$$\|\pi_{W_i}Cu^*f\| = \|\pi_{W_i}u^*Cf\| = \|\pi_{W_i}u^*\pi_{uW_i}C^*f\| \le \|u\|\|\pi_{uW_i}C^*f\|.$$