$$f_{\infty}(N^m x) \le C e^{Nmx^{\frac{1}{m}}}.$$