$$S[\,a_\alpha\,,N] = V\,!\int_{t_i}^{t_f} N\mathrm{d}t\, L\,, \quad L = a^3\left[-f(a)\left(\frac{\dot{a}}{Na}\right)^2 + \frac{h(a)}{a^6}\right]\,.$$