$$H_{\text{int}} = \sum_{\alpha=1,2} \sum_k \hbar g_k^\alpha \left(\sigma_\alpha b_k^{(\alpha)\dagger} + \sigma_\alpha^\dagger b_k^\alpha \right),$$