$$1_K = \sum_{p=1}^{K} |\varphi_p^\sigma\rangle\langle\varphi_p^\sigma|.$$