$$|U(t) - \tilde{U}(t)| \le \int_0^T |\phi(U(s)) - \phi(\tilde{U}(s))| ds \le L_I \int_0^T |U(s) - \tilde{U}(s)| ds.$$