$$\mathcal{A}_1 \Phi_1^0 = -\mathcal{A}_0 \phi_k - k c_k \Phi_1^0,$$