

$$\tau = \int_{t_0}^t \frac{dt'}{t'^2} = \frac{1}{t_0} - \frac{1}{t}.$$