

$$|\nabla \log E_n(x)| \lesssim 2^n \kappa_n + 2^{n/2} q^n \kappa_n + 2^n \kappa_n^2 |x| + \frac{M^{2^n}}{e^{\frac{\beta_{\min}}{2} q^{2n}}} e^{2^{2n+1} \kappa_n^2 \beta |x|^2},$$