$$-Ric(\nabla_\nu e_2, e_2) + Ric(\nabla_\nu e_1, e_1) = h_{22}R_{22} - h_{11}R_{11} - \frac{4L_{22}}{|\nabla f|^3} = HS|\nabla f| - \frac{4L_{22}}{|\nabla f|^3}.$$