

$$f_{\infty}(nm x) \leq C e^{Nm x} \frac{1}{m}.$$