

$$\int_{\Omega} \omega \tilde{\psi} dx dy \leq \left( \int_{\Omega} \frac{|\omega|^2}{g'(\psi_0)} dx dy \right)^{1/2} \left( \int_{\Omega} g'(\psi_0) |\tilde{\psi}|^2 dx dy \right)^{1/2} \leq C |\omega|_{x_0} |\tilde{\psi}|_{x_0}$$