$$P_X g_1^2 + P_X g_2^2 \lesssim P_X \left| m_0'(\theta_0^\top X) X^\top (\check{\theta}_n - \theta_0) + (\check{m}_n - m_0)(\theta_0^\top X) \right|^2$$