

$$\|P_M \int_0^t \partial_x U(t-s) f(\bullet, s) ds\|_{L_x^2 L_{yzI}^\infty} \lesssim (\ln^+ M)^2 M \|P_M f\|_{L_x^1 L_{yzI}^2},$$