$$\partial_r f \partial_r \beta - \beta \partial_r \partial_r f + 2f \partial_r \partial_r \beta = 0$$
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