$$|a(t)|_{L^\infty(\mathbb{R}^d)} \leq C(d,p) \, |f(t)|_{L^1(\mathbb{R}^d)}^{\frac{p}{p-1}\left(\frac{2}{d}-\frac{1}{p}\right)} \, |f(t)|_{L^p(\mathbb{R}^d)}^{\frac{p}{p-1}\left(1-\frac{2}{d}\right)}.$$