$$\chi^{\alpha} * \chi^{\beta} = \frac{\delta_{\alpha\beta}\chi^{\alpha}}{d_{\alpha}}.$$