

$$1_K = \sum_{p=1}^K |\varphi_p^\sigma\rangle \langle \varphi_p^\sigma|.$$