$$\hat{\rho}^{\text{full}}(t) = \sum_{\mathbf{p}} \hat{\rho}_{\mathbf{p}}(t) \otimes \frac{|\text{l}, \mathbf{p}\rangle\langle\text{l}, \mathbf{p}| + |\text{u}, \mathbf{p}\rangle\langle\text{u}, \mathbf{p}|}{2} \otimes \hat{\rho}_{\text{tl}}.$$