$$f_\infty(N^m x) \leq C \, e^{N m x^{\frac{1}{m}}}.$$