$$\begin{aligned}
i_{E^*}(T) \cap P|\zeta &= i_E(T) \cap S|\zeta \\
&= T \cap S|\zeta \\
&= T \cap P|\zeta.
\end{aligned}$$