$$(H\Omega)^T z = (\widetilde{H}\Omega)^T (\beta Z^2 + \mu_y I)^{-2} z = 0.$$