$$|p(\nabla u)^T \colon \nabla u|_1 \le |p|_q |\nabla u|_2 |\nabla u|_r \le C |p|_{H^1} |\nabla u|_2 |\nabla u|_r$$