$$f^{\pm}(w) = \frac{1}{2} \left(f(w) \pm \alpha_{j + \frac{1}{2}} w \right).$$