$$\mathbf{H}(\mathbf{u}\mathbf{u}^{\mathrm{T}} + \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}})\mathbf{w}$$

$$= \mathbf{H}(\frac{c_1}{\sqrt{n}}\mathbf{1}_n + c_2\mathbf{u}) = \frac{c_1(m-1)}{\sqrt{n}}\mathbf{u}^{\circledast m-2} - \frac{c_1\alpha}{\sqrt{n}}\mathbf{1}_n + c_2(m-2)\alpha\mathbf{u} + c_2(m-1)\beta\mathbf{1}_n$$

$$= (c_2(m-1)\beta - \frac{c_1\alpha}{\sqrt{n}})\mathbf{1}_n + c_2(m-2)\alpha\mathbf{u} + \frac{c_1(m-1)}{\sqrt{n}}\mathbf{u}^{\circledast m-2}$$