

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T r_t(\pi^*(X_t)) - r_t(\hat{a}_t) \leq 0, \quad (a.s.).$$