$$1_K = \sum_{p=1}^K |\varphi_p^{\sigma}\rangle\langle\varphi_p^{\sigma}|.$$