$$\omega \sin^2 \phi + \omega_+ \cos^2 \phi + \sqrt{2}g \sin 2\phi = \frac{1}{2}\left([\omega_+ + \omega] + \sqrt{8g^2 + (\omega_+ - \omega)^2}\right)$$
$$= \Omega_2.$$