

$$\mathbb{P}\left(|S_n| \geq c\sigma \sqrt{n \log n}\right) \leq 2 \exp\left(-\frac{c^2}{2} \log n\right).$$