] - \hat{z}_k\right\}^T \tag{20}$$

Finally, the propagated covariance is given by

$$P_{k+1} = P_k - K_k[P_k^{zz} + \sigma_{k+1}^2(\hat{\mathbf{x}}_k)]K_k^T \tag{21}$$

New sigma points can now be calculated using P_{k+1} for the sequential UF process.

Another approach for the UF uses the measurement noise model of Eq. (3) with an augmented vector given by the state and $\boldsymbol{\epsilon}$ in Eq. (4c). Therefore, a decomposition of a 12×12 matrix is now required. In the strictest sense this approach is more optimal than the first approach because the effect of the nonlinear-appearing measurement noise is directly used in the UF. But, the computational requirements are vastly increased due to the decomposition of a higher dimensional augmented matrix. Also, from numerous simulation trials no apparent advantages to using the augmented approach in the UF is seen. More details on this UF formulation for magnetometer calibration can be found in Ref. 11.

Simulated and Real Data Results

In this section results of the EKF and UF formulations are shown using both simulated and real data. The simulated spacecraft is modelled after the Tropical Rainfall Measurement Mission (TRMM) spacecraft. This is an Earth-pointing spacecraft (rotating about its y-axis) in low-Earth orbit (currently near-circular at 402 km), with an inclination of 35°.[12] The geomagnetic field is simulated using a 10$^{\text{th}}$-order International Geomagnetic Reference Field model.[13] The magnetometer-body and geomagnetic-reference vectors for the simulated runs each have a magnitude of about 500 milliGauss (mG). The measurement noise is assumed to be white and Gaussian, and the covariance is taken to be isotropic with a standard deviation of 0.5 mG. The measurements are sampled every 10 seconds over an 8-hour span. The true values for the bias $\mathbf{b}$ and elements of the D matrix are shown in Table 1. Large values for the biases are used to test the robustness of the sequential centered, EKF and UF algorithms.

Thirty runs have been executed, which provide a Monte-Carlo type simulation. Shown in Table 1 are the averaged batch solutions given by the TWOSTEP and centered algorithms, each with their maximum deviations obtained. Since the TWOSTEP approach is the most rigorous, all comparisons are made with respect to this algorithm. The centered algorithm does a fairly good job at estimating all parameters, with the exception of b_2. This parameter corresponds to the least observable variable, which results in a wide variation from the averaged value.

The EKF and UF are both executed at time $t = 0$ using initial conditions of zeros for all states. The initial covariance matrix is diagonal, given by

$$P_0 = \begin{bmatrix} 500I_{3\times3} & 0_{3\times6} \\ \\ 0_{6\times3} & 0.001I_{6\times6} \end{bmatrix} \tag{22}$$

This assumes a 3σ bound on the initial bias estimates to be about 70 mG and a 3σ bound on the initial estimates for the elements of the D matrix to be about 0.1. The parameters used in the UF are $\alpha = 0.1$, $\beta = 2$, $\kappa = 3 - n$, and $n = 9$. The EKF and UF solutions at the final time are shown in Table 1. The EKF does not converge to the correct solution for many of the parameters, while the UF gave results that are just as good as the TWOSTEP solutions. Also, the maximum deviations for the UF are much smaller than for the EKF and centered algorithm. Even though the mean values of the centered algorithm are better than the UF results, it is important to note the $+/-$ values are more important because they represent the variability over the 30 Monte-Carlo runs.

Figures 1 and 2 show the EKF and UF estimates for the parameter b_3 for a typical case. The EKF does not converge to the correct solution during the 8-hour simulated run. This is due to the fact that the first-order approximation in the EKF does not adequately capture the large initial errors. The biggest concern with the EKF solutions is the confidence of the results dictated by the 3σ bounds, with b_3 shown in Figure 3. In fact, this plot shows that the EKF is performing better than the UF. This can certainly provide some misleading results. However, unlike the EKF, the maximum deviations associated with the UF shown in Table 1 are within the 3σ bounds for all the parameters, also shown in Figure 2 for the parameter b_3. This indicates that the UF is performing in an optimal manner. But, the UF algorithm comes with a computational cost, mainly due to the covariance decomposition. Our experience has shown that the UF algorithm is about 2 times slower than the EKF algorithm. Still, the performance enhancements of the UF over the EKF may outweigh the increased computational costs.

The robustness of the real-time algorithms is now tested by adding colored noise to the measurements, which more closely models the actual geomagnetic field errors. This noise is modelled using a first-order Markov process driven by white noise, where the "time constant" corresponds to an orbital arc length of 18 degrees[4] and the standard deviation of the output magnitude has a steady-state value of 2 mG for each axis. The initial conditions and covariances are the same as the previous simulation. Shown in Table 2 are the averaged batch solutions given by the TWOSTEP, sequential centered, EKF and UF algorithms, each with their maximum deviations obtained. Larger deviations are present due to the colored-noise process. We should also note that the actual errors are outside the bounds computed from the covariance of all the estimators based on the now incorrect assumptions of the measurement noise.[4] Clearly, b_2 is not well estimated for any of the algorithms. Moreover, the sequential centered algorithm gave the worst results for b_2. A surprising outcome is given for the EKF algorithm because it is now performing nearly as well as the TWOSTEP and

UF algorithms in terms of the final solutions, which contradicts the results of the previous simulation using white-noise errors only. This may be due to an increased observability from the artificial "motion" induced by the colored noise. Figure 4 shows the convergence of b_3 for both the EKF and UF. Even though the EKF estimates converge to nearly the same value as the UF estimates, the UF converges near the true value of 60 mG much faster than the EKF. Similar results are seen in the other parameters as well. Both simulation results, one using white-noise errors only and the other using colored-noise errors, indicate that the UF provides the most robust real-time algorithm in terms of both overall accuracy and convergence properties.

Next, results using real data from the TRACE spacecraft are shown. This is an Sun-synchronous spacecraft in low-Earth orbit (currently near-circular at 402 km). The data collected for the spacecraft is given during an inertial pointing mode. The errors associated with the geomagnetic field model are typically spacially correlated and may be non-Gaussian in nature.[14] This violates the assumptions for all the estimators shown in this paper. We still assume that the measurement noise is white and Gaussian, but the standard deviation is now increased to a value of 3 mG, which bounds the errors in a practical sense. The measurements are sampled every 3 seconds over a 6-hour span.

The EKF and UF are both executed at time $t = 0$ using initial conditions of zeros for all states. The initial covariance matrix is diagonal, given by

$$P_0 = \begin{bmatrix} 10 I_{3\times 3} & 0_{3\times 6} \\ & \\ 0_{6\times 3} & 0.001 I_{6\times 6} \end{bmatrix} \tag{23}$$

This assumes a 3σ bound on the initial bias estimates to be about 10 mG and a 3σ bound on the initial estimates for the elements of the D matrix to be about 0.1. The parameters used in the UF are $\alpha = 0.1$, $\beta = 2$, $\kappa = 3 - n$, and $n = 9$. For the experimental data the solutions obtained using TWOSTEP, and the EKF and UF algorithms are nearly identical.

This is most likely due to the well-behaved nature of the data (i.e., the calibration errors are small). However, the sequential centered algorithm gave slightly different results. The centered algorithm final results are given by

$$\mathbf{b}^* = \begin{bmatrix} 1.4007 & -8.7350 & -3.7927 \end{bmatrix}^T \tag{24a}$$

$$D^* = \begin{bmatrix} 0.0086 & 0.0437 & 0.0065 & 0.0006 & 0.0035 & -0.0120 \end{bmatrix}^T \tag{24b}$$

The TWOSTEP, EKF and UF final results are given by

$$\mathbf{b}^* = \begin{bmatrix} 1.6056 & -8.4140 & -4.6123 \end{bmatrix}^T \tag{25a}$$

$$D^* = \begin{bmatrix} 0.0123 & 0.0181 & 0.0040 & -0.0005 & 0.0038 & -0.0019 \end{bmatrix}^T \tag{25b}$$

Figure 5 shows EKF estimates for the bias vector $\mathbf{b}$. Another advantage of a real-time approach is the convergence properties of the particular estimator. From Figure 5 good convergence is seen for all the parameters, which indicates that the calibration parameters are well behaved (i.e., truly constant in a practical sense). Figure 6 shows the 3σ bounds for the bias estimates. This at least qualitatively indicates that good parameter estimates are achieved since these bounds are fairly small compared to the TAM measurements. Similar results are obtained for the D matrix parameters.

An investigation of the residuals between the norm of the estimated vector, using the calibrated parameters, and the geomagnetic-reference vector is useful to check the consistency of the results. A plot of these residuals is shown in Figure 7. A spectrum analysis shows the presence of sinusoidal motions with periods equivalent to the orbital period ($\approx$ 90 min) and higher-order harmonics (see Ref. 14 for a model of this process). The mean value for the sequential centered residuals is 0.60 mG, while the mean value for the EKF and UF residuals is only 0.02 mG. Also, the magnitudes of the EKF and UF residuals are smaller than the centered residuals.

Conclusions

In this paper three real-time algorithms were developed for the calibration of three-axis magnetometers. The first algorithm was derived from a linear least squares approach based on a centering approximation. The other algorithms were derived using the extended Kalman filter and Unscented filter. Simulated Monte-Carlo test cases showed that the Unscented filter gave accurate results with the least amount of variation compared to the other real-time algorithms, and is very robust to realistic non-white noise errors. Results using real data indicated that the residuals from the extended Kalman filter and Unscented filter algorithms have mean closer to zero and have smaller magnitudes than the residuals from the from the sequential centered algorithm. Taken together, the simulation and real data results indicate that the Unscented filter provided the most robust real-time algorithm in terms of both overall accuracy and convergence properties. Therefore, this algorithm is recommended for actual implementation when computational requirements are not burdensome.

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List of Table Captions

	Truth	TWOSTEP	Centered	EKF	UF
b_1	50 mG	49.9620 ± 0.4320	49.9700 ± 0.5525	49.4029 ± 2.2625	49.7289 ± 0.4328
b_2	30 mG	29.8174 ± 0.4258	29.5937 ± 5.5779	19.5297 ± 2.2044	29.4657 ± 0.3598
b_3	60 mG	60.0331 ± 0.3165	60.0456 ± 0.3165	50.3248 ± 1.2340	59.4561 ± 0.2331
D_{11}	0.05	0.0500 ± 0.0001	0.0500 ± 0.0001	0.0492 ± 0.0008	0.0499 ± 0.0001
D_{22}	0.10	0.0993 ± 0.0014	0.0988 ± 0.0123	0.0736 ± 0.0075	0.0949 ± 0.0013
D_{33}	0.05	0.0500 ± 0.0001	0.0500 ± 0.0001	0.0481 ± 0.0004	0.0499 ± 0.0001
D_{12}	0.05	0.0499 ± 0.0010	0.0499 ± 0.0011	0.0313 ± 0.0051	0.0486 ± 0.0010
D_{13}	0.05	0.0499 ± 0.0001	0.0499 ± 0.0001	0.0440 ± 0.0006	0.0495 ± 0.0002
D_{23}	0.05	0.0501 ± 0.0007	0.0501 ± 0.0007	0.0287 ± 0.0030	0.0487 ± 0.0006

	Truth	TWOSTEP	Centered	EKF	UF
b_1	50 mG	49.4639 ± 5.8713	48.8273 ± 5.7715	47.8364 ± 5.7580	48.4291 ± 5.6001
b_2	30 mG	25.6061 ± 6.8879	41.7811 ± 76.5934	25.0715 ± 7.7910	25.7210 ± 6.6465
b_3	60 mG	59.8327 ± 5.6847	58.8314 ± 7.1579	57.6702 ± 6.0841	59.0407 ± 5.9030
D_{11}	0.05	0.0506 ± 0.0042	0.0509 ± 0.0094	0.0475 ± 0.0039	0.0483 ± 0.0040
D_{22}	0.10	0.0836 ± 0.0270	0.1350 ± 0.1452	0.0824 ± 0.0311	0.0882 ± 0.0278
D_{33}	0.05	0.0508 ± 0.0042	0.0526 ± 0.0200	0.0480 ± 0.0040	0.0486 ± 0.0036
D_{12}	0.05	0.0482 ± 0.0124	0.0452 ± 0.0306	0.0448 ± 0.0145	0.0460 ± 0.0136
D_{13}	0.05	0.0478 ± 0.0028	0.0488 ± 0.0117	0.0497 ± 0.0024	0.0498 ± 0.0019
D_{23}	0.05	0.0494 ± 0.0118	0.0431 ± 0.0359	0.0445 ± 0.0156	0.0473 ± 0.0136

List of Figure Captions

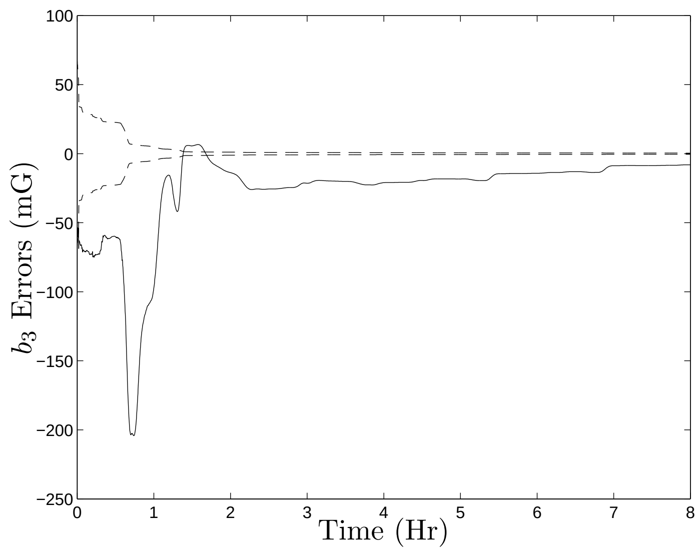
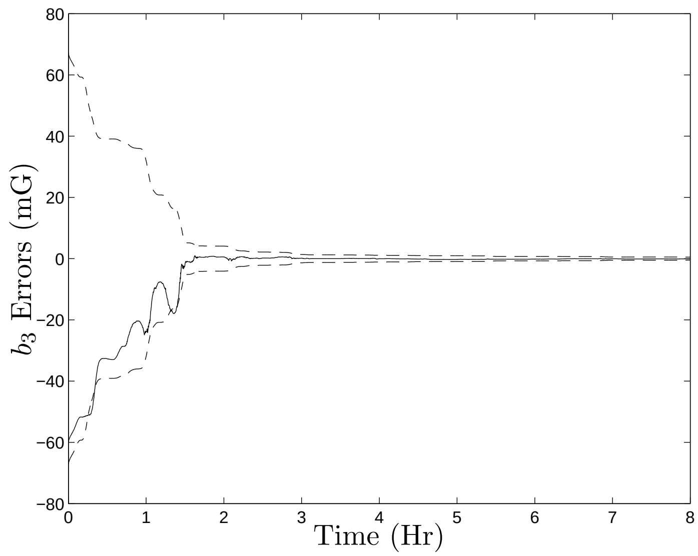
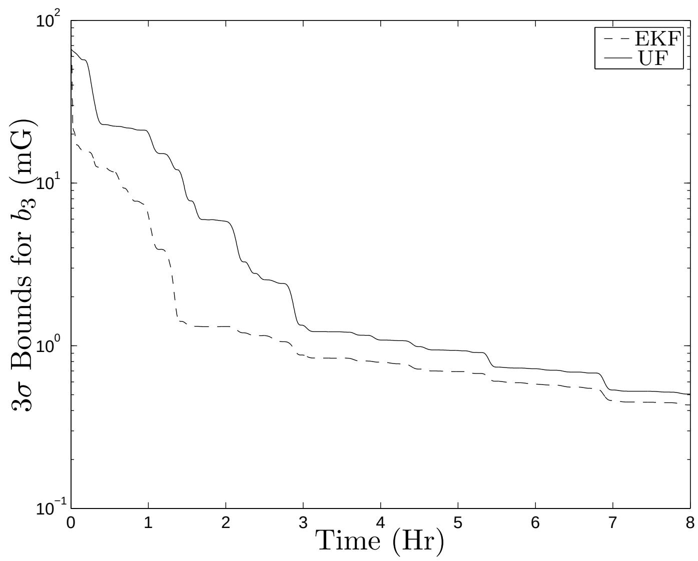


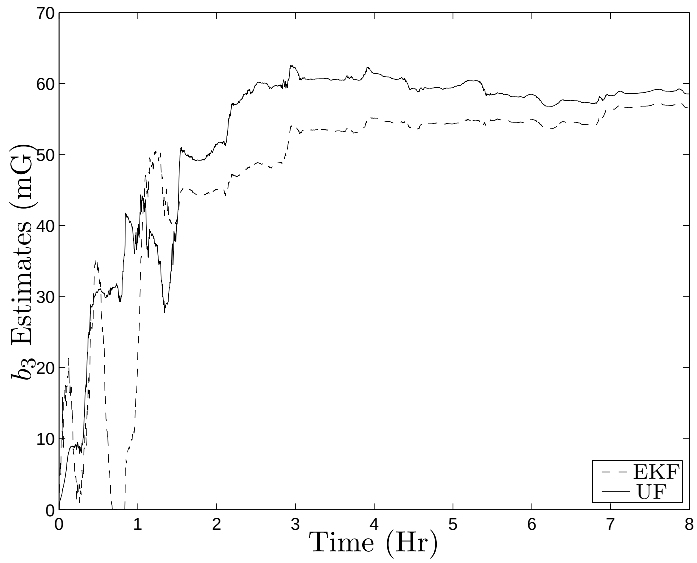
 FIGURE 1:


80
60
40
20
0
−20
−40
−60
−80
b_3 Errors (mG)
0
1
2
3
4
5
6
7
8
Time (Hr)

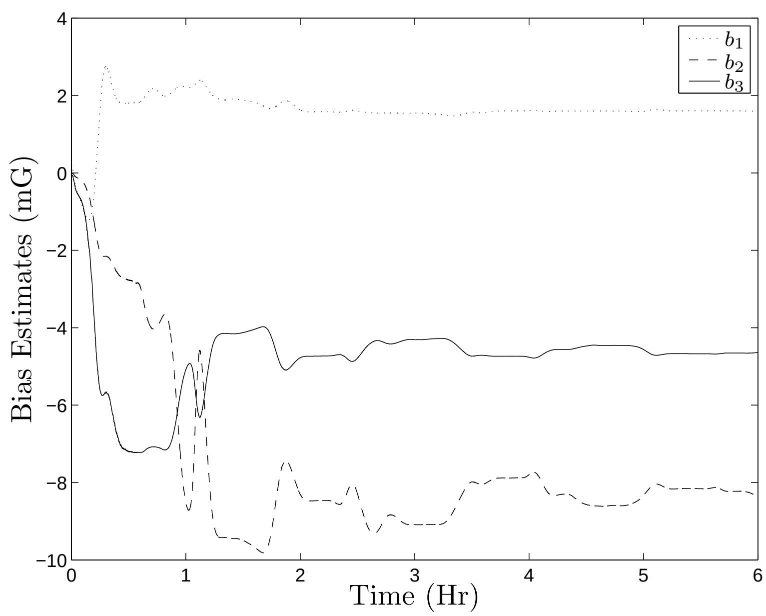


3σ Bounds for b_3 (mG)
Time (Hr)
EKF
UF
10^2
10^1
10^0
10^{-1}
0
1
2
3
4
5
6
7
8

 Figure 3:



70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
b_3 Estimates (mG)
Time (Hr)
EKF
UF



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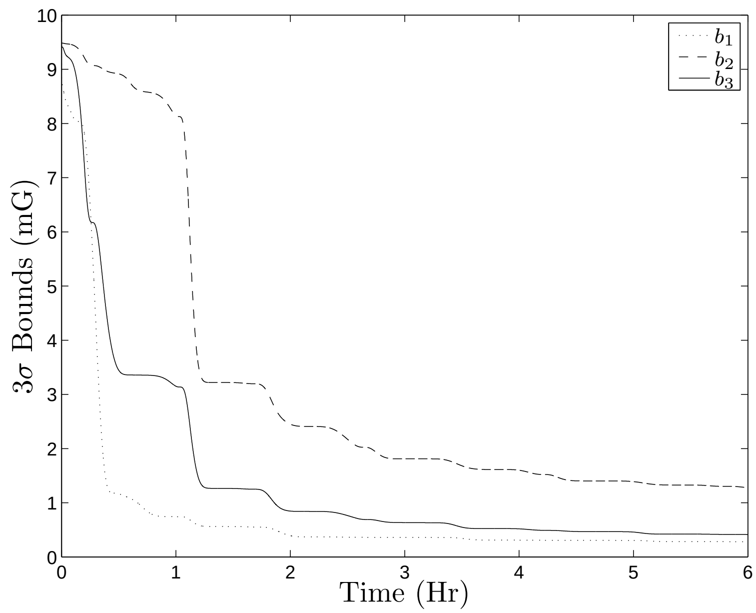
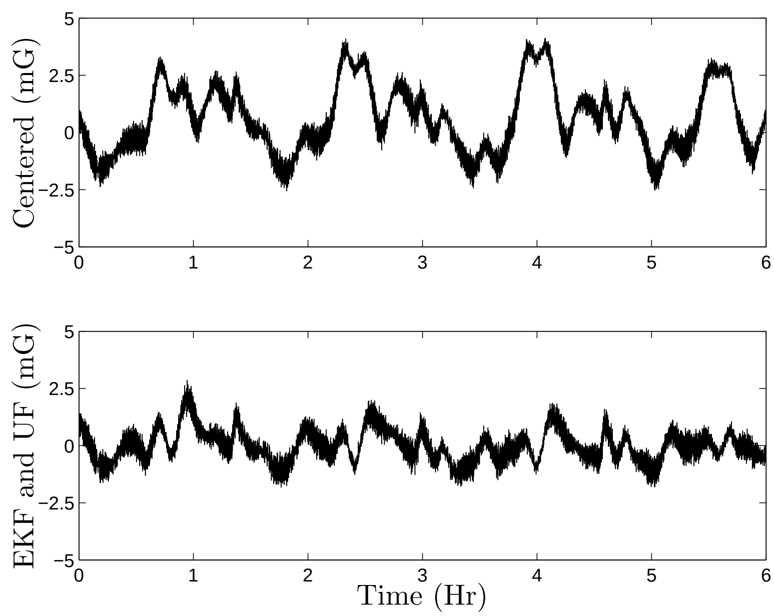


Figure 6:



Centered (mG)
5
2.5
0
−2.5
−5
0
1
2
3
4
5
6
EKF and UF (mG)
5
2.5
0
−2.5
−5
0
1
2
3
4
5
6
Time (Hr)