

$$\max_x \{y, Ax\} - f(x) = (y, Ax^*(A^T y)) - f(x^*(A^T y))$$