$$Z(z) = e^{\alpha \tau} \prod_{k} (1 - \frac{\tau}{\tau_k})$$