$$\mathbb{P}\left(|S_n| \geq c\sigma\sqrt{n\log n}\right) \leq 2\exp\left(-\frac{c^2}{2}\log n\right).$$