$$S(\ell,d)=\frac{1}{6}\log\frac{R}{\epsilon}-\frac{i\pi}{2}\int_0^\infty ds\frac{s}{\sinh^2(\pi s)}\log\left(\frac{4^{is}\Gamma\left[is\right]\Gamma\left[-1+d/2+\ell-is\right]}{\Gamma\left[-is\right]\Gamma\left[-1+d/2+\ell+is\right]}\right),$$