3. Take conjugate of output sequence of FFT. This gives $8x(n)$.
4. Divide the sequence obtained in step 3 by 8. The resultant sequence is $x(n)$.

The flow graph for computation of N = 8-point IDFT using DIT FFT algorithm is shown in Figure 7.18.

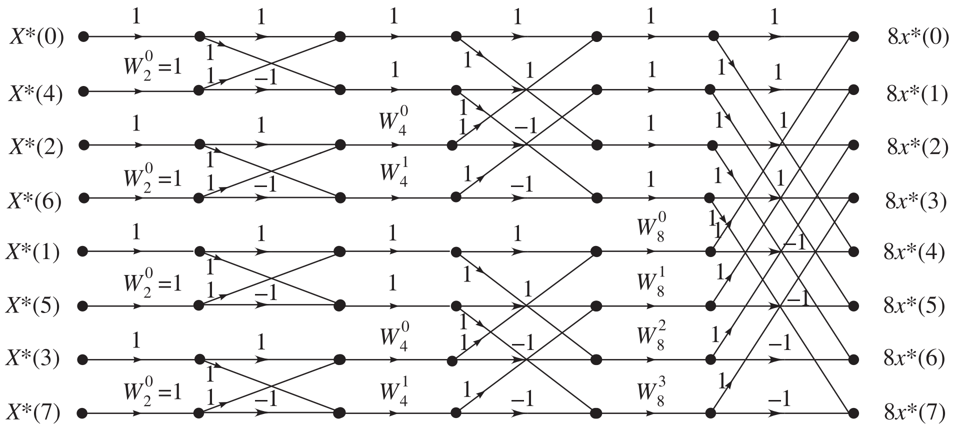

Figure 7.18 Computation of 8-point DFT of $X^*(k)$ by radix-2, DIT FFT.

From Figure 7.18, we get the 8-point DFT of $X^*(k)$ by DIT FFT as

$$8x^*(n) = \{8x^*(0), 8x^*(1), 8x^*(2), 8x^*(3), 8x^*(4), 8x^*(5), 8x^*(6), 8x^*(7)\}$$

$$\therefore \qquad x(n) = \frac{1}{8}\{8x^*(0), 8x^*(1), 8x^*(2), 8x^*(3), 8x^*(4), 8x^*(5), 8x^*(6), 8x^*(7)\}^*$$

EXAMPLE 7.6 Find the 4-point DFT of the sequence $x(n) = \{2, 1, 4, 3\}$ by
(a) DIT FFT algorithm (b) DIF FFT algorithm. Also plot the magnitude and phase plot.

Solution: (a) To compute the 4-point DFT by DIT FFT algorithm, first the given sequence $x(n) = \{x(0), x(1), x(2), x(3)\}$ is to be written in bit reversed order as $x_r(n) = \{x(0), x(2), x(1), x(3)\}$. The output will be in normal order. The given $x(n)$ in bit reversed order is $x_r(n) = \{2, 4, 1, 3\}$. The 4-point DFT of $x(n)$ using DIT FFT algorithm is computed as shown in Figure 7.19.

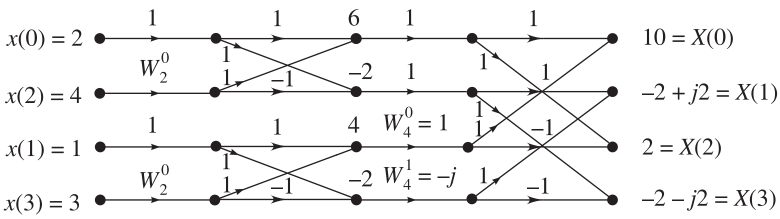

Figure 7.19 4-point DFT by DIT FFT.

From Figure 7.19, the 4-point DFT of $x(n)$ by radix-2, DIT FFT algorithm is

$$X(k) = \{10, -2 + j2, 2, -2, -j2\}$$

(b) To compute the DFT by DIF FFT, the input sequence is to be in normal order and the output sequence will be in bit reversed order. Figure 7.20 shows the computation of 4-point DFT of $x(n)$ by radix-2 DIF FFT algorithm.

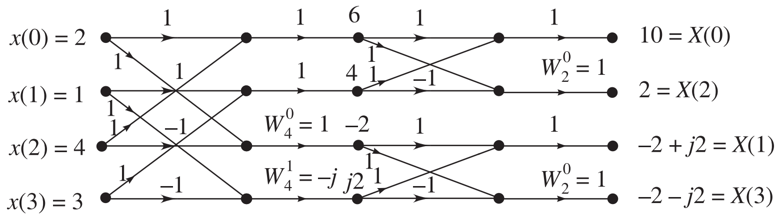

Figure 7.20 4-point DFT by DIF FFT.

From Figure 7.20, the 4-point DFT of $x(n)$ is $X(k) = \{10, -2 + j2, 2, -2 - j2\}$.

To draw the magnitude and phase plot, we have

$$X(0) = 10 \quad \therefore |X(0)| = 10 \text{ and } \underline{|X(0)} = 0° = 0 \text{ rad}$$

$$X(1) = -2 + j2 \quad \therefore |X(1)| = \sqrt{2^2 + 2^2} = 2.828 \text{ and } \underline{|X(1)} = 135° = 2.356 \text{ rad}$$

$$X(2) = 2 \quad \therefore |X(2)| = 2 \text{ and } \underline{|X(2)} = 0° = 0 \text{ rad}$$

$$X(3) = -2 - j2 \quad \therefore |X(3)| = \sqrt{2^2 + 2^2} = 2.828 \text{ and } \underline{|X(3)} = -135° = -2.356 \text{ rad}$$

The magnitude and phase spectrum are shown in Figure 7.21.

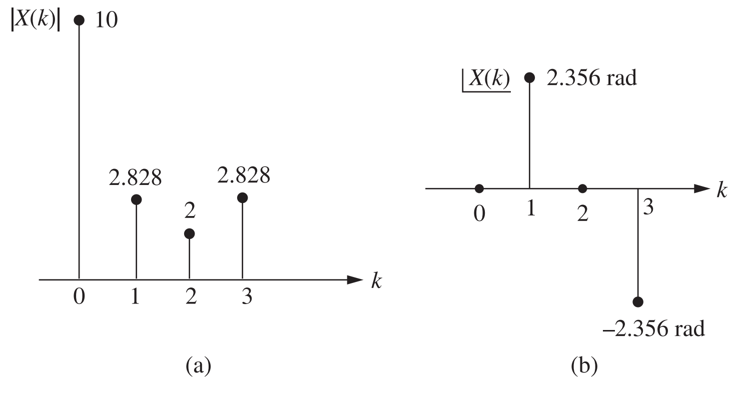

Figure 7.21 (a) Magnitude spectrum (b) Phase spectrum.

EXAMPLE 7.7 Compute the circular convolution of the two sequences $x_1(n) = \{1, 2, 0, 1\}$ and $x_2(n) = \{2, 2, 1, 1\}$ using DFT approach.

Solution: To compute the circular convolution of the two sequences $x_1(n)$ and $x_2(n)$, i.e. to compute $x_1(n) \circledast x_2(n)$ using DFT approach, first the DFTs of the two sequences, i.e. $X_1(k)$

and $X_2(k)$ are to be determined independently, then their product $Y(k) = X_1(k)X_2(k)$ is to be determined and then the IDFT of $Y(k)$, i.e. $y(n)$ is to be determined. Here DFTs and IDFT are performed via FFT. For DIT FFT, the input sequence is in bit reversed order and output sequence is in normal order. The computation of 4-point DFTs of $x_1(n)$ and $x_2(n)$ using radix-2 DIT FFT algorithm is shown in Figures 7.22(a) and (b).

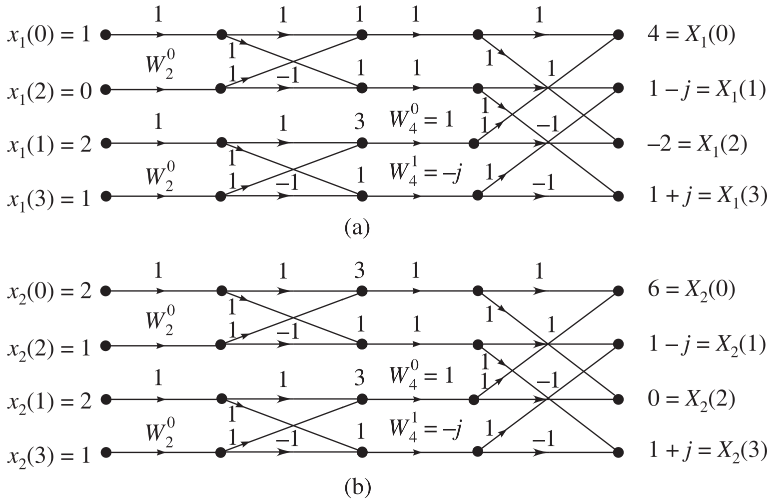

Figure 7.22 Computation of (a) DFT of $x_1(n)$; (b) DFT of $x_2(n)$ by DIT FFT.

From Figures 7.22(a) and (b), we have

$$X_1(k) = \{4,\ 1-j,\ -2,\ 1+j\} \text{ and } X_2(k) = \{6,\ 1-j,\ 0,\ 1+j\}$$

$\therefore$ The product sequence $X(k) = X_1(k)X_2(k)$

$$\therefore \qquad X(k) = \{4,\ 1-j, -2,\ 1+j\}\{6,\ 1-j,\ 0,\ 1+j\} = \{24, -j2,\ 0, j2\}$$

$$\therefore \qquad X^*(k) = \{24, j2, 0,\ -j2\} \text{ in normal order}$$

$$X^*(k) \text{ in bit reverse order is } X_r^*(k) = \{24, 0, j2,\ -j2\}$$

The computation of 4-point DFT of $X^*(k)$ using radix-2 DIT FFT algorithm is shown in Figure 7.23. For DIT FFT, the input is in bit reversed order and the output is in normal order.

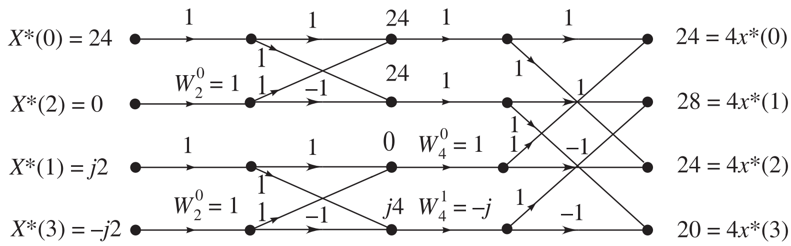

Figure 7.23 Computation of 4-point DFT of $X^*(k)$ by radix-2 DIT FFT.

From Figure 7.23, we have $4x^*(n) = \{24, 28, 24, 20\}$

$$\therefore \qquad x^*(n) = \frac{1}{4}\{24, 28, 24, 20\} = \{6, 7, 6, 5\}$$

$$x(n) = \{6, 7, 6, 5\}^* = \{6, 7, 6, 5\}$$

$$\therefore \qquad [x_1(n) = \{1, 2, 0, 1\}] \oplus [x_2(n) = \{2, 2, 1, 1\}] = [x(n) = \{6, 7, 6, 5\}]$$

EXAMPLE 7.8 Compute the DFT of the square wave sequence

$$x(n) = \begin{cases} 1, & 0 \le n \le \left(\dfrac{N}{2} - 1\right) \\ -1, & \dfrac{N}{2} \le n \le N \end{cases}$$

where N is even.

Solution: It is given that N is even, but the value of N is not given. Let us take $N = 4$.

$$\therefore \qquad x(n) = \{1, 1, -1, -1\}$$

Let us calculate $X(k)$ using 4-point radix-2 DIT FFT algorithm as shown in Figure 7.24. For DIT FFT, the input is in bit reversed order and output is in normal order. $x(n)$ in bit reversed order is $x_r(n) = \{1, -1, 1, -1\}$.

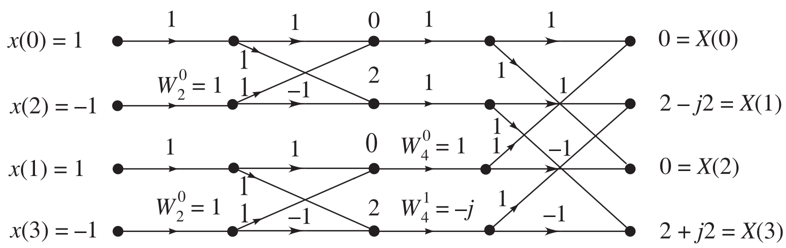

Figure 7.24 4-point DFT of x(n) by radix-2 DIT FFT.

From Figure 7.24, we have $X(k) = \{0, 2 - j2, 0, 2 + j2\}$.

EXAMPLE 7.9 In an LTI system, the input $x(n) = \{2, 2, 2\}$ and the impulse response $h(n) = \{-2, -2\}$. Determine the response of LTI system by radix-2, DIT FFT.

Solution: The response $y(n)$ of LTI system is given by the linear convolution of input $x(n)$ and impulse response $h(n)$.

$$\therefore \qquad \text{Response or output } y(n) = x(n) * h(n)$$

The DFT (or FFT) supports only circular convolution. Hence, to get the result of linear convolution from circular convolution, the sequences $x(n)$ and $h(n)$ should be converted to the size of $y(n)$ by appending with zeros and circular convolution of $x(n)$ and $h(n)$ is performed.

The length of $x(n)$ is 3 and the length of $h(n)$ is 2. Hence, the length of $y(n)$ is $3 + 2 - 1 = 4$. Therefore, the given sequences $x(n)$ and $h(n)$ are converted into 4-point sequences by appending zeros.

$$\therefore \quad x(n) = \{2, 2, 2, 0\} \text{ and } h(n) = \{-2, -2, 0, 0\}$$

Now the response $y(n)$ is given by $y(n) = x(n) \circledast h(n)$

Let $\quad \text{DFT}\{x(n)\} = X(k)$, $\text{DFT}\{h(n)\} = H(k)$, and $\text{DFT}\{y(n)\} = Y(k)$

By convolution theorem of DFT, we get

$$\text{DFT}\{x(n) \circledast h(n)\} = X(k)H(k)$$

$$\therefore \quad y(n) = \text{IDFT}\{X(k)H(k)\} = \text{IDFT}\{Y(k)\}$$

The various steps in computing $y(n)$ are

Step 1: Determine $X(k)$ using radix-2 DIT FFT algorithm
Step 2: Determine $H(k)$ using radix-2 DIT FFT algorithm
Step 3: Determine the product $X(k)H(k)$
Step 4: Take IDFT of the product $X(k)H(k)$ using radix-2 DIT FFT algorithm.

1. The 4-point DFT of $x(n)$, i.e. $X(k)$ is determined using radix-2, DIT FFT algorithm as shown in Figure 7.25. For DIT FFT, the input is in bit reversed order and output is in normal order.

$$x(n) = \{2, 2, 2, 0\};\ x_r(n) = \{2, 2, 2, 0\}$$

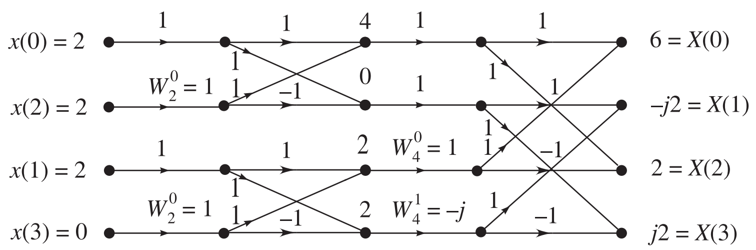

Figure 7.25 Computation of 4-point DFT of x(n) by radix-2, DIT FFT.

From Figure 7.25, $X(k) = \{6, -j2, 2, j2\}$.

2. The 4-point DFT of $h(n)$, i.e. $H(k)$ using 4-point DIT FFT is determined as shown in Figure 7.26.

$$h(n) = \{-2, -2, 0, 0\};\ h_r(n) = \{-2, 0, -2, 0\}$$

From Figure 7.26, $H(k) = \{-4, -2 + j2, 0, -2 - j2\}$

3. $Y(k) = X(k)H(k) = \{6, -j2, 2, j2\}\ \{-4, -2 + j2, 0, -2 - j2\}$
$= \{-24, 4 + j4, 0, 4 - j4\}$

4. $Y^*(k) = \{-24, 4 - j4, 0, 4 + j4\};\ Y_r^*(k) = \{-24, 0, 4 - j4, 4 + j4\}$

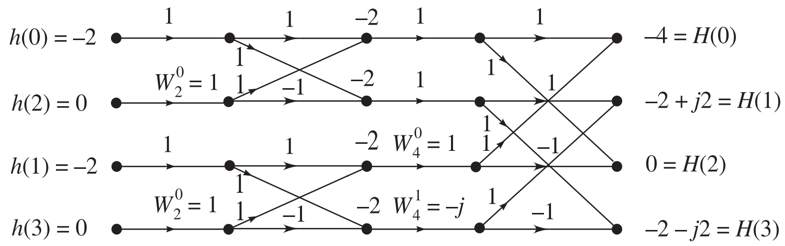

Figure 7.26 Computation of 4-point DFT of $h(n)$ by radix-2, DIT FFT.

The 4-point DFT of $Y^*(k)$ using radix-2, DIT FFT is computed as shown in Figure 7.27.

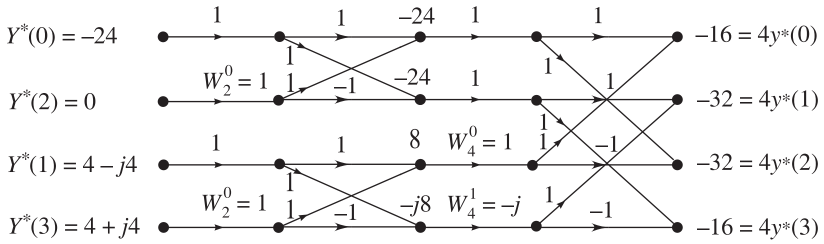

Figure 7.27 Computation of 4-point DFT of $Y^*(k)$ by radix-2, DIT FFT.

From Figure 7.27, we get $4y^*(n) = \{-16, -32, -32, -16\}$.

$$\text{Therefore,} \quad y^*(n) = \frac{1}{4}\{-16, -32, -32, -16\} = \{-4, -8, -8, -4\}$$

$$y(n) = \{-4, -8, -8, -4\}^* = \{-4, -8, -8, -4\}$$

EXAMPLE 7.10 Find the response of LTI system with impulse response $h(n) = \{0.5, 1\}$ when the input sequence $x(n) = \{1, 0.5, 0\}$ is applied to it by radix-2 DIT FFT.

Solution: The response $y(n)$ of LTI system is given by linear convolution of input sequence $x(n)$ and impulse response $h(n)$, i.e. $y(n) = x(n) * h(n)$.

The length of $x(n) = 3$ and the length of $h(n) = 2$. So the length of output sequence $y(n)$ is $3 + 2 + (-1) = 4$.

Since the DFT supports only circular convolution, to get the result of linear convolution from circular convolution, the sequences $x(n)$ and $h(n)$ are to be converted to the size of $y(n)$[i.e., $N = 4$] by appending with zeros and the circular convolution of $x(n)$ and $h(n)$ is performed.

$x(n)$ and $h(n)$ as 4-point sequences are $x(n) = \{1, 0.5, 0, 0\}$ and $h(n) = \{0.5, 1, 0, 0\}$.

Now to find $y(n)$ by FFT, we have to find $X(k)$ and $H(k)$ by 4-point radix-2 DIT FFT, get $Y(k) = X(k)H(k)$, find DFT of $Y^*(k)$, take the conjugate of that and divide by 4. The entire procedure is as given below:

Step 1: Determination of $X(k)$

Given $x(n) = \{1, 0.5, 0, 0\}$, $x(n)$ in bit reversed order is $x_r(n) = \{1, 0, 0.5, 0\}$.

The computation of 4-point DFT of $x(n)$, $X(k)$ by radix-2 DIT FFT algorithm is shown in Figure 7.28.

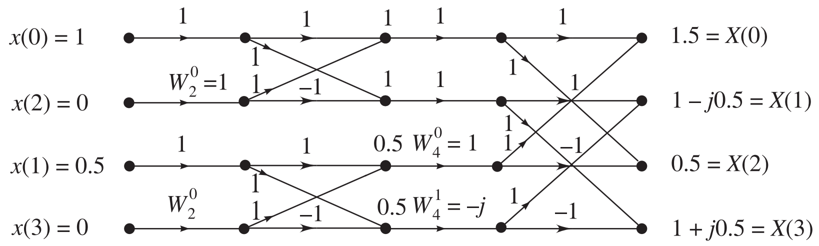

Figure 7.28 Computation of 4-point DFT of $x(n)$ by DIT FFT.

From Figure 7.28, the 4-point DFT of $x(n)$ is $X(k) = \{1.5,\ 1 - j0.5,\ 0.5,\ 1 + j0.5\}$.

Step 2: Computation of $H(k)$

Given $h(n) = \{0.5, 1, 0, 0\}$, $h(n)$ in bit reverse order is $h_r(n) = \{0.5, 0, 1, 0)$.

The computation of 4-point DFT of $h(n)$, i.e. $H(k)$ using radix-2, DIT FFT algorithm is shown in Figure 7.29.

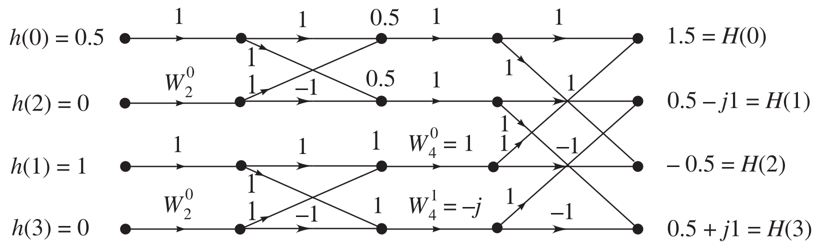

Figure 7.29 Computation of 4-point DFT of $h(n)$ by DIT FFT.

From Figure 7.29, the 4-point DFT of $h(n)$ is $H(k) = \{1.5,\ 0.5 - j1,\ -0.5,\ 0.5 + j1\}$.

Step 3: Computation of $Y(k)$

$$Y(k) = X(k)H(k) = \{1.5, 1 - j0.5, 0.5, 1 + j0.5\}\{1.5, 0.5 - j1, -0.5, 0.5 + j1\}$$
$$= \{2.25,\ -j1.25, -0.25, j1.25\}$$

$$\therefore \qquad Y^*(k) = \{2.25, j1.25,\ -0.25,\ -j1.25\}$$

Step 4: $Y^*(k)$ in bit reverse order is $Y_r^*(k) = \{2.25,\ -0.25, j1.25,\ -j1.25\}$

Computation of 4-point DFT of $Y^*(k)$ using radix-2 DIT FFT is shown in Figure 7.30.

From Figure 7.30, we get $4y^*(n) = \{2, 5, 2, 0\}$.

$$\therefore \qquad y^*(n) = \frac{1}{4}\{2, 5, 2, 0\} = \{0.5, 1.25, 0.5, 0\}$$

$$\therefore \qquad y(n) = \{0.5, 1.25, 0.5, 0\}^* = \{0.5, 1.25, 0.5, 0\}$$

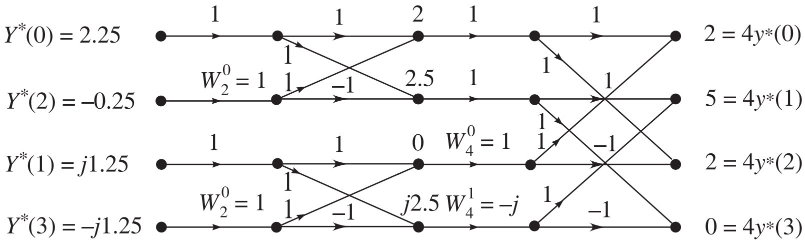

Figure 7.30 Computation of 4-point DFT of $Y^*(k)$ by DIT FFT.

EXAMPLE 7.11 Compute the DFT of the sequence $x(n) = \{1, 0, 0, 0, 0, 0, 0, 0\}$ (a) directly, (b) by FFT.

Solution: (a) Direct computation of DFT
The given sequence is $x(n) = \{1, 0, 0, 0, 0, 0, 0, 0\}$. We have to compute 8-point DFT. So $N = 8$.

$$\text{DFT}\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x(n)\,W_N^{nk} = \sum_{n=0}^{7} x(n)\,W_8^{nk}$$

$$= x(0)W_8^{\,0} + x(1)W_8^{\,1} + x(2)W_8^{\,2} + x(3)W_8^{\,3} + x(4)W_8^{\,4} + x(5)W_8^{\,5} + x(6)W_8^{\,6} + x(7)W_8^{\,7}$$

$$= (1)(1) + (0)(W_8^{\,1}) + (0)W_8^2 + (0)W_8^{\,3} + (0)W_8^{\,4} + (0)W_8^{\,5} + (0)W_8^{\,6} + (0)W_8^{\,7} = 1$$

$\therefore \quad X(k) = 1$ for all k

$\therefore \quad X(0) = 1,\ X(1) = 1,\ X(2) = 1,\ X(3) = 1,\ X(4) = 1,\ X(5) = 1,\ X(6) = 1,\ X(7) = 1$

$\therefore \quad X(k) = \{1, 1, 1, 1, 1, 1, 1, 1\}$

(b) Computation by FFT. Here $N = 8 = 2^3$
The computation of 8-point DFT of $x(n) = \{1, 0, 0, 0, 0, 0, 0, 0\}$ by radix-2 DIT FFT algorithm is shown in Figure 7.31. $x(n)$ in bit reverse order is

$$x_r(n) = \{x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7)\}$$
$$= \{1, 0, 0, 0, 0, 0, 0, 0\}$$

For DIT FFT input is in bit reversed order and output is in normal order.
From Figure 7.31, the 8-point DFT of the given $x(n)$ is $X(k) = \{1, 1, 1, 1, 1, 1, 1, 1\}$

EXAMPLE 7.12 An 8-point sequence is given by $x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$.
Compute the 8-point DFT of $x(n)$ by

(a) Radix-2 DIT FFT algorithm
(b) Radix-2 DIF FFT algorithm

Also sketch the magnitude and phase spectrum.