

$$\frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{d}{dt} \left( \frac{1}{2} m \frac{dx}{dt} \frac{dx}{dt} \right) = m \frac{dx}{dt} \frac{d^2x}{dt^2} = m v \frac{dv}{dt} = \frac{d}{dt} \left( \frac{1}{2} m v^2 \right)$$