$$\chi = -g_{\mu\nu}U^{\mu}U^{\nu},$$