

$$\dot{v}v_t = -\frac{1}{2}(v_{\varphi\varphi} + avv - evv^*)$$