$\mathbb{P}\left(|S_n| \ge c\sigma\sqrt{n\log n}\right) \le 2\exp\left(-\frac{c^2}{2}\log n\right).$