$$\frac{d}{dt}m(t) = \langle f(\cdot, m(t), u(t, \cdot)), \nabla \rangle m(t).$$