$$\widehat{\nabla}\Phi(x_k) = \nabla_x f(x_k, y_k^T) - \nabla_x \nabla_y g(x_k, y_k^T) v_N,$$