$$|K_{\infty,L}(z)|_2 \le |K_{\infty}(z)|_2 \le \frac{1}{2|\sqrt{z}|} |V|_1.$$