

$$\begin{aligned}
\left| \tilde{U}_{1:k}^T U_{k+1:d} \right| &\leq \frac{\left| \tilde{A} - A \right|}{\lambda_k - \tilde{\lambda}_{k+1}} \\
&= \frac{\left| \tilde{A} - A \right|}{\lambda_k - \lambda_{k+1} - \lambda_{k+1} - \tilde{\lambda}_{k+1}} \\
&\leq \frac{\left| \tilde{A} - A \right|}{\lambda_k - \lambda_{k+1} - \left| \tilde{A} - A \right|}.
\end{aligned}$$