

$$e^{\sqrt{\alpha}(\epsilon \lambda A_{\lambda} + \eta \lambda A_{\lambda}^{\dagger})} = e^{-\sqrt{\alpha} \lambda \frac{\epsilon + \eta}{2} \coth \frac{\beta \hbar \omega}{2}}.$$