$$\left(I - \frac{1}{2}P_Y\right)\nabla^2 h(YY^T)[Y\xi_Y^T + \xi_Y Y^T]Y(Y^TY)^{-1} = \eta_{\uparrow_Y},$$