

$$\left| \sum_{k, \ell=1}^n \frac{1}{\sqrt{k\ell}} H_p^{*2i} \left( \log \frac{k}{\ell} \right) - \int_0^n du \int_0^\infty dv \frac{1}{uv} H_p^{*2i} \left( \log \frac{u}{v} \right) \right| \leq C^i.$$