

$$\rho_{SE} = \rho_S \otimes \rho_E = \frac{1}{4} (\mathbb{I} + a \sigma_x) \otimes (\mathbb{I} + \kappa \sigma_z).$$