

$$\lambda^3 = \sum_{i=1}^l (-\lambda_i)^3 + \sum_{i=l+1}^k (-\lambda_i)^3 - (n - k - 1) \geq \frac{(-\lambda_1 - \cdots - \lambda_l)^3}{l^2} - (k - l) - (n - k - 1)$$

$$\geq \frac{(n - d - 1)^3}{(d + 1)^2} - n.$$