

$$\log_p \pi_p'(k) \sim \log_p \pi_p(k) + k - \log_p(k) - \log_p \log_p(k)$$

$$\sim \frac{\psi(k)}{\ln p} + k - \log_p(k),$$